RAILROAD CONSTRUCTION.

THEORY AND PRACTICE.

A TEXT-BOOK FOR THE USE OF STUDENTS IN COLLEGES AND TECHNICAL SCHOOLS.

BY

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etc.

FIRST EDITION.

FIRST THOUSAND.

PREFACE.

THE preparation of this book was begun several years ago, when much of the subject-matter treated was not to be found in print, or was scattered through many books and pamphlets, and was hence unavailable for student use. Portions of the book have already been printed by the mimeograph process or have been used as lecture-notes, and hence have been subjected to the refining process of classroom use.

The author would call special attention to the following features:

a. Transition curves; the multiform-compound-curve method is used, which has been followed by many railroads in this country; the particular curves here developed have the great advantage of being exceedingly simple, and although the method is not theoretically exact, it is demonstrable that the differences are so small that they may safely be neglected.

b. A system of earthwork computations by means of a slide-rule (which accompanies the volume) which enables one to compute readily the volume of the most complicated earthwork forms with an accuracy only limited by the precision of the cross-sectioning.

c. The "mass curve" in earthwork; the theory and use of this very valuable process.

d. Tables I, II, III, and IV have been computed *ab novo*. Tables I and II were checked (after computation) with other tables, which are generally considered as standard, and all discrepancies were further examined. They are believed to be perfect.

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e. Tables V, VI, VII, and IX have been borrowed, by per-
mission, from '' Ludlow's Mathematical Tables.'' It is believed
that five-place tables give as accurate results as actual field prac-
tice requires. Tables VIII and X have been compiled to con-
form with Ludlow's system.

The author wishes to acknowledge his indebtedness to Mr.
Chas. A. Sims, civil engineer and railroad contractor, for read-
ing and revising the portions relating to the cost of earthwork.

Since the book is written primarily for students of railroad
engineering in technical institutions, the author has assumed the
usual previous preparation in algebra, geometry, and trigo-
nometry.

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CHAPTER I.

RAILROAD SURVEYS.

THE proper conduct of railroad surveys presupposes an adequate knowledge of almost the whole subject of railroad engineering, and particularly of some of the complicated questions of Railroad Economics, which are not generally studied except at the latter part of a course in railroad engineering, if at all. This chapter will therefore be chiefly devoted to methods of instrumental work, and the problem of choosing a general route will be considered only as it is influenced by the topography or by the application of those elementary principles of Railroad Economics which are self-evident or which may be accepted by the student until he has had an opportunity of studying those principles in detail.

RECONNOISSANCE SURVEYS.

1. Character of a reconnoissance survey. A reconnoissance survey is a very hasty examination of a belt of country to determine which of all possible or suggested routes is the most promising and best worthy of a more detailed survey. It is essentially very rough and rapid. It aims to discover those salient features which instantly stamp one route as distinctly superior to another and so narrow the choice to routes which are so nearly equal in value that a more detailed survey is necessary to decide between them.

2. Selection of a general route. The general question of running a railroad between two towns is usually a financial rather than an engineering question. Financial considerations usually determine that a road *must* pass through certain more or less important towns between its termini. When a railroad runs through a thickly settled and very flat country, where, from a topographical standpoint, the road may be run by any desired route, the "right-of-way agent" sometimes has a greater influence in locating the road than the engineer. But such modifications of alignment, on account of business considerations, are foreign to the engineer's side of the subject, and it will be hereafter assumed that topography alone determines the location of the line. The consideration of those larger questions combining finance and engineering (such as passing by a town on account of the necessary introduction of heavy grades in order to reach it) is likewise ignored.

3. Valley route. This is perhaps the simplest problem. If the two towns to be connected lie in the same valley, it is frequently only necessary to run a line which shall have a nearly uniform grade. The reconnoissance problem consists largely in determining the difference of elevation of the two termini of this division and the approximate horizontal distance so that the proper grade may be chosen. If there is a large river running through the valley, the road will probably remain on one side or the other throughout the whole distance, and both banks should be examined by the reconnoissance party to determine which is preferable. If the river may be easily bridged, both banks may be alternately used, especially when better alignment is thereby secured. A river valley has usually a steeper slope in the upper part than in the lower part. A uniform grade throughout the valley will therefore require that the road climbs up the side slopes in the lower part of the valley. In case the "ruling grade" * for the whole road is as great as or greater

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* The *ruling grade* may here be loosely defined as the maximum grade which is permissible. This definition is not strictly true, as may be seen later when studying Railroad Economics, but it may here serve the purpose.

than the steepest natural valley slope, more freedom may be used in adopting that alignment which has the least cost—regardless of grade. The natural slope of large rivers is almost invariably so low that grade has no influence in determining the choice of location. When bridging is necessary, the river banks should be examined for suitable locations for abutments and piers. If the soil is soft and treacherous much difficulty may be experienced and the choice of route may be largely determined by the difficulty of bridging the river except at certain favorable places.

4. Cross-country route. A cross-country route always has one or more summits to be crossed. The problem becomes more complex on account of the greater number of possible solutions and the difficulty of properly weighing the advantages and disadvantages of each. The general aim should be to choose the lowest summits and the highest stream crossings, provided that by so doing the grades between these determining points shall be as low as possible and shall not be greater than the ruling grade of the road. Nearly all railroads combine cross-country and valley routes to some extent. Usually the steepest natural slopes are to be found on the cross-country routes, and also the greatest difficulty in securing a low through grade. An approximate determination of the ruling grade is usually made during the reconnoissance. If the ruling grade has been previously decided on by other considerations, the leading feature of the reconnoissance survey will be the determination of a general route along which it will be possible to survey a line whose maximum grade shall not exceed the ruling grade.

5. Mountain route. The streams of a mountainous region frequently have a slope exceeding the desired ruling grade. In such cases there is no possibility of securing the desired grade by following the streams. The penetration of such a region may only be accomplished by " development "—accompanied perhaps by tunneling. " Development " consists in deliberately increasing the length of the road between two extremes of elevation so that the rate of grade shall be as low as desired.

The usual method of accomplishing this is to take advantage of some convenient formation of the ground to introduce some lateral deviation. The methods may be somewhat classified as follows:

(a) Running the line up a convenient lateral valley, turning a sharp curve and working back up the opposite slope. As shown in Fig. 1, the considerable rise between A and B was
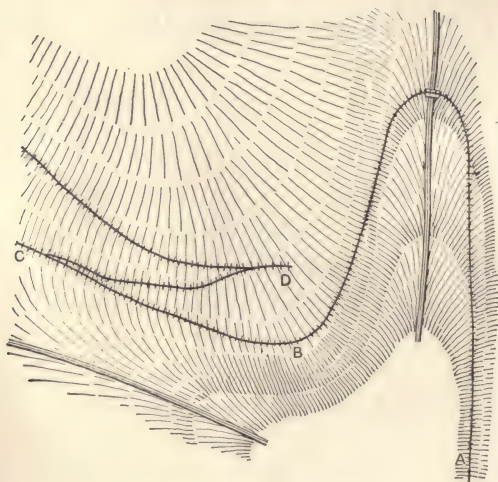
FIG. 1.

surmounted by starting off in a very different direction from the general direction of the road; then, when about one-half of the desired rise had been obtained, the line crossed the valley and continued the climb along the opposite slope. (b) *Switch-back*. On the steep side-hill BCD (Fig. 1) a very considerable gain in elevation was accomplished by the switchback CD. The gain in elevation from B to D is very great. On the other hand, the speed must always be slow; there are two complete stoppages of the train for each run; all trains must run backward from C to D. (c) *Bridge spiral*. When a valley is so narrow at some point that a bridge or viaduct of reasonable length can span the valley at a considerable elevation above the bottom of the valley, a bridge spiral may be desirable. In

Fig. 2 the line ascends the stream valley past A, crosses the stream at B, works back to the narrow place at C, and there crosses itself, having gained perhaps 100 feet in elevation. (d) *Tunnel spiral*. This is the reverse of the previous plan.
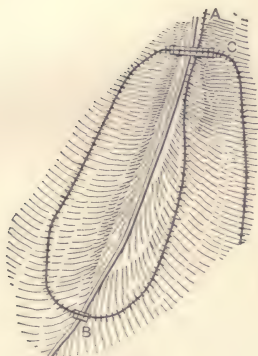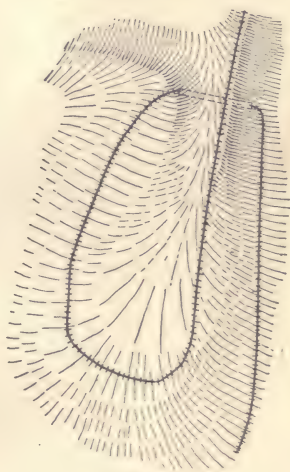
Fig. 2.   Fig. 3.

It implies a thin steep ridge, so thin at some place that a tunnel through it will not be excessively long.   Switchbacks and spirals are sometimes necessary in mountainous countries, but they should not be considered as normal types of construction. A region must be very difficult if these devices cannot be avoided.

Rack railways and cable roads, although types of mountain railroad construction, will not be here considered.

6. Existing maps. The maps of the U. S. Geological Survey are exceedingly valuable as far as they have been completed. So far as topographical considerations are concerned, they almost dispense with the necessity for the reconnoissance and "first preliminary" surveys.   Some of the State Survey maps will give practically the same information.   County and township maps can often be used for considerable information as to the relative *horizontal* position of governing points, and even some

approximate data regarding elevations may be obtained by a
study of the streams. Of course such information will not dis-
pense with surveys, but will assist in so planning them as to
obtain the best information with the least work. When the
relative horizontal positions of points are reliably indicated on a
map, the reconnoissance may be reduced to the determination
of the relative elevations of the governing points of the route.

7. Determination of relative elevations. A recent description
of European methods includes spirit-leveling in the reconnois-
sance work. This may be due to the fact that, as indicated
above, previous topographical surveys have rendered unnecessary
the " exploratory " survey which is required in a new country,
and that their reconnoissance really corresponds more nearly to
our preliminary.

The perfection to which barometrical methods have been
brought has rendered it possible to determine differences of
elevation with sufficient accuracy for reconnoissance purposes
by the combined use of a mercurial and an aneroid barometer.
The mercurial barometer should be kept at " headquarters," and
readings should be taken on it at such frequent intervals that
any fluctuation is noted, and throughout the period that observa-
tions with the aneroid are taken in the field. At each observa-
tion there should also be recorded the time, the reading of the
attached thermometer, and the temperature of the external
air. For uniformity, the mercurial readings should then be
" reduced to 32° F." Before starting out, a reading of the
aneroid should be taken at headquarters coincident with a read-
ing of the mercurial. The difference is one value of the correc-
tion to the aneroid. · As soon as the aneroid is brought back
another comparison of readings should be made. Even though
there has been considerable rise or fall of pressure in the interval,
the *difference* in readings (the correction) should be substantially
the same provided the aneroid is a good instrument. The best
aneroids read directly to $\frac{1}{100}$ of an inch of mercury and may be
estimated to $\frac{1}{1000}$ of an inch—which corresponds to about 0.9
foot difference of elevation. In the field there should be read,

at each point whose elevation is desired, the aneroid, the time, and the temperature. These readings, corrected by the mean value of the correction between the aneroid and the mercurial, should then be combined with the reading of the mercurial (interpolated if necessary) for the times of the aneroid observations and the difference of elevation obtained. [See the author's "Problems in the Use and Adjustment of Engineering Instruments," Prob. 22.] Important points should be observed more than once if possible. Such duplicate observations will be found to give surprisingly concordant results even when a general fluctuation of atmospheric pressure so modifies the tabulated readings that an agreement is not at first apparent. Variations of pressure produced by high winds, thunder-storms, etc., will generally vitiate possible accuracy by this method. By "headquarters" is meant any place whose elevation above any given datum is known and where the mercurial may be placed and observed while observations within a range of several miles are made with the aneroid. If necessary the elevation of a new headquarters may be determined by the above method, but there should be if possible several independent observations whose accordance will give a fair idea of their accuracy.

The above method should be neither slighted nor used for more than it is worth. When properly used, the errors are compensating rather than cumulative. When used, for example, to determine that a pass B is 260 feet higher than a determined bridge crossing at A which is six miles distant, and that another pass C is 310 feet higher than A and is ten miles distant, the figures, even with all necessary allowances for inaccuracy, will give an engineer a good idea as to the choice of route especially as affected by ruling grade. There is no comparison between the time and labor involved in obtaining the above information by barometric and by spirit-leveling methods, and *for reconnoissance purposes* the added accuracy of the spirit-leveling method is hardly worth its cost.

8. Horizontal measurements, bearings, etc. When there is no map which may be depended on, or when only a skeleton

map is obtainable, a rapid survey, sufficiently accurate for the purpose, may be made by using a pocket compass for bearings and a telemeter, odometer, or pedometer for distances. The telemeter [stadia] is more accurate, but it requires a definite clear sight from station to station, which may be difficult through a wooded country. The odometer, which records the revolutions of a wheel of known circumference, may be used even in rough and wooded country, and the results may be depended on to a small percentage. The pedometer (or pace-measurer) depends for its accuracy on the actual movement of the mechanism for each pace and on the uniformity of the pacing. Its results are necessarily rough and approximate, but it may be used to fill in some intermediate points in a large skeleton map. A hand-level is also useful in determining the relative elevation of various topographical features which may have some bearing on the proper location of the road.

9. Importance of a good reconnoissance. The foregoing instruments and methods should be considered only as aids in exercising an educated common sense, without which a proper location cannot be made. The reconnoissance survey should command the best talent and the greatest experience available. If the general route is properly chosen, a comparatively low order of engineering skill can fill in a location which will prove a paying railroad property; but if the general route is so chosen that the ruling grades are high and the business obtained is small and subject to competition, no amount of perfection in detailed alignment or roadbed construction can make the road a profitable investment.

PRELIMINARY SURVEYS.

10. Character of survey. A preliminary railroad survey is properly a topographical survey of a belt of country which has been selected during the reconnoissance and within which it is estimated that the located line will lie. The width of this belt will depend on the character of the country. When a railroad

is to follow a river having very steep banks the choice of loca-
tion is sometimes limited at places to a very few feet of width
and the belt to be surveyed may be correspondingly narrowed.
In very flat country the desired width may be only limited by the
ability to survey points with sufficient accuracy at a considerable
distance from what may be called the " backbone line " of the
survey.

11. Cross-section method. This is the only feasible method
in a wooded country, and is employed by many for all kinds
of country. The *backbone* line is surveyed either by observ-
ing magnetic bearings with a compass or by carrying forward
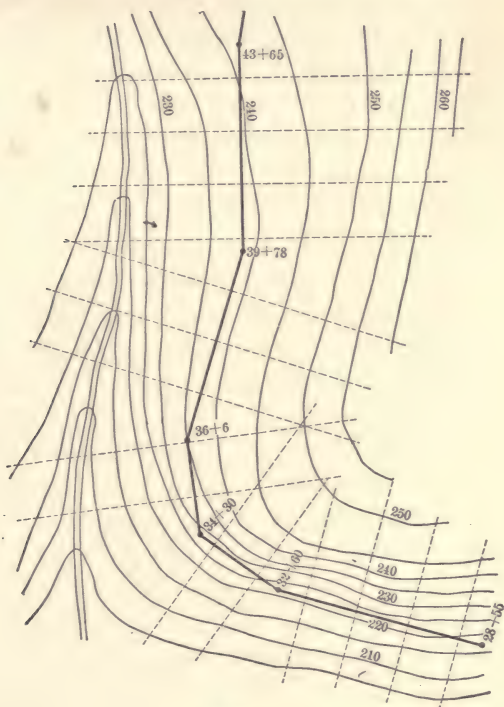
Fig. 4.

absolute azimuths with a transit. The compass method has
the disadvantages of limited accuracy and the possibility of

considerable local error owing to local attraction.   On the other hand there are the advantages of greater simplicity, no necessity for a back rodman, and the fact that the errors are purely local and not cumulative, and may be so limited, with care, that they will cause no vital error in the subsequent location survey. The transit method is essentially more accurate, but is liable to be more laborious and troublesome.   If a large tree is encountered, either it must be cut down or a troublesome operation of offsetting must be used.   If the compass is employed under these circumstances, it need only be set up on the far side of the tree and the former bearing produced.   An error in reading a transit azimuth will be carried on throughout the survey.   An error of only five minutes of arc will cause an offset of nearly eight feet in a mile.   Large azimuth errors may, however, be avoided by immediately checking each new azimuth with a needle reading.   It is advisable to obtain true azimuth at the beginning of the survey by an observation on the sun or Polaris, and to check the azimuths every few miles by azimuth observations.   Distances along the backbone line should be measured with a chain or steel tape and stakes set every 100 feet.   When a course ends at a substation, as is usually the case, the remaining portion of the 100 feet should be measured along the next course.   The level party should immediately obtain the elevations (to the nearest tenth of a foot) of all stations, and also of the lowest points of all streams crossed and even of dry gullies which would require culverts.

12. Cross-sectioning.   It is usually desirable to obtain contours at five-foot intervals.   This may readily be done by the use of a Locke level (which should be held on top of a simple five-foot stick), a tape, and a rod ten feet in length graduated to feet and tenths.   The method of use may perhaps be best explained by an example.   Let Fig. 5 represent a section *perpendicular* to the survey line—such a section as would be made by the dotted lines in Fig. 4.   *C* represents the station point. Its elevation as determined by the level is, say, 158.3 above datum.   When the Locke level on its five-foot rod is placed at

C, the level has an elevation of 163.3. Therefore when a point is found (as at *a*) where the level will read 3.3 on the rod, that
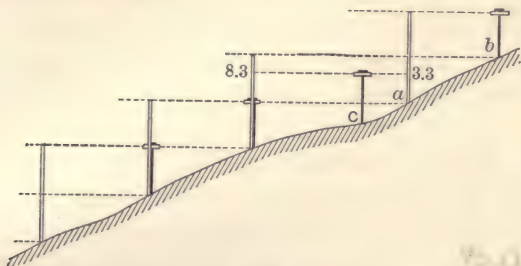
Fig. 5.

point has an elevation of 160.0 and its distance from the center gives the position of the 160-foot contour. Leaving the long rod at that point (*a*), carry the level to some point (*b*) such that the level will sight at the *top* of the rod. *b* is then on the 165-
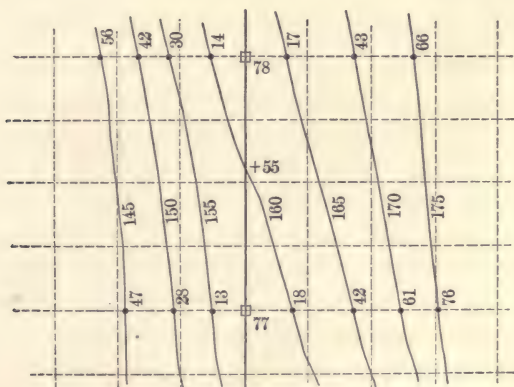
Fig. 6.

foot contour, and the *horizontal* distance *ab* added to the horizontal distance *ac* gives the position of that contour from the center. The contours on the lower side are found similarly. The first rod reading will be 8.3, giving the 155-foot contour. Plot the results in a note-book which is ruled in quarter-inch squares, using a scale of 100 feet per inch in both directions.

Plot the work UP the page; then when looking ahead along the line, the work is properly oriented. When a contour crosses the survey line, the place of crossing may be similarly determined. If the ground flattens out so that five-foot contours are very far apart, the absolute elevations of points at even fifty-foot distances from the center should be determined. The method is exceedingly rapid. Whatever error or inaccuracy occurs is confined in its effect to the one station where it occurs. The work being thus plotted in the field, unusually irregular topography may be plotted with greater certainty and no great error can occur without detection. It would even be possible by this method to detect a gross error that might have been made by the level party.

13. Stadia method. This method is best adapted to fairly open country where a "shot" to any desired point may be taken without clearing. The *backbone* survey line is the same as in the previous method except that each course is limited to the practicable length of a stadia sight. The distance between stations should be checked by foresight and backsight—also the vertical angle. Azimuths should be checked by the needle. Considering the vital importance of leveling on a railroad survey it might be considered desirable to run a line of levels over the stadia stations in order that the leveling may be as precise as possible; but when it is considered that a preliminary survey is a somewhat hasty survey of a route that *may* be abandoned, and that the errors of leveling by the stadia method (which are compensating) may be so minimized that no proposed route would be abandoned on account of such small error, and that the effect of such an error may be easily neutralized by a slight change in the location, it may be seen that excessive care in the leveling of the preliminary survey is hardly justifiable.

Since the students taking this work are assumed to be familiar with the methods of stadia topographical surveys, this part of the subject will not be further elaborated.

14. "First" and "second" preliminary surveys. Some engineers advocate two preliminary surveys. When this is done,

the first is a very rapid survey, made perhaps with a compass, and is only a better grade of reconnoissance. Its aim is to rapidly develop the facts which will decide for or against any proposed route, so that if a route is found to be unfavorable another more or less modified route may be adopted without having wasted considerable time in the survey of useless details. By this time the student should have grasped the fundamental idea that both the reconnoissance and preliminary surveys are not surveys of *lines* but of *areas*; that their aim is to survey only those topographical features which would have a deter- mining influence on any railroad line which might be constructed through that particular territory, and that the value of a locating engineer is largely measured by his ability to recognize those determining influences with the least amount of work from his surveying corps. Frequently too little time is spent on the comparative study of preliminary lines. A line will be hastily decided on after very little study; it will then be surveyed with minute detail and estimates carefully worked up, and the claims of any other suggested route will then be handicapped, if not disregarded, owing to an unwillingness to discredit and throw away a large amount of detailed surveying. The cost of two or three extra preliminary surveys (*at critical points* and not over the whole line) is utterly insignificant compared with the prob- able improvement in the "operating value" of a line located after such a comparative study of preliminary lines.

<div align="center">LOCATION SURVEYS.</div>

15. "Paper location." When the preliminary survey has been plotted to a scale of 200 feet per inch and the contours drawn in, a study may be made for the location survey. Disre- garding for the present the effect on location of transition curves, the alignment may be said to consist of straight lines (or "tan- gents") and circular curves. The "paper location" therefore consists in plotting on the preliminary map a succession of straight lines which are tangent to the circular curves connect-

ing them. The determining points should first be considered.
Such points are the termini of the road, the lowest practicable
point over a summit, a river-crossing, etc. So far as is possi-
ble, having due regard to other considerations, the road should
be a " surface " road, i.e., the cut and fill should be made as
small as possible. The maximum permissible grade must also
have been determined and duly considered. The method of
location differs radically according as the lines joining the deter-
mining points have a very low grade or have a grade that ap-
proaches the maximum permissible. With very low natural
grades it is only necessary to strike a proper balance between
the requirements for easy alignment and the avoidance of exces-
sive earthwork. When the grade between two determined
points approaches the maximum, a study of the location may be
begun by finding a strictly surface line which will connect those
points with a line at the given grade. For example, suppose
the required grade is 1.6% and that the contours are drawn at
5-foot intervals. It will require 312 feet of 1.6% grade to rise
5 feet. Set a pair of dividers at 312 feet and step off this in-
terval on successive contours. This line will in general be very
irregular, but in an easy country it may lie fairly close to the
proper location line, and even in difficult country such a surface
line will assist greatly in selecting a suitable location. When the
larger part of the line will evidently consist of tangents, the tan-
gents should be first located and should then be connected by
suitable curves. When the curves predominate, as they gener-
ally will in mountainous country, and particularly when the line
is purposely lengthened in order to reduce the grade, the curves
should be plotted first and the tangents may then be drawn
connecting them. Considering the ease with which such lines
may be drawn on the preliminary map, it is frequently advisable,
after making such a paper location, to begin all over, draw a
new line over some specially difficult section and compare re-
sults. Profiles of such lines may be readily drawn by noting their
intersection with each contour crossed. Drawing on each profile
the required grade line will furnish an approximate idea of the

comparative amount of earthwork required.   After deciding on
the paper location, the length of each tangent, the central angle
(see § 21), and the radius of each curve should be measured as
accurately as possible.   Since a slight error made in such meas-
urements, taken from a map with a scale of 200 feet per inch,
would by accumulation cause serious discrepancies between the
plotted location and the location as afterward surveyed in the
field, frequent tie lines and angles should be determined between
the plotted location line and the preliminary line, and the loca-
tion should be altered, as may prove necessary, by changing the
length of a tangent or changing the central angle or radius of a
curve, so that the agreement of the check-points will be suffi-
ciently close.   The errors of an inaccurate preliminary survey
may thus be easily neutralized (see § 33).   When the pre-
liminary line has been properly run, its "backbone" line will
lie very near the location line and will probably cross it at fre-
quent intervals, thus rendering it easy to obtain short and nu-
merous tie lines.

16. Surveying methods.   A transit should be used for align-
ment, and only precise work is allowable.   The transit stations
should be centered with tacks and should be tied to witness-
stakes, which should be located outside of the range of the earth-
work, so that they will neither be dug up nor covered up.   All
original property lines lying within the limits of the right of way
should be surveyed with reference to the location line, so that
the right-of-way agent may have a proper basis for settlement.
When the property lines do not extend far outside of the re-
quired right of way they are frequently surveyed completely.

The leveler usually reads the target to the nearest thousandth
of a foot on turning-points and bench-marks, but reads to the
nearest tenth of a foot for the elevation of the ground at
stations.   Considering that $\frac{1}{1000}$ of a foot has an angular value
of only 7 seconds at a distance of 300 feet, and that one division
of a level-bubble is usually about 30 seconds, it may be seen that
it is a useless refinement to read to thousandths unless corre-
sponding care is taken in the use of the level.   The leveler

should also locate his bench-marks outside of the range of earthwork. A knob of rock protruding from the ground affords an excellent mark. A large nail, driven in the roots of a tree, which is not to be disturbed, is also a good mark. These marks should be clearly described in the note-book. The leveler should obtain the elevation of the ground at all station-points; also at all sudden breaks in the profile line, determining also the distance of these breaks from the previous even station. This will include the position and elevation of all streams, and even dry gullies, which are crossed.

Measurements should preferably be made with a steel tape, care being taken on steep ground to insure horizontal measurements. Stakes are set each 100 feet, and also at the beginning and end of all curves. Transit-points (sometimes called "plugs" or "hubs") should be driven flush with the ground, and a "witness-stake," having the "number" of the station, should be set three feet to the right. For example, the witness-stake might have on one side "137 + 69.92," and on the other side "P C 4° R," which would signify that the transit hub is 69.92 feet beyond station 137, or 13769.92 feet from the beginning of the line, and also that it is the "point of curve" of a "4°-curve" which turns to the *right*.

Alignment. The alignment is evidently a part of the location survey, but, on account of the magnitude and importance of the subject, it will be treated in a separate chapter.

17. Form of Notes. Although the Form of Notes cannot be thoroughly understood until after curves are studied, it is here introduced as being the most convenient place. The right-hand page should have a sketch showing all roads, streams, and property lines crossed with the bearings of those lines. This should be drawn to a scale of 100 feet per inch—the quarter-inch squares which are usually ruled in note-books giving convenient 25-foot spaces. This sketch will always be more or less distorted on curves, since the center line is always shown as *straight* regardless of curves. The station points ("Sta." in first column, left-hand page) should be placed opposite to their

sketched positions, which means that even stations will be
recorded on every *fourth* line. This allows three intermediate
lines for substations, which is ordinarily more than sufficient.
The notes should read ᴜᴘ the page, so that the sketch will be
properly oriented when looking ahead along the line. The
other columns on the left-hand page will be self-explanatory
when the subject of curves is understood. If the " calculated
bearings " are based on azimuthal observations, their agreement
(or *constant* difference) with the needle readings will form a
valuable check on the curve calculations and the instrumental
work.

FORM OF NOTES.

[Left-hand page.] [Right-hand page.]

Sta.	Align-ment	Vernier	Tang. Defl.	Calc. Bearing.	Needle.	
54						Bear Creek—10' wide 53+60
53 ☉ + 72.2	P.T.	9° 11'	18° 22'	N 54° 48' E	N 62° 15' 1	JAS. WILSON
52		7 57				52+18 N 7° 15' W
51	8° 24' curve to right for [18° 22'; tang. dist., 272.5]	6 15				
☉ 50		4 33				WM. BROWN
49		2 51				48+75 SMITHVILLE ROAD S 72° 20' E 71.1 48+42
48		1 09				
☉ + 32 47	P.C.	0°				JOHN JONES N 17° 40' E
46				N 36° 26' E	N 41° 0' E	46+31

CHAPTER II.

ALIGNMENT.

In this chapter the alignment of the *center line* only of a pair of rails is considered. When a railroad is crossing a summit in the grade line, although the horizontal projection of the alignment may be straight, the vertical projection will consist of two sloping lines joined by a curve. When a curve is on a grade, the center line is really a spiral, a curve of double curvature, although its horizontal projection is a circle. The center line therefore consists of straight lines and curves of single and double curvature. The simplest method of treating them is to consider their horizontal and vertical projections separately. In treating simple, compound, and transition curves, only the horizontal projections of those curves will be considered.

SIMPLE CURVES.

18. Designation of curves. A curve may be designated either by its radius or by the angle subtended by a chord of unit length. Such an angle is known as the " degree of curve " and is indicated by D. Since the curves that are practically used have very long radii, it is generally impracticable to make any use of the actual center, and the curve is located without reference to it. If AB in Fig. 7 represents a unit chord (C) of a curve of radius R, then by the above defini-
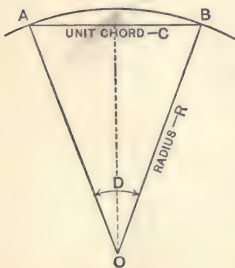
FIG. 7.

18

tion the angle AOB equals D. Then $AO \sin \frac{1}{2}D = \frac{1}{2}AB = \frac{1}{2}C$.

$$\therefore \ R = \frac{\frac{1}{2}C}{\sin \frac{1}{2}D} \quad \text{. . . . . (1)}$$

or, by inversion,

$$\sin \frac{1}{2}D = \frac{C}{2R} \quad \text{. . . . . . (2)}$$

The unit chord is variously taken throughout the world as 100 feet, 66 feet, and 20 meters. In the United States 100 feet is invariably used as the unit chord length, and throughout this work it will be so considered. Table I has been computed on this basis. It gives the radius, with its logarithm, of all curves from a 0° 01' curve up to a 10° curve, varying by single minutes. The sharper curves, which are seldom used, are given with larger intervals.

An approximate value of R may be readily found from the following simple rule, which should be memorized:

$$R = \frac{5730}{D}.$$

Although such values are not mathematically correct, since R does not strictly vary inversely as D, yet the resulting value is within a tenth of one per cent for all commonly used values of R, and is sufficiently close for many purposes, as will be shown later.

19. Length of a sub-chord. Since it is impracticable to measure along a curved arc, curves are always measured by laying off 100-foot chord lengths. This means that the actual arc is always a little longer than the chord. It also

FIG. 8.

means that a *subchord* (a chord shorter than the unit length) will be a little longer than the ratio of the angles subtended would call for. The truth of this may be seen without calcu-

lation by noting that two equal subchords, each subtending the angle $\frac{1}{2}D$, will evidently be slightly longer than 50 feet each. If c be the length of a subchord subtending the angle d, then, as in Eq. (2),
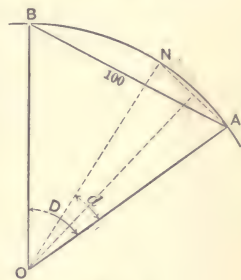
$$\sin \tfrac{1}{2}d = \frac{c}{2R},$$

or, by inversion,

$$c = 2R \sin \tfrac{1}{2}d. \quad . \quad . \quad . \quad . \quad . \quad (3)$$

The *nominal length* of a subchord $= 100\dfrac{d}{D}$. For example, a *nominal* subchord of 40 feet will subtend an angle of $\frac{40}{100}$ of D°; its *true length* will be slightly more than 40 feet, and may be computed by Eq. 3. The *difference* between the nominal and true lengths is maximum when the subchord is about 57 feet long, but with the low degrees of curvature ordinarily used the difference may be neglected. With a 10° curve and a nominal chord length of 60 feet, the true length is 60.049 feet. Very sharp curves should be laid off with 50-foot or even 25-foot chords (nominal length). In such cases especially the true lengths of these subchords should be computed and used instead of the nominal lengths.

20. Length of a curve. The length of a curve is always indicated by the quotient of $100\varDelta \div D$. If the quotient of $\varDelta \div D$ is a whole number, the length as thus indicated is the true length—*measured in 100-foot chord lengths.* If it is an odd number or if the curve begins and ends with a subchord (even though $\varDelta \div D$ is a whole number), theoretical accuracy requires that the *true* subchord lengths shall be used, although the difference may prove insignificant. The length of the arc (or the mean length of the two rails) is therefore always in excess of the length as given above. Ordinarily the amount of this excess is of no practical importance. It simply adds an insignificant amount to the length of rail required.

Example. Required the nominal and true lengths of a 3° 45′ curve having a central angle of 17° 25′. First reduce

the degrees and minutes to decimals of a degree. $(100 \times 17° 25')$ $\div 3° 45' = 1741.667 \div 3.75 = 464.444$. The curve has four 100-foot chords and a nominal chord of 64.444. The true chord should be 64.451. The actual arc is

$$17°.4167 \times \frac{\pi}{180°} \times R = 464.527.$$

The excess is therefore $464.527 - 464.451 = 0.076$ foot.

21. Elements of a curve. Considering the line as running from A toward B, the beginning of the curve, at A, is called the *point of curve* (PC). The other end of the *curve*, at B, is called the *point of tangency* (PT). The intersection of the tangents is called the *vertex* (V). The angle made by the tangents at V, which equals the angle made by the radii to the extremities of the curve, is called the *central angle* $(\varDelta)$. AV and BV, the two equal tangents from the vertex to the PC and PT, are called the *tangent distances* (T). The chord AB is called the *long chord* (LC). The intercept HG from the middle

FIG. 9.

of the long chord to the middle of the arc is called the *middle ordinate* (M). That part of the secant GV from the middle of the arc to the vertex is called the *external distance* (E). From the figure it is very easy to derive the following frequently used relations:
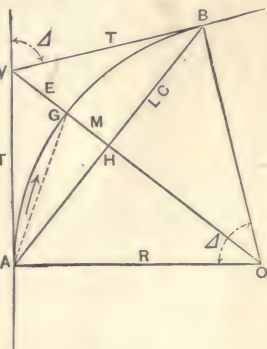
$$T = R \tan \tfrac{1}{2}\varDelta \quad . \quad . \quad . \quad . \quad . \quad . \quad (4)$$
$$LC = 2R \sin \tfrac{1}{2}\varDelta \quad . \quad . \quad . \quad . \quad . \quad (5)$$
$$M = R \text{ vers } \tfrac{1}{2}\varDelta \quad . \quad . \quad . \quad . \quad . \quad (6)$$
$$E = R \text{ exsec } \tfrac{1}{2}\varDelta \quad . \quad . \quad . \quad . \quad . \quad (7)$$

22. Relation between T, E, and $\varDelta$. Join A and G in Fig. 9. The angle $VAG = \tfrac{1}{4}\varDelta$, since it is measured by one half of the

arc AG between the secant and tangent. $AGO = 90° − \frac{1}{4}\varDelta$.

$$AV: VG:: \sin AGV: \sin VAG;$$

$$\sin AGV = \sin AGO = \cos \tfrac{1}{4}\varDelta;$$

$$T: E:: \cos \tfrac{1}{4}\varDelta: \sin \tfrac{1}{4}\varDelta;$$

$$T = E \cot \tfrac{1}{4}\varDelta. \quad . \quad . \quad . \quad . \quad . \quad (8)$$

The same relation may be obtained by dividing Eq. 4 by Eq. 7, since $\tan \alpha \div \mathrm{exsec}\ \alpha = \cot \tfrac{1}{2}\alpha$.

23. Elements of a 1° curve. From Eqs. 1 to 8 it is seen that the elements of a curve vary directly as R. It is also seen to be very nearly true that R varies inversely as D. If the elements of a 1° curve for various central angles are calculated and tabulated, the elements of a curve of $D°$ curvature may be approximately found by dividing by D the corresponding elements of a 1° curve having the same central angle. For small central angles and low degrees of curvature the errors involved by the approximation are insignificant, and even for larger angles the errors are so small that *for many purposes* they may be disregarded.

In Table II is given the value of the tangent distances, external distances, and long chords for a 1° curve for various central angles. The student should familiarize himself with the degree of approximation involved by solving a large number of cases under various conditions by the exact and approximate methods, in order that he may know when the approximate method is sufficiently exact for the intended purpose. The approximate method also gives a ready check on the exact method.

24. Exercises. (*a*) What is the tangent distance of a 4° 20′ curve having a central angle of 18° 24′ ?

(*b*) Given a 3° 30′ curve and a central angle of 16° 20′, how far will the curve pass from the vertex ? [Use Eq. 7.]

(*c*) An 18° curve is to be laid off using 25-foot (nominal) chord lengths. What is the true length of the subchords ?

(*d*) Given two tangents making a central angle of 15° 24′. It is desired to connect these tangents by a curve which shall pass 16.2 feet from their intersection. How far down the tangent will the curve begin and what will be its radius ? (Use Eq. 8 and then use Eq. 4 inverted.)

25. Curve location by deflections. The angle between a secant and a tangent (or between two secants intersecting on an arc) is measured by one half of the intercepted arc. Beginning at the *PC* (*A* in Fig. 10), if the first chord is to be a full chord we may deflect an angle VAa ($= \frac{1}{2}D$), and the point *a*, which is 100 feet from *A*, is a point on the curve. For the next station, *b*, deflect an *additional* angle bAa ($= \frac{1}{2}D$) and, with one end of the tape at *a*, swing the other end until the 100-foot point is on the line *Ab*. The point *b* is then on the curve. If the final chord *cB* is a subchord, its *additional deflection* ($\frac{1}{2}\alpha$) is something less than $\frac{1}{2}D$. The last *deflection*

FIG. 10.

(*BAV*) is of course $\frac{1}{2}\Delta$. It is particularly important, when a curve begins or ends with a subchord and the deflections are odd quantities, that the last additional deflection should be carefully computed and added to the previous deflection, to check the mathematical work by the agreement of this last computed deflection with $\frac{1}{2}\Delta$.
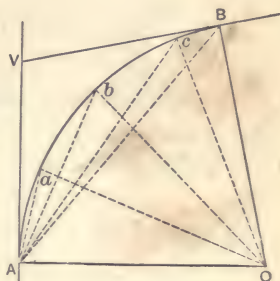
Example. Given a 3° 24′ curve having a central angle of 18° 22′ and beginning at sta. 47 + 32, to compute the deflections. The nominal length of curve is 18° 22′ ÷ 3° 24′ = 18.367 ÷ 3.40 = 5.402 stations or 540.2 feet. The curve therefore ends at sta. 52 + 72.2. The deflection for sta. 48 is $\frac{68}{100} \times \frac{1}{2}(3° 24')$ = 0.68 × 1°.7 = 1°.156 = 1° 09′ nearly. For each additional 100 feet it is 1° 42′ additional. The final additional deflection for the final subchord of 72.2 feet is

$$\frac{72.2}{100} \times \frac{1}{2}(3° 24') = 1°.2274 = 1° 14' \text{ nearly.}$$

The deflections are

P. C ... Sta. $47 + 32$ ....$0°$

48 ....$0°$ $+ 1° \; 09' = 1° \; 09'$

49 . ...$1° \; 09' + 1° \; 42' = 2° \; 51'$

50 ....$2° \; 51' + 1° \; 42' = 4° \; 33'$

51............$4° \; 33' + 1° \; 42' = 6° \; 15'$

52 ....$6° \; 15' + 1° \; 42' = 7° \; 57'$

P. T.........$52 + 72.2$....$7° \; 57' + 1° \; 14' = 9° \; 11'$

As a check $9° \; 11' = \frac{1}{2}(18° \; 22') = \frac{1}{2}\varDelta$.   (See the Form of Notes in § 17.)

26. Instrumental work. It is generally impracticable to locate more than 500 to 600 feet of a curve from one station. Obstructions will sometimes require that the transit be moved up every 200 or 300 feet. There are two methods of setting off the angles when the transit has been moved up from the *PC*.

(a) The transit may be sighted at the previous transit station with a reading on the plates equal to the deflection angle from that station to the station occupied, but with the angle set off on the *other* side of $0°$, so that when the telescope is turned to $0°$ it will sight along the tangent at the station occupied. Plunging the telescope, the forward stations may be set off by deflecting the proper deflections from the tangent at the station occupied. This is a very common method and, when the degree of curvature is an even number of degrees and when the transit is only set at even stations, there is but little objection to it. But the degree of curvature is sometimes an odd quantity, and the exigencies of difficult location frequently require that substations be occupied as transit stations. Method (*a*) will then require the recalculation of all deflections for each new station occupied. The mathematical work is largely increased and the probability of error is very greatly increased and not so easily detected. Method (*b*) is just as simple as method (*a*) even for the most simple cases, and for the more difficult cases just referred to the superiority is very great.

(b) Calculate the deflection for each station and substation throughout the curve as though the whole curve were to be located from the *P C*. The computations may thus be completed and *checked* (as above) before beginning the instrumental work. If it unexpectedly becomes necessary to introduce a substation at any point, its deflection from the *P C* may be readily interpolated. The stations actually set from the *P C* are located as usual. **RULE. When the transit is set on any forward station, backsight to ANY previous station with the plates set at the deflection angle for the station sighted at. Plunge the telescope and sight at any forward station with the deflection angle originally computed for that station.** When the plates read the deflection angle for the station occupied, the telescope is sighting along the tangent at that station—which is the method of getting the forward tangent when occupying the *P T*. Even though the station occupied is an unexpected substation, when the instrument is properly oriented at that station, the angle reading for *any* station, forward or back, is that originally computed for it from the *P C*. In difficult work, where there are obstructions, a valuable check on the accuracy may be found by sighting backward at *any* visible station and noting whether its deflection agrees with that originally computed. As a numerical illustration, assume a 4° curve, with 28° curvature, with stations 0, 2, 4, and 7 occupied. After setting stations 1 and 2, set up the transit at sta. 2 and backsight to sta. 0 with the deflection for sta. 0, which is 0°. The reading on sta. 1 is 2°; when the reading is 4° the telescope is tangent to the curve, and when sighting at 3 and 4 the deflections will be 6° and 8°.
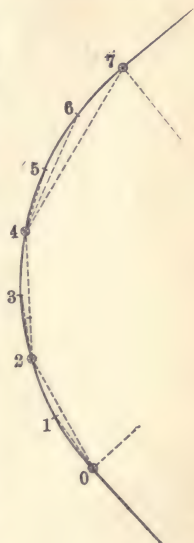
Fig. 11.

Occupy 4; sight to 2 with a reading of 4°. When the reading is 8° the telescope is tangent to the curve and, by plunging the telescope, 5, 6, and 7 may be located with the originally com-

puted deflections of 10°, 12°, and 14°. When occupying 7 a backsight may be taken to any visible station with the plates reading the deflection for that station; then when the plates read 14° the telescope will point along the forward tangent.

The location of curves by deflection angles is the normal method. A few other methods, to be described, should be considered as exceptional.

27. Curve location by two transits. A curve might be located more or less on a swamp where accurate chaining would be exceedingly difficult if not impossible. The long chord AB may be determined by triangulation or otherwise, and the elements of
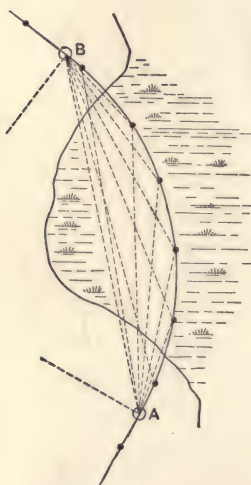
FIG. 12.   FIG. 13.

the curve computed, including (possibly) subchords at each end. The deflection from A and B to each point may be computed. A rodman may then be sent (by whatever means) to locate long stakes at points determined by the simultaneous sightings of the two transits.

28. Curve location by tangential offsets. When a curve is very flat and no transit is at hand the following method may be

used: Produce the back tangent as far forward as necessary. Compute the ordinates Oa', Ob', Oc', etc., and the abscissæ $a'a$, $b'b$, $c'c$, etc. If Oa is a full station (100 feet), then

$$
\left.
\begin{aligned}
Oa' = Oa' &= 100 \cos \tfrac{1}{2}D, \ \text{also} = R \sin D; \\
Ob' = Oa' + a'b' &= 100 \cos \tfrac{1}{2}D + 100 \cos \tfrac{3}{2}D, \\
&\quad \text{also} = R \sin 2D; \\
Oc' = Oa' + a'b' + b'c' &= 100(\cos \tfrac{1}{2}D + \cos \tfrac{3}{2}D + \cos \tfrac{5}{2}D), \\
&\quad \text{also} = R \sin 3D;
\end{aligned}
\right\} \quad (9)
$$

etc.

$$
\left.
\begin{aligned}
a'a = &\quad 100 \sin \tfrac{1}{2}D, \ \text{also} = R \operatorname{vers} D; \\
b'b = a'a + b''b &= 100 \sin \tfrac{1}{2}D + 100 \sin \tfrac{3}{2}D, \\
&\quad \text{also} = R\operatorname{vers}2D; \\
c'c = b'b + c''c &= 100(\sin \tfrac{1}{2}D + \sin \tfrac{3}{2}D + \sin \tfrac{5}{2}D), \\
&\quad \text{also} = R\operatorname{vers}3D;
\end{aligned}
\right\} \quad (10)
$$

etc.

The functions $\tfrac{1}{2}D$, $\tfrac{3}{2}D$, etc., may be more conveniently used *without* logarithms, by adding the several *natural* trigonometrical functions and pointing off two decimal places. It may also be noted that ob' (for example) is one half of the long chord for four stations; also that $b'b$ is the middle ordinate for four stations. If the engineer is provided with tables giving the long chords and middle ordinates for various degrees of curvature, these quantities may be taken (perhaps by interpolation) from such tables.

If the curve begins or ends at a substation, the angles and terms will be correspondingly altered. The modifications may be readily deduced on the same principles as above, and should be worked out as an exercise by the student.

29. Curve location by middle ordinates. Take first the simpler case when the curve begins at an even station. If we consider (in Fig. 14) the curve produced back to z, the chord $za = 2 \times 100 \cos \tfrac{1}{2}D$, $A'a = 100 \cos \tfrac{1}{2}D$, and $A'A = am = zn = 100 \sin \tfrac{1}{2}D$. Set off AA' perpendicular to the tangent and $A'a$ parallel to the tangent. $AA' = aa' = bb' = cc'$, etc. $= 100 \sin \tfrac{1}{2}D$. Set off aa' perpendicular to $a'A$. Produce Aa'

until $a'b = A'a$, thus determining b. Succeeding points of the curve may thus be determined indefinitely.

Suppose the curve begins with a subchord. As before $ra = Am' = c' \cos \frac{1}{2}d'$, and $rA = am' = c' \sin \frac{1}{2}d'$. Also $sz = An' = c'' \cos \frac{1}{2}d''$, and $sA = zn' = c'' \sin \frac{1}{2}d''$, in which
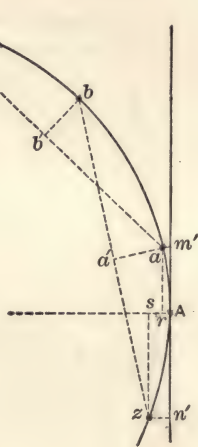
FIG. 14.   FIG. 15.

$(d' + d'') = D$. The points z and a being determined on the ground, aa' may be computed and set off as before and the curve continued in full stations. A subchord at the end of the curve may be located by a similar process.

30. Curve location by offsets from the long chord. (Fig. 16.) Consider at once the general case in which the curve commences with a subchord (curvature, d'), contains with one or more full chords (curvature of each, D), and ends with a subchord with curvature d''. The numerical work consists in computing first AB, then the various abscissæ and ordinates. $AB = 2R \sin \frac{1}{2}\Delta$.

$$\left.\begin{aligned}
Aa' = Aa' &\qquad= c' \cos \tfrac{1}{2}(\Delta - d'); \\
Ab' = Aa' + a'b' &\qquad= c' \cos \tfrac{1}{2}(\Delta - d') + 100 \cos \tfrac{1}{2}(\Delta - 2d' - D); \\
Ac' = Aa' + a'b' + b'c' &\qquad= c' \cos \tfrac{1}{2}(\Delta - d') + 100 \cos \tfrac{1}{2}(\Delta - 2d' - D) \\
&\qquad\qquad + 100 \cos \tfrac{1}{2}(\Delta - 2d'' - D); \\
\text{also} & \\
= AB - Bc' &\qquad= 2R \sin \tfrac{1}{2}\Delta - c'' \cos \tfrac{1}{2}(\Delta - d'').
\end{aligned}\right\} \quad (11)$$

$$a'a = a'a \qquad = c' \sin \tfrac{1}{2}(\varDelta - d');$$
$$b'b = a'a + mb = c' \sin \tfrac{1}{2}(\varDelta - d') + 100 \sin \tfrac{1}{2}(\varDelta - 2d' - D);$$
$$c'c = b'b - nb = c' \sin \tfrac{1}{2}(\varDelta - d') + 100 \sin \tfrac{1}{2}(\varDelta - 2d' - D) \qquad \qquad \left.\right\} \quad (12)$$
$$\qquad \qquad \qquad \qquad \qquad - 100 \sin \tfrac{1}{2}(\varDelta - 2d'' - D);$$
$$\text{also} \qquad = c'' \sin \tfrac{1}{2}(\varDelta - d'').$$

The above formulæ are considerably simplified when the curve begins and ends at even stations. When the curve is very long a regular law becomes very apparent in the formation of all terms between the first and last. There are too few terms in the above equations to show the law.

31. Use and value of the above methods. The chief value of the above methods lies in the possibility of doing the work without a transit. The same principles are sometimes employed, even when a transit is used, when obstacles prevent the use of the normal method (see § 32, c). If the terminal tangents have already been accurately determined, these methods are useful to locate points of the curve when rigid accuracy is not essential. Track foremen frequently use such methods to lay out unimportant sidings,

Fig. 16.

especially when the engineer and his transit are not at hand. Location by tangential offsets (or by offsets from the long chord) is to be preferred when the curve is flat (i.e., has a small central angle $\varDelta$) and there is no obstruction along the tangent, or long chord. Location by middle ordinates may be employed regardless of the length of the curve, and in cases when both the tangents and the long chord are obstructed. The above methods are but samples of a large number of similar methods which have been devised. The choice of the particular method to be adopted must be determined by the local conditions.
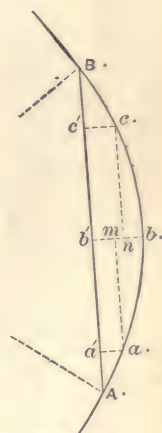
32. Obstacles to location. In this section will be given only a few of the principles involved in this class of problems, with illustrations. The engineer must decide in each case, which is

the best method to use, and it is frequently advisable to devise a special solution for some particular case.

a. When the vertex is inaccessible. As shown in § 26, it is not absolutely essential that the vertex of a curve should be located on the ground. But it is very evident that the angle between the terminal tangents is determined with far less probable error if it is measured by a single measurement at the vertex rather than as the result of numerous angle measurements along the curve, involving several positions of the transit and comparatively short sights. Sometimes the location of the tangents is already determined on the ground (as by bn and am, Fig. 17), and it is required to join the tangents by a curve of given radius. *Method.* Measure ab and the angles Vba and baV. $\varDelta$ is the sum of these angles. The distances bV and aV are computable from the above data. Given $\varDelta$ and R, the tan-
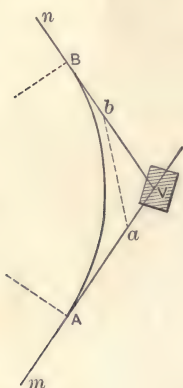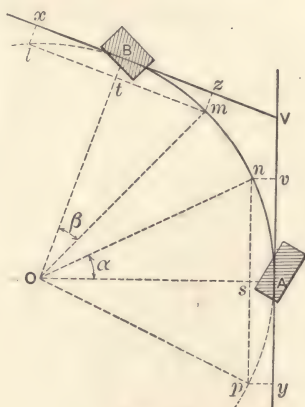
FIG. 17.   FIG. 18.

gent distances are computable, and then Bb and aA are found by subtracting bV and aV from the tangent distances. The curve may then be run from A, and the work may be checked by noting whether the curve as run ends at B—previously located from b.

b. When the point of curve (or point of tangency) is inaccessible. At some distance (As, Fig. 18) an unobstructed line pn

may be run parallel with AV. $nv = py = As = R$ vers α.

$$\therefore \quad \text{vers } \alpha = As \div R. \quad ns = ps = R \sin \alpha.$$

At y, which is at a distance ps back from the *computed* position of A, make an offset sA to p. Run pn parallel to the tangent. A tangent to the curve at n makes an angle of α with np. From n the curve is run in as usual.

If the point of tangency is obstructed, a similar process, somewhat reversed, may be used. β is that portion of $\mathit{\Delta}$ still to be laid off when m is reached. $tm = tl = R \sin \beta$. $mz = tB = lx = R$ vers β.

c. **When the central part of the curve is obstructed.** α is the central angle between two points of the curve between which a chord may be run. α may equal *any* angle, but it is preferable that α should be a multiple of D, the degree of curve, and that the points m and n should be on even stations. $mn = 2R \sin \frac{1}{2}\alpha$. A point s may be located by an offset ks from the chord mn by a similar method to that outlined in § 30.

The device of introducing the dotted curve mn having the same radius of curvature as the other, although neither necessary nor advisable in the case shown in Fig. 19, is sometimes the best method of surveying around an obstacle. The offset from any point on the dotted curve to the corresponding point on the true

FIG. 19.

curve is twice the "ordinate to the long chord," as computed in § 30.
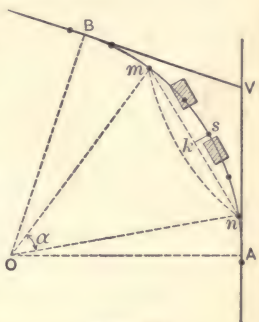
33. Modifications of location. The following methods may be used in allowing for the discrepancies between the "paper location" based on a more or less rough preliminary survey and the more accurate instrumental location. (See § 15.) They are also frequently used in locating new parallel tracks and modifying old tracks.

a. **To move the forward tangent parallel to itself a distance** x, **the point of curve** (A) **remaining fixed.** (Fig. 20.)

$$V'h = B'r = x'.$$

$$VV' = \frac{V'h}{\sin h\,VV'} = \frac{x'}{\sin \varDelta} \quad \cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot \quad (13)$$

$$AV' = AV + VV'.$$

The triangle BmB' is isosceles and $Bm = B'm$.

$$R' - R = O'O = mB = \frac{B'r}{\text{vers } B'mB} = \frac{x'}{\text{vers } \varDelta}$$

$$\therefore \quad R' = R + \frac{x'}{\text{vers } \varDelta} \quad \cdot \quad \cdot \quad \cdot \quad \cdot \quad (14)$$

The solution is very similar in case the tangent is moved inward to $V''B''$. Note that this method necessarily changes the
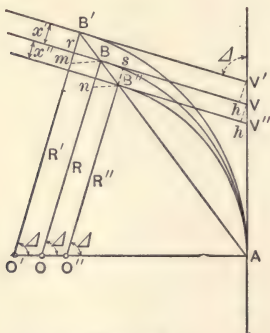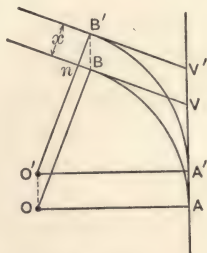
Fig. 20.   Fig. 21.

radius. If the radius is not to be changed, the point of curve must be altered as follows:

b. **To move the forward tangent parallel to itself a distance** x, **the radius being unchanged.** (Fig. 21.) In this case the whole

curve is moved bodily a distance $OO' = AA' = VV' = BB'$, and moved parallel to the first tangent $A V$.

$$BB' = \frac{B'n}{\sin nBB'} = \frac{x}{\sin \varDelta} = AA'. \quad . \quad . \quad (15)$$

c. To change the direction of the forward tangent at the point of tangency. (Fig. 22.) This problem involves a change (α) in the central angle and also requires a new radius. An error in the determination of the central angle furnishes an occasion for its use.

$R, \varDelta, \alpha, AV,$ and BV are known. $\varDelta' = \varDelta - \alpha.$

$Bs = R$ vers $\varDelta.$ $Bs = R'$ vers $\varDelta'.$

$$\therefore \quad R' = R \frac{\text{vers } \varDelta}{\text{vers } (\varDelta - \alpha)}. \quad . \quad . \quad . \quad (16)$$

$As = R \sin \varDelta.$ $A's = R' \sin \varDelta'.$

$$\therefore \quad AA' = A's - As = R' \sin \varDelta' - R \sin \varDelta. \quad . \quad (17)$$

The above solutions are given to illustrate a large class of problems which are constantly arising. All of the ordinary
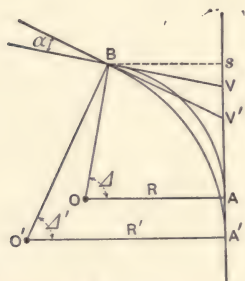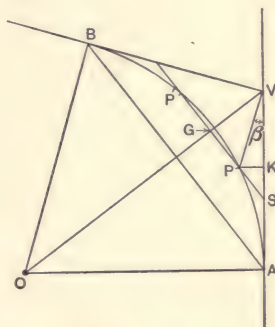
FIG. 22.   FIG. 23.

problems can be solved by the application of elementary geometry and trigonometry.

34. Limitations in location. It may be required to run a curve that shall join two given tangents and also pass through a given point. The point (P, Fig. 23) is assumed to be determined by its distance (VP) from the vertex and by the angle $AVP = \beta$.

It is required to determine the radius (R) and the tangent distance (AV). Δ is known.

$$PVG = \tfrac{1}{2}(180° - \Delta) - \beta = 90° - (\tfrac{1}{2}\Delta + \beta).$$

$$PP' = 2\,VP \sin PVG = 2\,VP \cos (\tfrac{1}{2}\Delta + \beta).$$

$$PSV = \tfrac{1}{2}\Delta. \qquad \therefore \quad SP = VP \frac{\sin \beta}{\sin \tfrac{1}{2}\Delta}.$$

$$AS = \sqrt{SP \times SP'} = \sqrt{SP(SP + PP')}.$$

$$= \sqrt{ VP\frac{\sin \beta}{\sin \tfrac{1}{2}\Delta}\left[VP\frac{\sin \beta}{\sin \tfrac{1}{2}\Delta} + 2\,VP \cos (\tfrac{1}{2}\Delta + \beta) \right]}$$

$$= VP \sqrt{ \frac{\sin^2 \beta}{\sin^2 \tfrac{1}{2}\Delta} + \frac{2 \sin \beta \cos (\tfrac{1}{2}\Delta + \beta)}{\sin \tfrac{1}{2}\Delta}}.$$

$$SV = VP\frac{\sin (\tfrac{1}{2}\Delta + \beta)}{\sin \tfrac{1}{2}\Delta}.$$

$$AV = AS + SV$$
$$= \frac{VP}{\sin \tfrac{1}{2}\Delta}[\sin (\tfrac{1}{2}\Delta + \beta) + \sqrt{\sin^2 \beta + 2 \sin \beta \sin \tfrac{1}{2}\Delta \cos (\tfrac{1}{2}\Delta + \beta)}]. \quad (18)$$

$$R = AV \cot \tfrac{1}{2}\Delta.$$

In the special case in which P is on the median line OV, $\beta = 90° - \tfrac{1}{2}\Delta$, and $(\tfrac{1}{2}\Delta + \beta) = 90°$. Eq. (18) then reduces to

$$AV = \frac{VP}{\sin \tfrac{1}{2}\Delta}(1 + \cos \tfrac{1}{2}\Delta) = VP \cot \tfrac{1}{4}\Delta,$$

as might have been immediately derived from Eq. (8).

In case the point P is given by the offset PK and by the distance VK, the triangle PKV may be readily solved, giving the distance VP and the angle β, and the remainder of the solution will be as above.

35. Determination of the curvature of existing track. (a) *Using a transit*.   Set up the transit at any point in the center of the track.   Measure in each direction 100 feet to points also in the center of the track.   Sight on one point with the plates at 0°. Plunge the telescope and sight at the other point.   The angle between the chords equals the degree of curvature.

(b) *Using a tape and string*.   Stretch a string (say 50 feet long) between two points on the inside of the head of the outer rail.   Measure the ordinate (x) between the *middle* of the string and the head of the rail.   Then

$$R = \frac{\text{chord}^2}{8x} \text{ (very nearly).} . . . . . \quad (19)$$

For, in Fig. 24, since the triangles AOE and ADC are similar, $AO : AE :: AD : DC$ or $R = \frac{1}{2}\overline{AD}^2 \div x$.   When, as is usual, the arc is very short compared with the radius, $AD = \frac{1}{2}AB$, very nearly.   Making this substitution we have Eq. (19).   With a chord of 50 feet and a 10° curve, the resulting difference in x is .0025 of an inch—far within the possible accuracy of such a method.   The above method gives the radius of the inner head

FIG. 24.

of the outer rail.   It should be diminished by $\frac{1}{2}g$ for the radius of the center of the track.   With easy curvature, however, this will not affect the result by more than one or two tenths of one per cent.
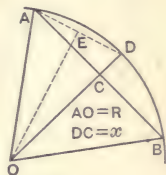
The inversion of this formula gives the required middle ordinate for a rail on a given curve.   For example, the middle ordinate of a 30-foot rail, bent for a 6° curve, is

$$x = 900 \div (8 \times 955) = .118 \text{ foot} = 1.4 \text{ inches.}$$

Another much used rule is to require the foreman to have a string, knotted at the centre, of such length that the middle ordinate, measured in inches, equals the degree of curve. To find that length, substitute (in eq. (19)) $5730 \div D$ for R and $D \div 12$ for x. Solving for *chord*, we obtain *chord* = 61.8 feet. The rule is not theoretically exact, but, considering the uncertain stretching of the string, the error is insignificant. In fact, the distance usually given is 62 feet, which is close enough for all purposes for which such a method should be used.

36. Problems. A systematic method of setting down the solution of a problem simplifies the work. Logarithms should always be used, and *all* the work should be so set down that a revision of the work to find a supposed error may be readily done. The value of such systematic work will become more apparent as the problems become more complicated. The two solutions given below will illustrate such work.

a. Given a 3° curve beginning at Sta. $27 + 60$ and running to Sta. $32 + 45$. Compute the ordinates and offsets used in locating the curve by tangential offsets.

b. With the same data as above, compute the distances to locate the curve by offsets from the long chord.

c. Assume that in Fig. 17 *ab* is measured as 217.6 feet, the angle *ab V* = 17° 42′, and the angle *ba V* = 21° 14′. Join the tangents by a 4° 30′ curve. Determine *bB* and *aA*.

d. Assume that in a case similar to Fig. 18 it was noted that a distance (*As*) equal to 12 feet would clear the building. Assume that $\Delta = 38° 20′$ and that $D = 4° 40′$. Required the value of α and the position of *n*. *Solution:*

vers $\alpha = As \div R$	$As = 12$	$\log = 1.07918$
	R (for 4° 40′ curve)	$\log = 3.08923$
	$\alpha = 8° 01′$	$\log \text{ vers } \alpha = \overline{7.98994}$
$ns = R \sin \alpha$		$\log \sin \alpha = 9.14445$
		$\log R = 3.08923$
	$ns = 171.27$	$\log = \overline{2.23369}$

e. Assume that the forward tangent of a 3° 20′ curve having a central angle of 16° 50′ must be moved 3.62 feet *inward*, without altering the *P.C.* Required the change in radius.

f. Given two tangents making an angle of 36° 18′. It is required to pass a curve through a point 93.2 feet from the vertex, the line from the vertex to the point making an angle of 42° 21′ with the tangent. Required the radius and tangent distance. *Solution :* Applying eq. (18), we have

$$
\begin{array}{rlrl}
& 2 & \log = & 0.30103 \\
\beta = & 42° \ 21' & \log \sin = & 9.82844 \\
\tfrac{1}{2}\varDelta = & 18° \ 09' & \log \sin = & 9.49346 \\
(\tfrac{1}{2}\varDelta + \beta) = & 60° \ 30' & \log \cos = & 9.69234 \\
& .20667 & & 9.31527
\end{array}
$$

$$
\begin{array}{rl}
\log \sin^2 \beta = 9.65688 \ldots \ldots \ldots & .45382 \\
2\,|\ 9.81987 \ldots \ldots \ldots & .66049 \\
9.90993 \ldots \ldots \ldots & .81271 \\
\text{nat sin } 60° \ 30' = .8703 & \\
\overline{\quad 1.6836\quad} &
\end{array}
$$

$$
\begin{array}{rl}
& \log = \ \ \overline{\ 0.22610} \\
VP = \ \ 93.2 & \log = \ \ 1.96941 \\
& \overline{\quad 2.19551 \quad} \\
& \log \sin \tfrac{1}{2}\varDelta = \ 9.49346 \\
\text{tang. dist. } A\,V = 503.56 & \log = \ \ \overline{\ 2.70205} \\
& \log \cot \tfrac{1}{2}\varDelta = 10.48437 \\
R = \underline{1536.1} & \overline{\quad 3.18642 \quad} \\
D = \overline{3° \ 44'} &
\end{array}
$$

COMPOUND CURVES.

37. Nature and use. Compound curves are formed by a succession of two or more simple curves of different curvature. The curves must have a common tangent at the point of compound curvature (*P.C.C.*). In mountainous regions there is frequently a necessity for compound curves having several changes of curvature. Such curves may be located separately as a succession of simple curves, but a combination of two

simple curves has special properties which are worth investigating and utilizing. In the following demonstrations R_2 always represents the *longer* radius and R_1 the *shorter*, no matter which succeeds the other. T_1 is the tangent adjacent to the curve of shorter radius (R_1), and is invariably the shorter tangent. Δ_1 is the central angle of the curve of radius R_1, but it may be greater or less than Δ_2.

38. Mutual relations of the parts of a compound curve having two branches. In Fig. 25, AC and CB are the two branches of
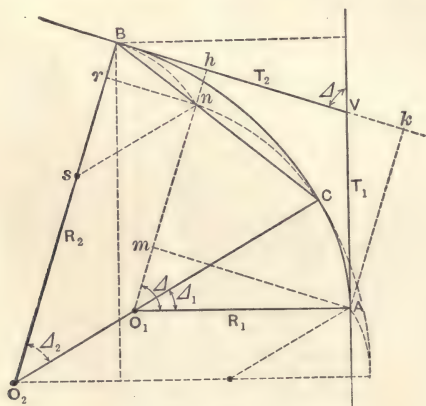
FIG 25.

the compound curve having radii of R_1 and R_2 and central angles of Δ_1 and Δ_2. Produce the arc AC to n so that $Ao_1n = \Delta$. The chord Cn produced *must* intersect B. The line ns, parallel to CO_2, will intersect BO_2 so that $Bs = sn = O_2O_1 = R_2 - R_1$. Draw Am perpendicular to O_1n. It will be parallel to hk.

$$Br = sn \text{ vers } Bsn \qquad = (R_2 - R_1) \text{ vers } \Delta_2;$$
$$mn = AO_1 \text{ vers } AO_1n \quad = R_1 \text{ vers } \Delta;$$
$$Ak = AV \sin AVk \qquad = T_1 \sin \Delta;$$
$$Ak = hm = mn + nh = mn + Br.$$
$$\therefore T_1 \sin \Delta = R_1 \text{ vers } \Delta + (R_2 - R_1) \text{ vers } \Delta_2. \quad . \quad (20)$$

Similarly it may be shown that

$$T_2 \sin \varDelta = R_2 \text{ vers } \varDelta - (R_2 - R_1) \text{ vers } \varDelta_1. \quad . \quad (21)$$

The mutual relations of the elements of compound curves may be solved by these two equations. For example, assume the tangents as fixed ($\varDelta$ therefore known) and that a curve of given radius R_1 shall start from a given point at a distance T_1 from the vertex, and that the curve shall continue through a given angle $\varDelta_1$. Required the other parts of the curve. From Eq. (20) we have

$$R_2 - R_1 = \frac{T_1 \sin \varDelta - R_1 \text{ vers } \varDelta}{\text{vers } \varDelta_2}.$$

$$\therefore R_2 = R_1 + \frac{T_1 \sin \varDelta - R_1 \text{ vers } \varDelta}{\text{vers } (\varDelta - \varDelta_1)} \quad . \quad . \quad . \quad (22)$$

T_2 may then be obtained from Eq. (21).

As another problem, given the location of the two tangents, with the two tangent distances (thereby locating the PC and PT), and the central angle of each curve; required the two radii. Solving Eq. (20) for R_1, we have

$$R_1 = \frac{T_1 \sin \varDelta - R_2 \text{ vers } \varDelta_2}{\text{vers } \varDelta - \text{vers } \varDelta_2}.$$

Similarly from Eq. (21) we may derive

$$R_1 = \frac{T_2 \sin \varDelta - R_2 (\text{vers } \varDelta - \text{vers } \varDelta_1)}{\text{vers } \varDelta_1}.$$

Equating these, reducing, and solving for R_2, we have

$$R_2 = \frac{T_1 \sin \varDelta \text{ vers } \varDelta_1 - T_2 \sin \varDelta (\text{vers } \varDelta - \text{vers } \varDelta_2)}{\text{vers } \varDelta_2 \text{ vers } \varDelta_1 - (\text{vers } \varDelta - \text{vers } \varDelta_1)(\text{vers } \varDelta - \text{vers } \varDelta_2)}. \quad (23)$$

Although the various elements may be chosen as above with considerable freedom, there are limitations. For example, in Eq. (22), since R_2 is always greater than R_1, the term to be added to R_1 must be essentially positive—i.e., $T_1 \sin \varDelta$ must be

greater than R_1 vers $\varDelta$. This means that $T_1 > R_1 \dfrac{\text{vers } \varDelta}{\sin \varDelta}$, or that $T_1 > R_1 \tan \frac{1}{2}\varDelta$, or that T_1 is greater than the corresponding tangent on a simple curve. Similarly it may be shown that T_2 is *less* than $R_2 \tan \frac{1}{2}\varDelta$ or *less* than the corresponding tangent on a simple curve. Nevertheless T_2 is always greater than T_1. In the limiting case when $R_2 = R_1$, $T_2 = T_1$ and $\varDelta_2 = \varDelta_1$.

39. Modifications of location. Some of these modifications may be solved by the methods used for simple curves. For example :

a. It is desired to move the tangent VB, Fig. 26, parallel to itself to $V'B'$. Run a new curve from the $P.C.C.$ which shall reach the new tangent at B', where the chord of the old curve
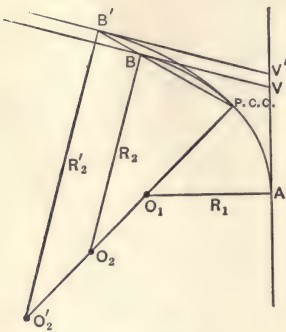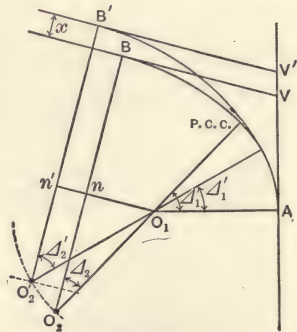
FIG. 26.   FIG. 27.

intersects the new tangent. The solution is almost identical with that in § 33, *a*.

b. Assume that it is desired to change the forward tangent (as above) but to retain the same radius. In Fig. 27

$$(R_2 - R_1) \cos \varDelta_2 = O_2 n;$$
$$(R_2 - R_1) \cos \varDelta_2' = O_2'n'.$$
$$x = O_2 n - O_2'n' = (R_2 - R_1)(\cos \varDelta_2 - \cos \varDelta_2').$$

$$\cos \varDelta_2' = \cos \varDelta_2 - \frac{x}{R_2 - R_1}. \quad . \quad . \quad . \quad (24)$$

The *P.C.C.* is moved *backward* along the sharper curve an angular distance of $\Delta_2' - \Delta_2 = \Delta_1 - \Delta_1'$.

In case the tangent is moved inward rather than outward, the solution will apply by transposing Δ_2 and Δ_2'. Then we will have

$$\cos \Delta_2' = \cos \Delta_2 + \frac{x}{R_2 - R_1}. \quad \cdot \quad \cdot \quad \cdot \quad (25)$$

The *P.C.C.* is then moved *forward*.

c. Assume the same case as (b) except that the larger radius comes first and that the tangent adjacent to the smaller radius is moved. In Fig. 28

$$(R_2 - R_1) \cos \Delta_1 = O_1 n;$$

$$(R_2 - R_1) \cos \Delta_1' = O_1' n'.$$
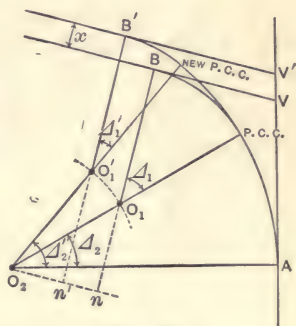
FIG. 28.

$$x = O_1' n' - O_1 n = (R_2 - R_1)(\cos \Delta_1' - \cos \Delta_1).$$

$$\cos \Delta_1' = \cos \Delta_1 + \frac{x}{R_2 - R_1}. \quad \cdot \quad \cdot \quad \cdot \quad (26)$$

The *P.C.C.* is moved *forward* along the easier curve an angular distance of $\Delta_1' - \Delta_1 = \Delta_2 - \Delta_2'$.

In case the tangent is moved *inward*, transpose as before and we have

$$\cos \Delta_1' = \cos \Delta_1 - \frac{x}{R_2 - R_1}. \quad \cdot \quad \cdot \quad \cdot \quad (27)$$

The *P.C.C.* is moved *backward*.

d. Assume that the radius of one curve is to be altered without changing either tangent. Assume conditions as in Fig. 29. For the diagrammatic solution assume that R_2 is to be in-

creased by O_2S. Then, since R_2' must pass through O_1 and extend beyond O_1 a distance O_1S, the locus of the new center must lie on the arc drawn about O_1 as center and with OS as radius. The locus of O_2' is also given by a line $O_2'p$ parallel to BV and at a distance of R_2' (equal to $S\ldots P.C.C.$) from it. The new center is therefore at the intersection O_2'. An arc with radius R_2' will therefore be tangent at B' and tangent to the old curve *produced* at NEW $P.C.C.$ Draw O_1n' perpendicular to O_2B. With O_2 as center draw the arc O_1m, and with O_2' as center draw the arc O_1m'.
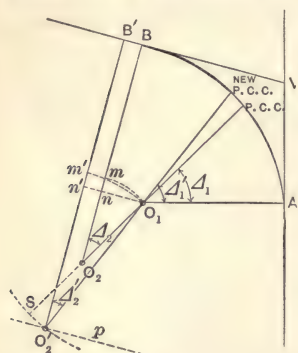
FIG. 29.

$mB = m'B' = R_1$. $\therefore mn = m'n' = (R_2' - R_1)$ vers $\Delta_2' = (R_2 - R_1)$ vers Δ_2.

$$\therefore \text{vers } \Delta_2' = \frac{(R_2 - R_1)}{(R_2' - R_1)} \text{ vers } \Delta_2. \quad \cdot \quad \cdot \quad \cdot \quad (28)$$

$$O_1n = (R_2 - R_1) \sin \Delta_2;$$

$$O_1n' = (R_2' - R_1) \sin \Delta_2'.$$

$$BB' = O_1n' - O_1n = (R_2' - R_1) \sin \Delta_2' - (R_2 - R_1) \sin \Delta_2. \quad (29)$$

This problem may be further modified by assuming that the radius of the curve is decreased rather than increased, or that the smaller radius follows the larger. The solution is similar and is suggested as a profitable exercise.

It might also be assumed that, instead of making a given change in the radius R_2, a given change BB' is to be made. Δ_2' and R_2' are required. Eliminate R_2' from Eqs. 28 and 29 and solve the resulting equation for Δ_2'. Then determine R_2' by a suitable inversion of either Eq. 28 or 29.

As in §§ 32 and 33, the above problems are but a few, although perhaps the most common, of the problems the engineer may meet with in compound curves. All of the ordinary problems may be solved by these and similar methods.

40. Problems. *a.* Assume that the two tangents of a compound curve are to be 348 feet and 624 feet, and that $\varDelta_1 = 22° 16'$ and $\varDelta_2 = 28° 20'$.   Required the radii.

$$[\textit{Ans.}\ \ R_1 = 326.92;\ \ R_2 = 1574.85.]$$

b. A line crosses a valley by a compound curve which is first a 6° curve for 46° 30′ and then a 9° 30′ curve for 84° 16′. It is afterward decided that the last tangent should be 6 feet farther up the hill. What are the required changes ?   [*Note.* The second tangent is evidently moved *outward*. The solution corresponds to that in the first part of § 39, *c.* The *P. C. C.* is moved forward 16.39 feet. If it is desired to know how far the *P. T.* is moved in the direction of the tangent (i.e., the *projection* of BB', Fig. 28, on $V'B'$), it may be found by observing that it is equal to $nn' = (R_2 - R_1)(\sin \varDelta_1 - \sin \varDelta_1')$. In this case it equals 0.65 foot, which is very small because $\varDelta_1$ is nearly 90°. The value of $\varDelta_2$ (46° 30′) is not used, since the solution is independent of the value of $\varDelta_2$. The student should learn to recognize which quantities are mutually related and therefore essential to a solution, and which are independent and non-essential.]

<center>TRANSITION CURVES.</center>

41. Superelevation of the outer rail on curves. When a mass is moved in a circular path it requires a centripetal force to keep it moving in that path. By the principles of mechanics we know that this force equals $Gv^2 \div gR$, in which G is the weight, v the velocity in feet per second, g the acceleration of gravity in feet per second in a second, and R the radius of curvature. If the two rails of a curved track were laid on a level (transversely), this centripetal force could only be furnished by the

pressure of the wheel-flanges against the rails.   As this is very objectionable, the outer rail is elevated so that the reaction of
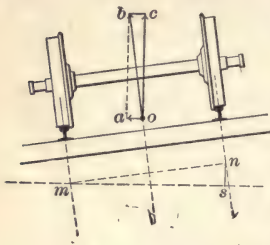
Fig. 30.

the rails against the wheels shall contain a horizontal component equal to the required centripetal force.   In Fig. 30, if *ob* represents the reaction, *oc* will represent the weight G, and *ao* will represent the required centripetal force.   From similar triangles we may write $sn : sm :: ao : oc$.   Call $g = 32.17$.   Call $R = 5730 \div D$, which is sufficiently accurate

for this purpose (see § 19).   Call $v = 5280 V \div 3600$, in which V is the velocity in miles per hour.   *mn* is the distance between rail centers, which, for an 80-lb. rail and standard gauge, is 4.916 feet.   *sm* is slightly less than this.   As an average value we may call it 4.900, which is its exact value when the superelevation is $4\frac{3}{4}$ inches.   Calling $sn = e$, we have

$$e = sm\frac{ao}{oc} = 4.9\frac{Gv^2}{gR}\frac{1}{G} = \frac{4.9 \times 5280^2\, V^2 D}{32.17 \times 3600^2 \times 5730}.$$

$$e = .0000572\, V^2 D. \quad . \quad . \quad . \quad . \quad . \quad (30)$$

It should be noticed that, according to this formula, the required superelevation varies as the *square* of the velocity, which means that a change of velocity of only 10% would call for a change of superelevation of 21%.   Since the velocities of trains over any road are extremely variable, it is impossible to adopt any superelevation which will fit all velocities even approximately.   The above fact also shows why any over-refinement in the calculations is useless and why the above approximations, which are really small, are amply justifiable. For example, the above formula contains the approximation that $R = 5730 \div D$.   In the extreme case of a 10° curve the error involved would be about 1%.   A change of about $\frac{1}{2}$ of 1% in

the velocity, or say from 40 to 40.2 miles per hour, would mean as much. The error in e due to the assumed constant value of sm is never more than a very small fraction of 1%. The rail-laying is not done closer than this. The following tabular form is based on Eq. 30:

SUPERELEVATION OF THE OUTER RAIL (IN FEET) FOR VARIOUS VELOCITIES AND DEGREES OF CURVATURE.

Velocity in Miles per Hour.	Degree of Curve.									
	1°	2°	3°	4°	5°	6°	7°	8°	9°	10°
30	.05	.10	.15	.20	.26	.31	.36	.41	.46	.51
40	.09	.18	.27	.37	.46	.55	.64	.73	.82	
50	.14	.29	.43	.57	.71	.86				
60	.20	.41	.62	.82						

42. Practical rules for superelevation. A much used rule for superelevation is to "elevate one half an inch for each degree of curvature." The rule is rational in that e in Eq. 30 varies directly as D. The above rule therefore agrees with Eq. 30 when V is about 27 miles per hour. However applicable the rule may have been in the days of low velocities, the elevation thus computed is too small now.

Another (and better) rule is to "elevate for the speed of the fastest trains." This rule is further justified by the fact that a four-wheeled truck, having two parallel axles, will always tend to run to the outer rail and will require considerable flange pressure to guide it along the curve. The effect of an excess of superelevation on the slower trains will only be to relieve this flange pressure somewhat. This rule is coupled with the limitation that the elevation should never exceed a limit of six inches —sometimes eight inches. This limitation implies that locomotive engineers must reduce the speed of fast trains around sharp curves until the speed does not exceed that for which the actual superelevation used is suitable. The heavy line in the tabular form (§ 41) shows the six-inch limitation.

Some roads furnish their track foremen with a list of the superelevations to be used on each curve in their sections. This method has the advantage that each location may be separately studied, and the proper velocity, as affected by local conditions (*e.g.*, proximity to a stopping-place for all trains), may be determined and applied.

Another method is to allow the foremen to determine the superelevation for each curve by a simple measurement taken at the curve. The rule is developed as follows: By an inversion of Eq. 19 we have

$$x = chord^2 \div 8R \quad . \quad . \quad . \quad . \quad (31)$$

Putting x equal to e in Eq. 30 and solving for "*chord*," we have

$$chord^2 = .0000572 V^2 D 8 R$$
$$= 2.621 V^2.$$
$$chord = 1.62 V. \quad . \quad . \quad . \quad . \quad . \quad (32)$$

To apply the rule, assume that 50 miles per hour is fixed as the velocity from which the superelevation is to be computed. Then $1.62 V = 1.62 \times 50 = 81$ feet, which is the distance given to the trackmen. Stretch a tape (or even a string) with a length of 81 feet between two points on the inside head of the outer rail or the outer head of the inner rail. The ordinate at the middle point then equals the superelevation. The values of this chord length for varying velocities are given in the accompanying tabular form.

Velocity in miles per hour......	20	25	30	35	40	45	50	55	60
Chord length in feet	32.4	40.5	48.6	56.7	64.8	72.9	81.0	89.1	97.2

43. Transition from level to inclined track. On curves the track is inclined transversely; on tangents it is level. The transition from one condition to the other must be made gradu-

ally. If there is no transition curve, there must be either in-
clined track on the tangent or insufficiently inclined track on the
curve or both. Sometimes the full superelevation is continued
through the total length of the curve and the "run-off"
(having a length of 100 to 200 feet) is located entirely on the
tangents at each end. In other practice it is located partly on
the tangent and partly on the curve. Whatever the method,
the superelevation is correct at only one point of the run-off.
At all other points it is too great or too small. This (and other
causes) produces objectionable lurches and resistances when
entering and leaving curves. The object of transition curves is
to obviate these resistances.

44. Fundamental principle of transition curves. If a curve
has variable curvature, beginning at the tangent with a curve of
infinite radius, and the curvature gradually sharpens until it
equals the curvature of the required simple curve and there
becomes tangent to it, the superelevation of such a transition
curve may begin at zero at the tangent, gradually increase to
the required superelevation for the simple curve, and yet have
at every point the superelevation required by the curvature at
that point. Since in Eq. (30) e is directly proportional to D,
the required curve must be one in which the degree of curve
increases directly as the distance along the curve. The mathe-
matical development of such a curve is quite complicated. It
has, however, been developed, and tables have been computed for
its use, by Prof. C. L. Crandall. The following method has
the advantage of great simplicity, while its agreement with the
true transition curve is as close as need be, as will be shown.

45. Multiform compound curves. If the transition curve
commences with a very flat curve and at regular even chord
lengths compounds into a curve of sharper curvature until the
desired curvature is reached, the increase in curvature at each
chord point being uniform, it is plain that such a curve is a
close approximation to the true spiral, especially since the rails
as laid will *gradually* change their curvature rather than main-
tain a uniform curvature throughout each chord length and

then abruptly change the curvature at the chord points. Such a curve, *as actually laid*, will be a much closer approximation to the true curve than the multiform compound curve by which it is set out. There will actually be a *gradual* increase in curvature which increases directly as the length of the curve.

46. Required length of spiral. The required length of spiral evidently depends on the amount of superelevation to be gained, and also depends somewhat on the speed. If the spiral is laid off in 25-foot chord lengths, with the first chord subtending a 1° curve, the second a 2° curve, etc., the fifth chord will subtend a 5° curve, and the increase from this last chord to a 6° curve is the same as the uniform increase of curvature between the chords. The same spiral extended would run on to a 12° curve in $(12 - 1)25 = 275$ feet. The last chord of a spiral should have a smaller degree of curvature than the simple curve to which it is joined. If the curves are very sharp, such as are used in street work and even in suburban trolley work, an increase in degree of curvature of 1° per 25 feet will not be sufficiently rapid, as such a rate would require too long curves. 2°, 10°, or even 20° increase per 25 feet may be necessary, but then the chords should be reduced to 5 feet. Such a rapid rate of increase is justified by the necessary reduction in speed. On the other hand, very high speed will make a lower rate of increase desirable, and therefore a spiral whose degree of curvature increases only 0° 30′ per 25 feet may be used. Such a spiral would require a length of 375 feet to run on to an 8° curve, which is inconveniently long, but it might be used to run on to a 4° curve, where its length would be only 175 feet. Three spirals have been developed in Table IV, each with chords of 25 feet, the rate of increase in the degree of curvature being 0° 30′, 1° and 2° per chord. One of these will be suitable for any curvature found on ordinary steam-railroads.

47. To find the ordinates of a 1°-per-25-feet spiral. Since the first chord subtends a 1° curve, its central angle is 0° 15′ and the angle aQV (Fig. 31) is 7′ 30″. The tangent at a makes an angle of 15′ with VQ. The angle between the chord ba and

the tangent at a is $\frac{1}{2}(30') = 15'$, and the angle $bab'' = \frac{1}{2}(30') + 15'$ $= 30'$. Similarly the angle $cbc'' = \frac{1}{2}(45') + 30' + 15' = 67'\ 30''$ $= 1°\ 07'\ 30''$, and the angle dcd'' is $2°\ 0'$. The ordinate aa' $= 25 \sin 7'\ 30''$, and $Qa' = 25 \cos 7'\ 30''$. $Qb' = Qa' + a'b'$ $= Qa' + ab'' = 25\ (\cos 7'\ 30'' + \cos 30')$. $bb' = b'b'' + bb''$ $= 25\ (\sin 7'\ 30'' + \sin 30')$. Similarly the ordinates of c, d, etc., may be obtained.
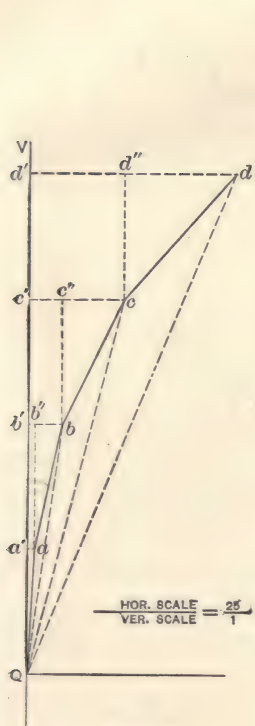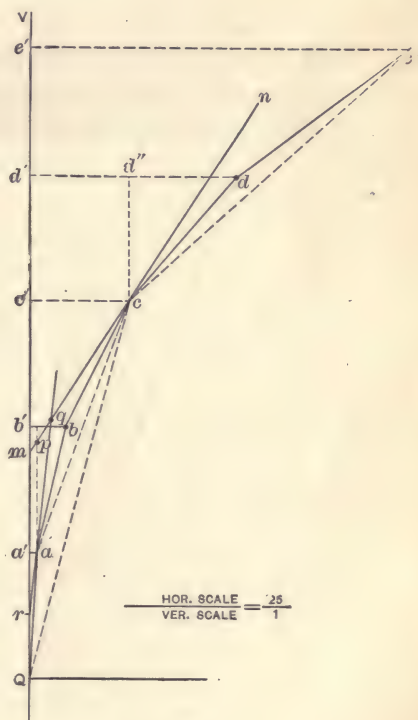
FIG. 31.   FIG. 32.

48. To find the deflections from any point of the spiral. $aQV = 7'\ 30''$. Tan $bQV = bb' \div Qb'$; tan $cQV = cc' \div Qc'$; etc. Thus we are enabled to find the deflection angles from the tangent at Q to any point of the spiral.

The tangent to the curve at c (Fig. 32) makes an angle of $1°\ 30'$ with QV, or $cmV = 1°\ 30'$. $Qcm = cmV - cQm$. The

value of cQm is known from previous work. The deflection from c to Q then becomes known.

$acm = cmV - cap = cmV - caq - qap$. caq is the deflection angle to c from the tangent at a and will have been previously computed numerically. $qap = 15'$. acm therefore becomes known.

$$bcm = \tfrac{1}{2} \text{ of } 45' = 22' \ 30'';$$
$$dcn = \tfrac{1}{2} \text{ of } 60' = 30'.$$

$ecn = ecd'' - ncd''$, $ncd'' = cmV$, $\tan ecd'' = (ee' - d''d') \div c'e'$, all of which are known from the previous work.

By this method the deflections from the tangent at any

Fig. 33.

point of the curve to any other point are determinable. These values are compiled in Table IV. The corresponding values of these angles when the increase in the degree of curvature per chord length is 30′, and when it is 2°, are also given in Table IV.

49. Connection of spiral with circular curve and with tangent.
See Fig. 33.* Let AV and BV be the tangents to be connected
by a $D°$ curve, having a suitable spiral at each end. If no
spirals were to be used, the problem would be solved as in
simple curves giving the curve AMB. Introducing the spiral
has the effect of throwing the curve away from the vertex a
distance MM' and reducing the central angle of the $D°$ curve
by 2ϕ. Continuing the curve beyond Z and Z' to A' and B',
we will have $AA' = BB' = MM'$. $ZK =$ the x ordinate and
is therefore known. Call $MM' = m$. $A'N = x - R$ vers ϕ.
Then

$$m = MM' = AA' = \frac{A'N}{\cos \frac{1}{2}\varDelta} = \frac{x - R \text{ vers } \phi}{\cos \frac{1}{2}\varDelta}. \quad . \quad . \quad (33)$$

$$NA = AA' \sin \tfrac{1}{2}\varDelta = (x - R \text{ vers } \phi) \tan \tfrac{1}{2}\varDelta.$$

$$VQ = QK - KN + NA + AV$$
$$= y - R \sin \phi + (x - R \text{ vers } \phi) \tan \tfrac{1}{2}\varDelta + R \tan \tfrac{1}{2}\varDelta$$
$$= y - R \sin \phi + x \tan \tfrac{1}{2}\varDelta + R \cos \phi \tan \tfrac{1}{2}\varDelta. \quad . \quad (34)$$

When $A'N$ has already been computed, it may be more con-
venient to write

$$VQ = y + R (\tan \tfrac{1}{2}\varDelta - \sin \phi) + A'N \tan \tfrac{1}{2}\varDelta. \quad (35)$$

$$VM' = VM + MM'$$

$$= R \text{ exsec } \tfrac{1}{2}\varDelta + \frac{x}{\cos \frac{1}{2}\varDelta} - \frac{R \text{ vers } \phi}{\cos \frac{1}{2}\varDelta}. \quad . \quad . \quad (36)$$

$$AQ = VQ - AV$$
$$= y - R \sin \phi + (x - R \text{ vers } \phi) \tan \tfrac{1}{2}\varDelta. \quad (37)$$

Example. To join two tangents making an angle of $34° 20'$
by a $5° 40'$ curve and suitable spirals. Use $1°$-per-25-feet

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* The student should at once appreciate the fact of the necessary distor-
tion of the figure. The distance MM' in Fig. 33 is perhaps 100 times its real
proportional value.

spirals with five chords. Then $\phi = 3° 45'$, $x = 2.999$, $\frac{1}{2}\varDelta$
$= 17° 10'$, and $y = 124.942$.

(Eq. 33)				R	3.00497
				vers ϕ	7.33063
			2.166		$\overline{0.33560}$
		$x =$	2.999		
		$A'N =$	$\overline{0.833}$		9.92064̂
				cos $\frac{1}{2}\varDelta$	9.98021
	$m = MM' = AA' =$		0.872		$\overline{9.94043̂}$
(Eq. 36)				R	$\overline{3.00497}$
				exsec $\frac{1}{2}\varDelta$	8.66863̂
		$VM =$	47.164		$\overline{1.67363̂}$
		$m =$	0.872		
		$VM' =$	$\overline{48.036}$		
(Eq. 35)	$y = 124.942$	nat. $\tan \frac{1}{2}\varDelta =$	.30891		
		nat. sin $\phi =$	.06540		
			$\overline{.24351}$		9.38651̂
			R		3.00497
	246.314				$\overline{2.39148̂}$
		[See above]	$A'N$		$\overline{9.92064̂}$
			tan $\frac{1}{2}\varDelta$		9.48984
	0.257		AN		$\overline{9.41048̂}$
	$VQ = \overline{371.513}$				
(Eq. 37)				R	3.00497
				tan $\frac{1}{2}\varDelta$	9.48984
	312.471			AV	$\overline{2.49481}$
	$AQ = \overline{59.042}$				

50. Field-work. When the spiral is designed during the
original location, the tangent distance VQ should be computed
and the point Q located. It is hardly necessary to locate all of
the points of the spiral until the track is to be laid. The
extremities should be located, and as there will usually be one
and perhaps two full station points on the spiral, these should

also be located. Z may be located by setting off $QK = y$ and $KZ = x$, or else by the tabular deflection for Z from Q and the distance ZQ, which is the long chord. Setting up the instrument at Z and sighting back at Q with the proper deflection, the tangent at Z may be found and the circular curve located as usual, its central angle being $\varDelta - 2\phi$. A similar operation will locate Q' from Z'.

To locate points on the spiral. Set up at Q, with the plates reading $0°$ when the telescope sights along VQ. Set off from Q the deflections given in Table IV for the instrument at Q, using a chord length of 25 feet, the process being like the method for simple curves except that the deflections are irregular. If a full station-point occurs within the spiral, interpolate between the deflections for the adjacent spiral-points. For example, a spiral begins at Sta. $56 + 15$. Sta. 57 comes 10 feet beyond the third spiral point. The deflection for the third point is $35'\ 0''$; for the fourth it is $56'\ 15''$. $\frac{10}{25}$ of the difference $(21'\ 15'')$ is $8'\ 30''$; the deflection for Sta. 57 is therefore $43'\ 30''$. This method is not theoretically accurate, but the error is small. Arriving at z, the forward alignment may be obtained by sighting back at Q (or at any other point) with the given deflection for that point from the station occupied. Then when the plates read $0°$ the telescope will be tangent to the spiral and to the succeeding curve. All rear points should be checked from z. If it is necessary to occupy an intermediate station, use the deflections given for that station, orienting as just explained for z, checking the back points and locating all forward points up to z if possible.

After the center curve has been located and z' is reached, the other spiral must be located but *in reverse order*, *i.e.*, the sharp curvature of the spiral is at z' and the curvature decreases toward Q'.

51. To replace a simple curve by a curve with spirals. This *may* be done by the method of § 49, but it involves shifting the whole track a distance m, which in the given example equals 0.87 foot. Besides this the track is appreciably shortened,

which would require rail-cutting. But the track may be kept at practically the same length and the lateral deviation from the old track may be made very small by slightly sharpening the curvature of the old track, moving the new curve so that it is wholly or partially *outside* of the old curve, the remainder of it with the spirals being *inside* of the old curve. It is found by experience that a decrease in radius of from 1% to 5% will answer
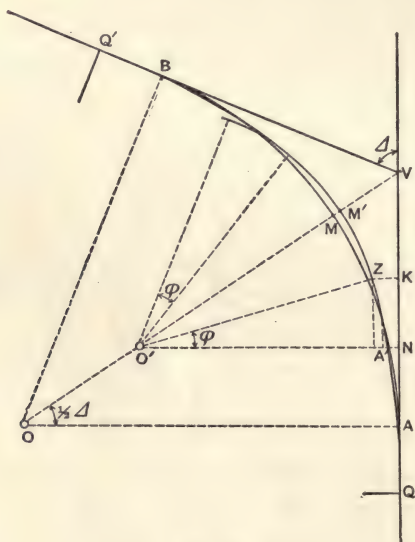
FIG. 34.

the purpose. The larger the central angle the less the change. The solution is as indicated in Fig. 34.

$$O'N = R' \cos \phi + x.$$
$$O'V = O'N \sec \tfrac{1}{2}\varDelta$$
$$\quad = R' \cos \phi \sec \tfrac{1}{2}\varDelta + x \sec \tfrac{1}{2}\varDelta.$$
$$m = MM' = MV - M'V$$
$$\quad = R \operatorname{exsec} \tfrac{1}{2}\varDelta - (O'V - R')$$
$$\quad = R \operatorname{exsec} \tfrac{1}{2}\varDelta - R' \cos \phi \sec \tfrac{1}{2}\varDelta - x \sec \tfrac{1}{2}\varDelta + R'. \quad (38)$$
$$AQ = QK - KN + NV - VA$$
$$\quad = y - R' \sin \phi + (R' \cos \phi + x) \tan \tfrac{1}{2}\varDelta - R \tan \tfrac{1}{2}\varDelta$$
$$\quad = y - R' \sin \phi + R' \cos \phi \tan \tfrac{1}{2}\varDelta - (R - x) \tan \tfrac{1}{2}\varDelta. \quad (39)$$

The length of the old curve from Q to $Q' = 2AQ + 100\dfrac{\varDelta}{D}$.

The length of the new curve from Q to $Q' = 2L + 100\dfrac{\varDelta - 2\phi}{D'}$,

in which L is the length of each spiral.

Example. Suppose the old curve is a $7°\ 30'$ curve with a central angle of $38°\ 40'$. As a trial, compute the relative length of a new $8°$ curve with spirals of seven chords. $\phi = 7°\ 0'$; $\frac{1}{2}\varDelta = 19°\ 20'$; R (for the $7°\ 30'$ curve) $= 764.489$; R' (for the $8°$ curve) $= 716.779$; $x = 7.628$.

[Eq. 38]			R	2.88337
			exsec $\frac{1}{2}\varDelta$	8.77642
				1.65979
	$\begin{array}{r}45.687\\R' = 716.779\end{array}$			
	762.466		R'	2.85538
			cos ϕ	9.99675
			sec $\frac{1}{2}\varDelta$	0.02521
		753.953		2.87734
			x	0.88241
			sec $\frac{1}{2}\varDelta$	0.02521
		8.084		0.90762
	762.037	762.037		
	$m = 0.429$			
[Eq. 39]	$y = 174.722$		R'	2.85538
			sin ϕ	9.08589
		87.353		1.94128
			R'	2.85538
			cos ϕ	9.99675
			tan $\frac{1}{2}\varDelta$	9.54512
	249.606			2.39725
			$R = 764.489$	
			$x = 7.628$	
			756.861	2.87901
			tan $\frac{1}{2}\varDelta$	9.54512
		265.543		2.42413
	$\begin{array}{r}424.328\\352.896\end{array}$	352.896		
	$AQ = 71.432$			

The length of the old curve from Q to Q' is

$$100\frac{\varDelta}{D} = 100\frac{38.667}{7.5} = \qquad\qquad 515.556$$

$$2\varDelta Q = 2 \times 71.432 = \qquad\qquad 142.864$$

$$\overline{\qquad\qquad 658.420}$$

New curve: $100\dfrac{\varDelta - 2\phi}{D'} = 100\dfrac{38.667 - 14.000}{8.0} = 308.333$

$$2L = 2 \times 175 \qquad\qquad = 350.000$$

$$\overline{\qquad 658.333} \quad 658.333$$

$$\text{Difference in length} = \overline{\quad 0.087}$$

Considering that this difference may be divided among 22 joints (using 30-foot rails) no rail-cutting would be necessary. If the difference is too large, a slight variation in the value of the new radius R' will reduce the difference as much as necessary. A truer comparison of the lengths would be found by comparing the lengths of the arcs.

52. Application of transition curves to compound curves. Since compound curves are only employed when the location is limited by local conditions, the elements of the compound curve should be determined (as in §§ 38 and 39) regardless of the transition curves, depending on the fact that the lateral shifting of the curve when transition curves are introduced is very small. If the limitations are very close, an estimated allowance may be made for them.

Methods have been devised for inserting transition curves between the branches of a compound curve, but the device is complicated and usually needless, since when the train is once on a curve the wheels press against the outer rail steadily and a change in curvature will not produce a serious jar even though the superelevation is temporarily a little more or less than it should be.

If the easier curve of the compound curve is less than 3° or 4°, there may be no need for a transition curve off from that branch. This problem then has two cases according as transition curves are used at both ends or at one end only.

a. *With transition curves at both ends.* Adopting the method of § 49, calling $\varDelta_1 = \frac{1}{2}\varDelta$, we may compute $m_1 = MM_1'$. Similarly, calling $\varDelta_2 = \frac{1}{2}\varDelta$, we may compute $m_2 = MM_2'$. But
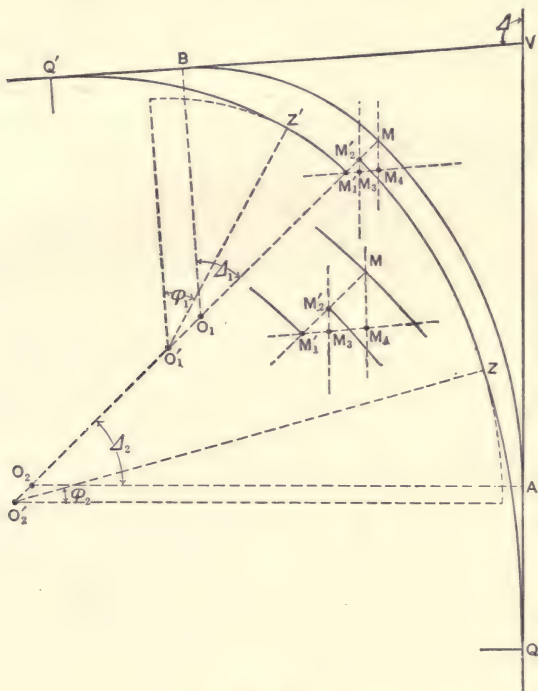
FIG. 35.

M_1' and M_2' must be made to coincide. This may be done by moving the curve $Z'M_1'$ and its transition curve parallel to $Q'V$ a distance $M_1'M_3$, and the other curve parallel to QV a distance $M_2'M_3$. In the triangle $M_1'M_3M_2'$, the angle at $M_1' = 90° - \varDelta_1$, the angle at $M_2' = 90° - \varDelta_2$, and the angle at $M_3 = \varDelta$.

$$\text{Then } M_1'M_3 = M_1'M_2\frac{\sin(90° - \varDelta_2)}{\sin \varDelta} = (m_1 - m_2)\frac{\cos \varDelta_2}{\sin \varDelta}. \left.\vphantom{\begin{matrix}1\\1\\1\end{matrix}}\right\}$$

$$\text{Similarly } M_2'M_3 = M_1'M_2\frac{\sin(90° - \varDelta_1)}{\sin \varDelta} = (m_1 - m_2)\frac{\cos \varDelta_1}{\sin \varDelta}. \left.\vphantom{\begin{matrix}1\\1\\1\end{matrix}}\right\} \quad (40)$$

b. *With a transition curve on the sharper curve only.* Compute $m_1 = MM_1'$ as before; then move the curve Z_1M_1' parallel to $Q'V$ a distance of

$$M_1'M_4 = m_1 \frac{\cos \varDelta_2}{\sin \varDelta} \quad . \quad . \quad . \quad . \quad (41)$$

The simple curve MA is moved parallel to VA a distance of

$$MM_4 = m_1 \frac{\cos \varDelta_1}{\sin \varDelta}. \quad . \quad . \quad . \quad (42)$$

If $\varDelta_1$ and $\varDelta_2$ are both small, $M_1'M_4$ and MM_4 may be more than m_1, but the lateral deviation of the new curve from the old will always be less than m_1.

53. To replace a compound curve by a curve with spirals. The solution is somewhat analogous to that of § 51. Compute m_1 for the sharper branch of the curve, placing $\varDelta_1 = \frac{1}{2}\varDelta$ in Eq. 38. Since m_1 and m_2 for the two branches of the curve must be identical, a value for R_2' must be found which will satisfy the determined value of $m_2 = m_1$. Solving Eq. 38 for R', we obtain

$$R' = \frac{R \text{ vers } \frac{1}{2}\varDelta - m \cos \frac{1}{2}\varDelta - x}{\cos \phi - \cos \frac{1}{2}\varDelta}. \quad . \quad . \quad (43)$$

Substituting in this equation the known value of $m_1 \, (= m_2)$ and calling $R' = R_2'$, $R = R_2$, and $\varDelta_2 = \frac{1}{2}\varDelta$, solve for R_2'. Obtain the value of AQ for each branch of the curve separately by Eq. 39, and compare the lengths of the old and new lines.

Example. Assume a compound curve with $D_1 = 8°$; $D_2 = 4°$; $\varDelta_1 = 36°$ and $\varDelta_2 = 32°$. Use 1°-per-25-feet spirals; $\phi_1 = 7° \, 0'$; $\phi_2 = 1° \, 30'$. Assume that the sharper curve is sharpened from $8° \, 0'$ to $8° \, 12'$.

[Eq. 38]

$$R_1 \quad 2.85538$$
$$\text{exsec } 36° \quad 9.37303$$

$$R_1' = \frac{169.209}{699.326}$$
$$\overline{868.535}$$

$$2.22842$$

$$R_1' \quad 2.84468$$
$$\cos \phi_1 \quad 9.99675$$
$$\sec \varDelta_1 \quad 0.09204$$

$$857.970 \qquad 2.93347$$

$$x_1 \quad 0.88241$$
$$\sec \varDelta_1 \quad 0.09204$$

$$\frac{9.429}{867.399} \qquad 0.97445$$

$$\frac{867.399}{m_1 = 1.136}$$

[Eq. 43]

$$R_2 \quad 3.15615$$
$$\text{vers } 32° \quad 9.18176$$

$$217.700 \qquad 2.33785$$

$$m_1 = 1.136 \quad 0.05538$$
$$\cos 32° \quad 9.92842$$

$$\frac{0.963}{x_2 = 0.763} \qquad 9.98380$$
$$\frac{1.726}{1.726}$$

$$215.974 \qquad 2.33440$$

$$\text{nat. } \cos \phi \ = .99966$$
$$\text{nat. } \cos \varDelta_2 = .84805$$

$$.15161 \quad 9.18073$$

$$R_2' = 1424.54 \quad [4° \ 1' \ 22''] \qquad 3.15367$$

[Eq. 39]
$$y_1 = 174.722$$

$$R_1' \quad 2.84468$$
$$\sin \phi_1 \quad 9.08586$$

$$85.226 \qquad 1.93057$$

$$R_1' \quad 2.84468$$
$$\cos \phi_1 \quad 9.99675$$
$$\tan \tfrac{1}{2}\varDelta[\varDelta_1 = 36°] \quad 9.86126$$

$$504.302 \qquad 2.70269$$

$$R_1 = 716.779$$
$$x_1 = \quad 7.628$$

$$\frac{709.151}{\tan \tfrac{1}{2}\varDelta} \quad \begin{matrix} 2.85074 \\ 9.86126 \end{matrix}$$

$$\frac{515.235}{600.461} \qquad 2.71206$$

$$\frac{679.024}{600.461}$$
$$AQ_1 = \overline{78.563}$$

[Eq. 39]

$y_2 = 74.994$

| | | R_2' | 3.15367 |
| | | $\sin \phi_2$ | 8.41792 |

37 290

| | | | 1.57156 |

		R_2'	3.15367
		$\cos \phi_2$	9.99985
		$\tan \frac{1}{2}\varDelta(\varDelta_2 = 32°)$	9.79579

389.843

| | | | 2.94931 |

$$R_2 = 1432.69$$
$$x_2 = \quad 0.76$$

| | | 1431.93 | 3.15592 |
| | | $\tan \frac{1}{2}\varDelta$ | 9.79579 |

894.770

| | | | 2.95171 |

964.837
932.060

932.060

$AQ_2 = \quad 32.777$

For the length of the old track we have:

$$100 \frac{\varDelta_1}{D_1} = 100 \frac{36°}{8°} = 450.$$
$$100 \frac{\varDelta_2}{D_2} = 100 \frac{32°}{4°} = 800.$$
$$AQ_1 = \quad 78.563$$
$$AQ_2 = \quad 32.777$$
$$\overline{\qquad 1361.340}$$

For the length of the new track we have:

$$100 \frac{\varDelta_1 - \phi_1}{D_1'} = 100 \frac{29°}{8°.20} = \quad 353.659$$

$$100 \frac{\varDelta_2 - \phi_2}{D_2'} = 100 \frac{30°.5}{4°.023} = \quad 758.140$$

Spiral on 8° 12' curve	175.000
" " 4° 01' 22'' "	75.
Length of new track	= 1361.799
" " old "	= 1361.340

Excess in length of new track =   0.459 feet.

Since the new track is slightly longer than the old, it shows that the new track runs too far *outside* the old track at the *P. C. C.* On the other hand the offset *m* is only 1.136. The maximum amount by which the new track comes *inside* of the old track at two points, presumably not far from Z' and Z, is very difficult to determine exactly. Since it is desirable that the maximum offsets (inside and outside) should be made as nearly equal as possible, this feature should not be sacrificed to an effort to make the two lines of precisely equal length so that the rails need not be cut. Therefore, if it is found that the offsets inside the old track are nearly equal to *m* (1.136), the above figures should stand. Otherwise *m* may be diminished (and the above excess in length of track diminished) by *increasing* R_1' very slightly and making the necessary consequent changes.

<div align="center">VERTICAL CURVES.</div>

54. Necessity for their use. Whenever there is a change in the rate of grade, it is necessary to eliminate the angle that would be formed at the point of change and to connect the two grades by a curve. This is especially necessary at a sag between two grades, since the shock caused by abruptly forcing an upward motion to a rapidly moving heavy train is very severe both to the track and to the rolling stock.

55. Required length. Theoretically the length should depend on the change in the rate of grade, the greater change requiring a longer curve. The importance of this was greater in the days when link couplers were in universal use and the " slack " in a long train was very great. Under such circumstances, when a train was moving down a heavy grade the cars would crowd ahead against the engine. Reaching the sag, the engine would begin to pull out, rapidly taking out the slack. Six inches of slack on each car would amount to several feet on a long train, and the resulting jerk on the couplers, especially those near the rear of the train, has frequently resulted in

broken couplers or even derailments. A vertical curve will
practically eliminate this danger if the curve is made long
enough, but the rapidly increasing adoption of close spring
couplers and air-brakes, even for freight trains, is obviating the
necessity for such very long curves.   Two hundred feet may be
considered sufficiently long for all ordinary changes of grade.
Four hundred feet would probably suffice for the greatest
change ever found in practice.

56. Form of curve.   In Fig. 36 assume that A and C, equi-

FIG. 36.

distant from B, are the extremities of the vertical curve.   Bi-
sect AC at e; draw Be and bisect it at h.   Bisect AB and BC
at k and l.   The line kl will pass through h.   A parabola may
be drawn with its vertex at h which will be tangent to AB and
BC at A and B.   It may readily be shown from the proper-
ties of a parabola that if an ordinate be drawn at *any* point (as
at n) we will have

$$sn : eh \text{ (or } hB) :: \overline{Am}^2 : \overline{Ae}^2,$$

or

$$sn = eh\frac{Am^2}{Ae^2}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (44)$$

Since the elevation of any point along AB or BC is readily
determinable, the elevation of any point on the curve may be
computed by adding the correction sn.

57. Numerical example.   Assume that B is located at Sta.
$16 + 20$; that the curve is to be 200 feet long; that the grade
of AB is -0.8%, and of $BC +1.2\%$; also that the elevation
of B above the datum plane is 162.6.   Then the elevation of
the various points is as follows: A, 163.4; C, 163.8; e,

$\frac{1}{2}(163.4 + 163.8) = 163.6$; h, $\frac{1}{2}(163.6 + 162.6) = 163.1$.   Then $eh = 0.5$.   The elevations of the points on the curve are:

Sta. **15 + 20**,	(A)	163.4
" **16** ,	$163.4 - (.80 \times 0.8) + (.80^2 \times 0.5) = 163.08$
" **17** ,	$162.6 + (.80 \times 1.2) + (.20^2 \times 0.5) = 163.58$
" **17 + 20**,	(C)	163.8

A theoretical inaccuracy in the above method lies in the fact that eh and all parallel lines are not truly vertical.   In the above case the variation from the vertical is $0° 07'$, while the effect of this variation on the elevations in this case (as in the most extreme cases) is absolutely inappreciable.   The grades in the figure are necessarily very greatly exaggerated, which increases the apparent inaccuracy.

CHAPTER III.

EARTHWORK.

FORM OF EXCAVATIONS AND EMBANKMENTS.

58. Usual form of cross-section in cut or fill. The normal form of cross-section in cut is as shown in Fig. 37, in which $e \ldots g$ represents the natural surface of the ground, no matter how irregular; ab represents the position and width of the re-
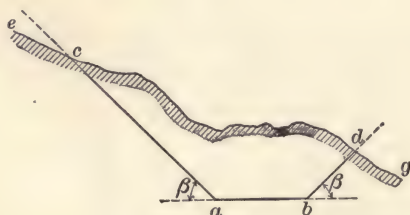
FIG. 37.

quired roadbed; ac and bd represent the "side slopes" which begin at a and b and which intersect the natural surface at such
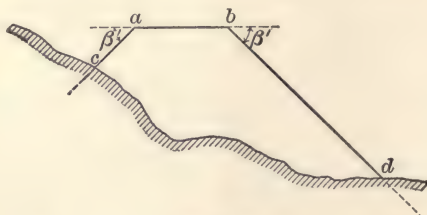
FIG. 38.

points (c and d) as will be determined by the required slope angle (β).

64

The normal section in fill is as shown in Fig. 38. The points *c* and *d* are likewise determined by the intersection of the required side slopes with the natural surface. In case the required roadbed (*ab* in Fig. 39) intersects the natural surface, both cut
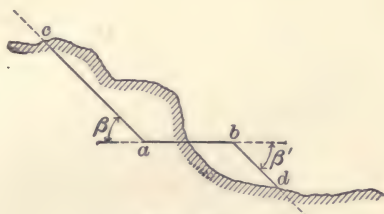
FIG. 39.

and fill are required, and the points *c* and *d* are determined as before. Note that β and β' are not necessarily equal. Their proper values will be discussed later.

59. Terminal pyramids and wedges. Fig. 40 illustrates the general form of cross-sections when there is a transition from cut to fill. *a . . . g* represents the grade line of the road which passes from cut to fill at *d*. *sdt* represents the surface profile. A cross-section taken at the point where either side of the roadbed *first* cuts the surface (the point *m* in this case) will usually be triangular if the ground is regular. A similar cross-section should be taken at *o*, where the other side of the roadbed cuts the surface. In general the earthwork of cut and fill terminates in two pyramids. In Fig. 40 the pyramid vertices are at *n* and *k*, and the bases are *lhm* and *opq*. The roadbed is generally wider in cut than in fill, and therefore the section *lhm* and the altitude *ln* are generally greater than the section *opq* and the altitude *pk*. When the line of intersection of the roadbed and natural surface (*nodkm*) becomes perpendicular to the axis of the roadbed (*ag*) the pyramids become wedges whose bases are the nearest convenient cross-sections.

60. Slopes. a. Cuttings. The required slopes for cuttings vary from perpendicular cuts, which may be used in hard rock which will not disintegrate by exposure, to a slope of perhaps

4 horizontal to 1 vertical in a soft material like quicksand or in a clayey soil which flows easily when saturated. For earthy materials a slope of 1 : 1 is the maximum allowable, and even this should only be used for firm material not easily affected by
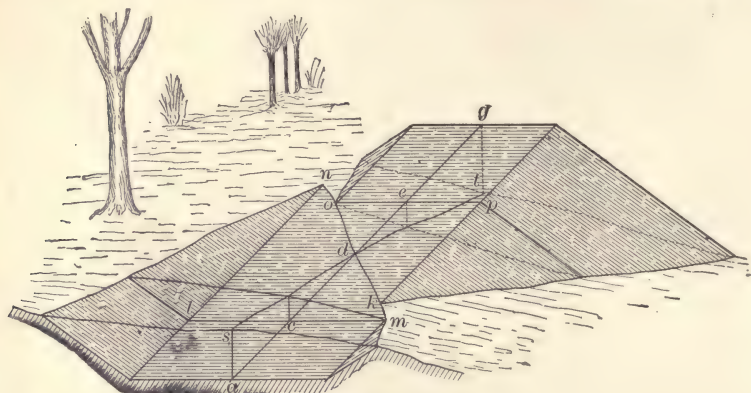
FIG. 40.

saturation. A slope of $1\frac{1}{2}$ horizontal to 1 vertical is a safer slope for average earthwork. It is a frequent blunder that slopes in cuts are made too steep, and it results in excessive work in clearing out from the ditches the material that slides down, at a much higher cost per yard than it would have cost to take it out at first, to say nothing of the danger of accidents from possible landslides.

b. Embankments. The slopes of an embankment vary from 1 : 1 to 1.5 : 1. A rock fill will stand at 1 : 1, and if some care is taken to form the larger pieces on the outside into a rough dry wall, a much steeper slope can be allowed. This method is sometimes a necessity in steep side-hill work. Earthwork embankments generally require a slope of $1\frac{1}{2}$ to 1. If made steeper at first, it generally results in the edges giving way, requiring repairs until the ultimate slope is nearly or quite $1\frac{1}{2}$: 1. The difficulty of incorporating the added material with the old embankment and preventing its sliding off frequently makes these repairs disproportionately costly.

61. Compound sections. When the cut consists partly of earth and partly of rock, a compound cross-section must be
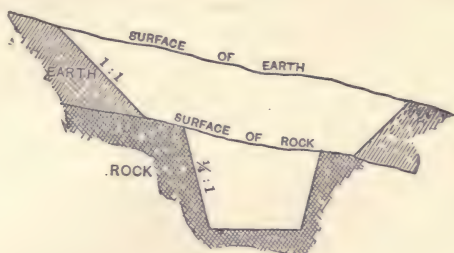
FIG. 41.

made. If borings have been made so that the contour of the rock surface is accurately known, then the true cross-section may be determined. The rock and earth should be calculated separately, and this will require an accurate knowledge of where the rock " runs out "—a difficult matter when it must be determined by boring. During construction the center part of the earth cut would be taken out first and the cut widened until a sufficient width of rock surface had been exposed so that the rock cut would have its proper width and side slopes. Then the earth slopes could be cut down at the proper angle. A " berm " of about three feet is usually left on the edges of the rock cut as a margin of safety against a possible sliding of the earth slopes. After the work is done, the amount of excavation that has been made is readily computable, but accurate preliminary estimates are difficult. The area of the cross-section of earth in the figure must be determined by a method similar to that developed for borrow-pits (see § 89).

62. Width of roadbed. Owing to the large and often disproportionate addition to volume of cut or fill caused by the addition of even one foot to the width of roadbed, there is a natural tendency to reduce the width until embankments become unsafe and cuts are too narrow for proper drainage. The cost of maintenance of roadbed is so largely dependent on the drainage of the roadbed that there is true economy in making an

ample allowance for it. The practice of some of the leading railroads of the country in this respect is given in the following table, in which are also given some data belonging more properly to the subject of superstructure.

WIDTH OF ROADBED FOR SINGLE AND DOUBLE TRACK—SLOPE RATIOS—DISTANCES BETWEEN TRACK CENTERS.

Road.	Single Track.		Double Track.		Slope Ratios.		Dist. between Track Centers.
	Cut.	Fill.	Cut.	Fill.	Cut.	Fill.	
A., T. & Santa Fé....	{ 28' earth / 22' rock	20	....	.... ..	1 : 1 / ¼ : 1	1.5 : 1	
Chi., Burl. & Quincy	14 + (2 × 5) *	16	28 + (2 × 5)	30	1.5 : 1	1.5 : 1	14'
Chi., Mil. & St. Paul..	18 + (2 × 6)	20 to 24	31 + (2 × 6)	33 to 37	1.5 : 1	1.5 : 1	13'
C., C., C. & St. Louis	20 + (2 × 4)	20	33 + (2 × 4)	33	1.5 : 1	1.5 : 1	13'
Illinois Central..	32.5	18	....	....	1.5 : 1	1.5 : 1	
Erie	20' 8½''	20' 8½''	33' 8½''	33' 8½''	1.5 : 1	1.5 : 1	13'
Lehigh Valley.......	14 + (2 × 3.5)	16	27 + (2 × 3.5)	30	1 : 1	1.5 : 1	13'
L. S. & Michigan So.	....	....	33 + (2 × 7.25)	32	1.5 : 1	1.5 : 1	13'
Louisville & Nashv..	13 + (2 × 4.5)	16	....	....	1 : 1	1.5 : 1	
Michigan Central...	....	....	33 + (2 × 2.5)	33	1.5 : 1	1.5 : 1	13'
N. Y. N. H. & H....	....	... ...	30	30	1.5 : 1	1.5 : 1	12'
Norfolk & Western...	{ 21' 2'' earth / 16' rock	17' 2'' /	34' 2'' earth / 29' rock	30' 2'' /	{ ¼ : 1 / ½ : 1	}	13'
Pennsylvania......	{ 19' 2'' light traffic / 27' 2'' heavy "	19' 2'' / 19' 2''	31' 4''+(2 × 4)	31' 4''	1.5 : 1	1.5 : 1	12' 2''
Union Pacific...	14 + (2 × 3.5)	16	....	....	1 : 1	1.5 : 1	

* (2 × 5) signifies two ditches each 5 feet wide: the following cases should be interpreted similarly.

It may be noted from the above table that the average width for an *earthwork* cut, single track, is about 24.7 feet, with a minimum of 19 feet 2 inches. The widths of fills, single track, average over 18 feet, with numerous minimums of 16 feet. The widths for double track may be found by adding the distance between track centers, which is usually 13 feet.

63. Form of subgrade. The stability of the roadbed depends largely on preventing the ballast and subsoil from becoming saturated with water. The ballast must be porous so that it will not retain water, and the subsoil must be so constructed that it will readily drain off the rain-water that soaks through the ballast. This is accomplished by giving the subsoil a curved form, convex

upward, or a surface made up of two or three planes, the two outer planes having a slope of about 1 : 24 (sometimes more and sometimes less, depending on the soil) and the middle plane, if three are used, being level. When a circular form is used, a crowning of 6 inches in a total width of 17 or 18 feet is generally used. Occasionally the subgrade is made level, especially in rock-cuts, but if the subsoil is previously compressed by rolling, as required on the N. Y. C. & H. R. R. R., or if the subsoil is drained by tile drains laid underneath the ditches, the necessity for slopes is not so great. Rock cuts are generally required to be excavated to one foot below subgrade and then filled up again to subgrade with the same material, if it is suitable.

64. Ditches. "The stability of the track depends upon the strength and permanence of the roadbed and structures upon which it rests; whatever will protect them from damage or prevent premature decay should be carefully observed. The worst enemy is WATER, and the further it can be kept away from the track, or the sooner it can be diverted from it, the better the track will be protected. Cold is damaging only by reason of the water which it freezes; therefore the first and most important provision for good track is drainage." (Rules of the Road Department, Illinois Central R. R.)

The form of ditch generally prescribed has a flat bottom 12″ to 24″ wide and with sides having a minimum slope, except in rock-work, of 1 : 1, more generally 1.5 : 1 and sometimes 2 : 1. Sometimes the ditches are made V-shaped, which is objectionable unless the slopes are low. The best form is evidently that which will cause the greatest flow for a given slope, and this will evidently be the form in which the ratio of area to wetted perimeter is the largest. The semicircle fulfills this condition better than any other form, but the nearly vertical sides would be difficult to maintain. (See Fig. 42.) A ditch,

Fig. 42.

with a flat bottom and such slopes as the soil requires, which approximates to the circular form will therefore be the best.

When the flow will probably be large and at times rapid it will be advisable to pave the ditches with stone, especially if the soil is easily washed away. Six-inch tile drains, placed 2' under the ditches, are prescribed on some roads. (See Fig. 43.) No better method could be devised to insure a dry subsoil. The ditches through cuts should be led off at the end of the cut so that the adjacent embankment will not be injured.

Wherever there is danger that the drainage from the land above a cut will drain down into the cut, a ditch should be made near the edge of the cut to intercept this drainage, and this ditch should be continued, and paved if necessary, to a point where the outflow will be harmless. Neglect of these simple and inexpensive precautions frequently causes the soil to be loosened on the shoulders of the slopes during the progress of a heavy rain, and results in a landslide which will cost more to repair than the ditches which would have prevented it for all time.

Ditches should be formed along the bases of embankments; they facilitate the drainage of water from the embankment, and may prevent a costly slip and disintegration of the embankment.

65. Effect of sodding the slopes, etc. Engineers are unanimously in favor of rounding off the shoulders and toes of embankments and slopes, sodding the slopes, paving the ditches, and providing tile drains for subsurface drainage, all to be put in during original construction. (See Fig. 43.) Some of the highest grade specifications call for the removal of the top layer of vegetable soil from cuts and from under proposed fills to some convenient place, from which it may be afterwards spread on the slopes, thus facilitating the formation of sod from grass-seed. But while engineers favor these measures and their economic value may be readily demonstrated, it is generally impossible to obtain the authorization of such specifications from railroad directors and promoters. The addition to the original cost of the roadbed is considerable, but is by no means as great as the capitalized value of the extra cost of maintenance resulting from the usual practice. Fig. 43 is a copy of
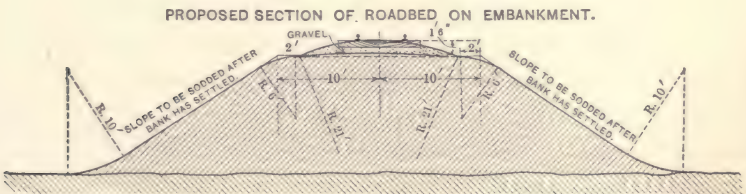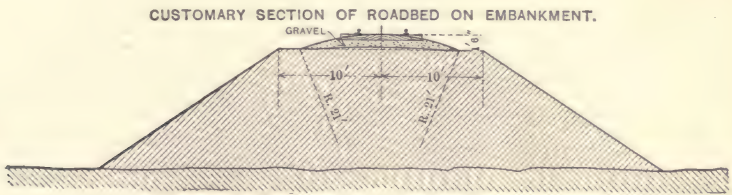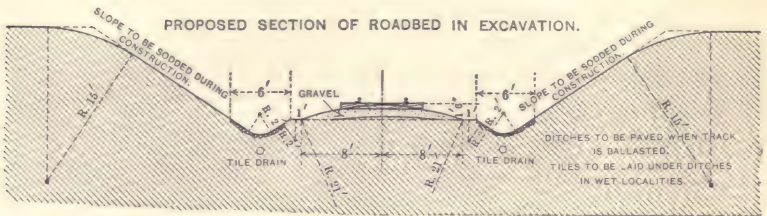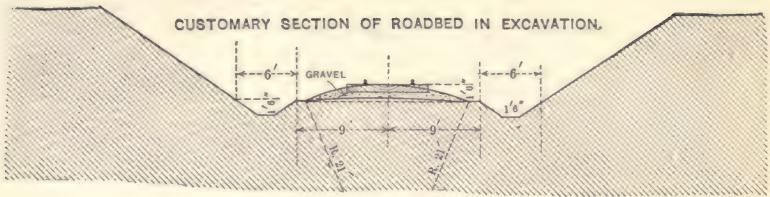
FIG. 43.—" WHITTEMORE ON RAILWAY EXCAVATION AND EMBANKMENTS,"
Trans. Am. Soc. C. E., Sept. 1894

designs * presented at a convention of the American Society of
Civil Engineers by Mr. D. J. Whittemore, Past President of
the Society and Chief Engineer of the Chi., Mil. & St. Paul
R.R. The "customary sections" represent what is, with some
variations of detail, the practice of many railroads. The "pro-
posed sections" elicited unanimous approval. They should be
adopted when not prohibited by financial considerations.

EARTHWORK SURVEYS.

66. Relation of actual volume to the numerical result. It
should be realized at the outset that the accuracy of the result
of computations of the volume of any given mass of earthwork
has but little relation to the accuracy of the mere numerical
work. The process of obtaining the volume consists of two
distinct parts. In the first place it is assumed that the volume
of the earthwork may be represented by a more or less com-
plicated geometrical form, and then, secondly, the volume of
such a geometrical form is computed. A desire for simplicity
(or a frank willingness to accept approximate results) will often
cause the cross-section men to assume that the volume may be
represented by a very simple geometrical form which is really
only a very rough approximation to the true volume. In such
a case, it is only a waste of time to compute the volume with
minute numerical accuracy. One of the first lessons to be
learned is that economy of time and effort requires that the
accuracy of the numerical work should be kept proportional to
the accuracy of the cross-sectioning work, and also that the
accuracy of both should be proportional to the use to be made
of the results. The subject is discussed further in § 94.

67. Prismoids. To compute the volume of earthwork, it is
necessary to assume that it has some geometric form whose vol-
ume is readily determinable. The general method is to consider

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* Trans. Am. Soc. Civil Eng., Sept. 1894.

the volume as consisting of a series of *prismoids*, which are
solids having parallel plane ends and bounded by surfaces which
may be formed by lines moving continuously along the edges of
the bases.   These surfaces may also be considered as the sur-
faces generated by lines moving along the edges joining the cor-
responding points of the bases, these edges being the directrices,
and the lines being always parallel to either base, which is a
plane director.   The surfaces thus developed may or may not
be planes.   The volume of such a prismoid is readily determi-
nable (as explained in § 70 *et seq.*), while its definition is so very
general that it may be applied to very rough ground.   The
" two plane ends " are sections perpendicular to the axis of the
road.   The roadbed and side slopes (also plane) form three of
the side surfaces.   The only approximation lies in the degree of
accuracy with which the plane (or warped) surfaces coincide with
the actual surface of the ground between these two sections.
This accuracy will depend (*a*) on the number of points which
are taken in each cross-section and the accuracy with which the
lines joining these points coincide with the actual cross-sections ;
(*b*) on the skill shown in selecting places for the cross-sections so
that the warped surfaces shall coincide as nearly as possible with
the surface of the ground.   In fairly smooth country, cross-
sections every 100 feet, placed at the even stations, are suf-
ficiently accurate, and such a method simplifies the computations
greatly ; but in rough country cross-sections must be inter-
polated as the surface demands.   As will be explained later,
carelessness or lack of judgment in cross-sectioning will introduce
errors of such magnitude that all refinements in the computations
are utterly wasted.

68. Cross-sectioning.   The process of cross-sectioning con-
sists in determining at any place the intersection by a vertical
plane of the prism of earth lying between the roadbed, the side
slopes, and the natural surface.   The intersection with the road-
bed and side slopes gives three straight lines.   The intersection
with the natural surface is in general an irregular line.   On
smooth regular ground or when approximate results are accept-

able this line is assumed to be straight. According to the irregularity of the ground and the accuracy desired more and more "intermediate points" are taken.

The distance (d in Fig. 44) of the roadbed below (or above) the natural surface at the center is known or determined from
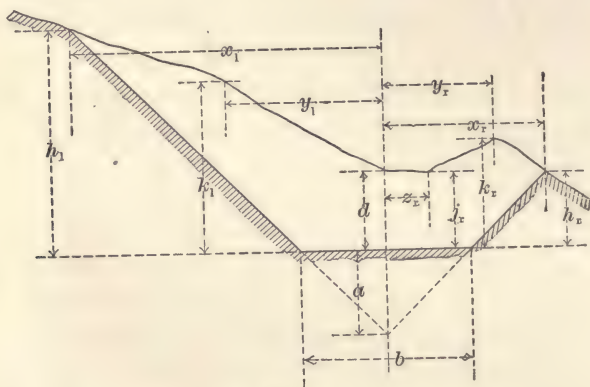
Fig. 44.

the profile or by the computed establishment of the grade line. The distances out from the center of all "breaks" are determined with a tape. To determine the elevations for a cut, set up a level at any convenient point so that the line of sight is higher than any point of the cross-section, and take a rod reading on the center point. This rod reading added to d gives the height of the instrument (H. I.) above the roadbed. Subtracting from H. I. the rod reading at any "break" gives the height of that point above the roadbed (h_l, k_l, h_r, etc.). This is true for all cases in excavation. For fill, the rod reading at center minus d equals the H. I., which may be positive or negative. When negative, add to the "H. I." the rod readings of the intermediate points to get their depths below "grade"; when positive, subtract the "H. I." from the rod readings.

The heights or depths of these intermediate points above or below grade need only be taken to the nearest tenth of a foot, and the distances out from the center will frequently be suffi-

ciently exact when taken to the nearest foot. The roughness of
the surface of farming land or woodland generally renders use-
less any attempt to compute the volume with any greater accu-
racy than these figures would imply unless the form of the ridges
and hollows is especially well defined. The position of the slope-
stake points is considered in the next section. Additional dis-
cussion regarding cross-sectioning is found in § 82.

69. Position of slope-stakes. The slope-stakes are set at the
intersection of the required side slopes with the natural surface,
which depends on the center cut or fill (d). The distance of
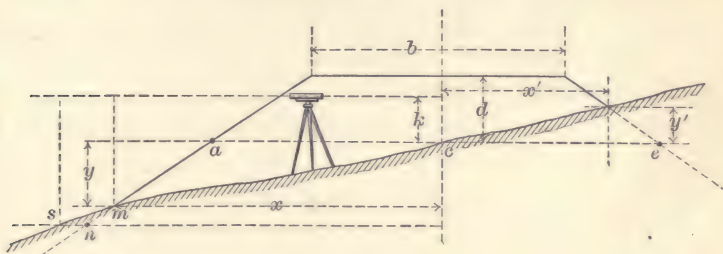
<center>Fig. 45.</center>

the slope-stake from the center for the lower side is $x = \frac{1}{2}b$
$+ s(d + y)$; for the up-hill side it is $x' = \frac{1}{2}b + s(d - y')$.
s is the "slope ratio" for the side slopes, the ratio of horizontal
to vertical. In the above equation both x and y are unknown.
Therefore some position must be found by trial which will sat-
isfy the equation. As a preliminary, the value of x for the
point $a = \frac{1}{2}b + sd$, which is the value of x for *level* cross-
sections. In the case of fills on sloping ground the value of x
on the *down-hill* side is *greater* than this; on the *up-hill* side it
is *less*. The difference in distance is s times the difference of
elevation. Take a numerical case corresponding with Fig. 45.
The rod reading on c is 2.9; $d = 4.2$; therefore the telescope is
$4.2 - 2.9 = 1.3$ *below* grade. $s = 1.5 : 1$, $b = 16$. Hence for
the point a (or for level ground) $x = \frac{1}{2} \times 16 + 1.5 \times 4.2 =$
14.3. At a distance out of 14.3 the ground is seen to be about 3
feet lower, which will not only require $1.5 \times 3 = 4.5$ more, but

enough additional distance so that the added distance shall be 1.5 times the additional drop. As a first trial the rod may be held at 24 feet out and a reading of, say, 8.3 is obtained. 8.3 + 1.3 = 9.6, the depth of the point below grade. The point on the slope line (*n*) which has this depth below grade is at a distance from the center $x = 8 + 1.5 \times 9.6 = 22.4$. The point on the surface (*s*) having that depth is 24 feet out. Therefore the true point (*m*) is nearer the center. A second trial at 20.5 feet out gives a rod reading of, say, 7.1 or a depth of 8.4 below grade. This corresponds to a distance out of 20.6. Since the natural soil (especially in farming lands or woods) is generally so rough that a difference of elevation of a tenth or so may be readily found by slightly varying the location of the rod (even though the distance from the center is the same), it is useless to attempt too much refinement, and so in a case like the above the combination of 8.4 below grade and 20.6 out from center may be taken to indicate the proper position of the slope-stake. This is usually indicated in the form of a fraction, the distance out being the denominator and the height above (or below) grade being the numerator; the fact of *cut* or *fill* may be indicated by *C* or *F*. Ordinarily a second trial will be sufficient to determine with sufficient accuracy the true position of the slope-stake. Experienced men will frequently estimate the required distance out to within a few tenths at the first trial. The left-hand pages of the note-book should have the station number, surface elevation, grade elevation, center cut or fill, and rate of grade. The right-hand pages should be divided in the center and show the distances out and heights above grade of all points, as is illustrated in § 84. The notes should read UP the page, so that when looking ahead along the line the figures are in their proper relative position. The "fractions" farthest from the center line represent the slope-stake points.

COMPUTATION OF VOLUME.

70. Prismoidal formula. Let Fig. 46 represent a triangular prismoid. The two triangles forming the ends lie in *parallel*

planes, but since the angles of one triangle are not equal to the corresponding angles of the other triangle, at least two of the surfaces must be *warped*. If a section, parallel to the bases, is
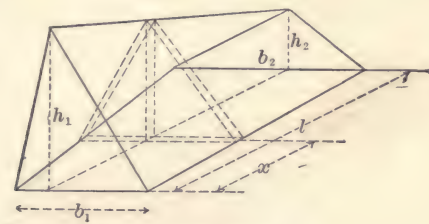
FIG. 46.

made at any point at a distance x from one end, the area of the section will evidently be

$$A_x = \tfrac{1}{2}b_x h_x = \tfrac{1}{2}\Big[b_1 + (b_2 - b_1)\frac{x}{l} \Big]\Big[h_1 + (h_2 - h_1)\frac{x}{l} \Big].$$

The volume of a section of infinitesimal length will be $A_x dx$, and the total volume of the prismoid will be *

$$\int_0^l A_x dx = \tfrac{1}{2}\int_0^l \Big[b_1 + (b_2 - b_1)\frac{x}{l} \Big]\Big[h_1 + (h_2 - h_1)\frac{x}{l} \Big]dx$$

$$= \tfrac{1}{2}\Big[b_1 h_1 x + (b_2 - b_1)h_1\frac{x^2}{2l} + b_1(h_2 - h_1)\frac{x^2}{2l}$$

$$+ (b_2 - b_1)(h_2 - h_1)\frac{x^3}{3l^2}\Big]_0^l$$

$$= \tfrac{1}{2}\Big\{ b_1 h_1 l + [(b_2 - b_1)h_1 + b_1(h_2 - h_1)]\frac{l}{2} + (b_2 - b_1)(h_2 - h_1)\frac{l}{3} \Big\},$$

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* Students unfamiliar with the Integral Calculus may take for granted the fundamental formulæ that $\int dx = x$, that $\int x\,dx = \tfrac{1}{2}x^2$, and that $\int x^2 dx = \tfrac{1}{3}x^3$; also that in integrating between the limits of l and 0 (zero), the value of the integral may be found by simply substituting l for x after integration.

$$\int_0^l A_x dx = \frac{l}{2}[\tfrac{1}{3}b_1h_1 + \tfrac{1}{6}b_1h_2 + \tfrac{1}{6}b_2h_1 + \tfrac{1}{3}b_2h_2]$$

$$= \frac{l}{6}[\tfrac{1}{2}b_1h_1 + \tfrac{1}{2}b_1(h_1+h_2) + \tfrac{1}{2}b_2(h_1+h_2) + \tfrac{1}{2}b_2h_2]$$

$$= \frac{l}{6}\left[\tfrac{1}{2}b_1h_1 + 4\left(\frac{1}{2}\cdot\frac{b_1+b_2}{2}\cdot\frac{h_1+h_2}{2}\right) + \tfrac{1}{2}b_2h_2\right]$$

$$= \frac{l}{6}[A_1 + 4A_m + A_2], \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (45)$$

in which A_1, A_2, and A_m are the areas respectively of the two bases and of the middle section. Note that A_m is *not* the *mean* of A_1 and A_2, although it does not necessarily differ very greatly from it.

The above proof is absolutely independent of the values, absolute or relative, of b_1, b_2, h_1 or h_2. For example, h_2 may be zero and the second base reduces to a line and the prismoid becomes wedge-shaped; or b_2 and h_2 may both vanish, the second base becoming a point and the prismoid reduces to a pyramid Since every prismoid (as defined in § 67) may be reduced to a combination of triangular prismoids, wedges, and pyramids, and since the formula is true for any one of them individually, it is true for all collectively; therefore it may be stated that *

The volume of a prismoid equals one sixth of the perpendicular distance between the bases multiplied by the sum of the areas of the two bases plus four times the area of the middle section.

While it is always possible to compute the volume of any prismoid by the above method, it becomes an extremely complicated and tedious operation to compute the true value of the middle section if the end sections are complicated in form. It

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* The student should note that the derivation of equation (45) does not complete the proof, but that the statements in the following paragraph are logically necessary for a general proof.

therefore becomes a simpler operation to compute volumes by approximate formulæ and apply, if necessary, a correction. The most common methods are as follows :

71. Averaging end areas. The volume of the triangular prismoid (Fig. 46), computed by averaging end areas, is $\frac{l}{2}[\frac{1}{2}b_1h_1 + \frac{1}{2}b_2h_2]$. Subtracting this from the true volume (as given in the equation above, Eq. (45)), we obtain the correction

$$\frac{l}{12}[(b_1 - b_2)(h_2 - h_1)]. \quad . \quad . \quad . \quad (46)$$

This shows that if either the h's or b's are equal, the correction vanishes; it also shows that if the bases are roughly similar and b varies roughly with h (which *usually* occurs, as will be seen later), the correction will be *negative*, which means that the method of averaging end areas *usually* gives *too large* results.

72. Middle areas. Sometimes the middle area is computed and the volume is assumed to be equal to the length times the middle area. This will equal $\frac{l}{2} \times \frac{b_1 + b_2}{2} \times \frac{h_1 + h_2}{2}$. Subtracting this from the true volume, we obtain the correction

$$\frac{l}{24}(b_1 - b_2)(h_1 - h_2). \quad . \quad . \quad . \quad (47)$$

As before, the form of the correction shows that if either the h's or b's are equal, the correction vanishes; also under the *usual* conditions, as before, the correction is *positive* and only one-half as large as by averaging end areas. Ordinarily the labor involved in the above method is no less than that of applying the exact prismoidal formula.

73. Two-level ground. When *approximate* computations of earthwork are sufficiently exact the field-work may be materially reduced by observing simply the center cut (or fill) and the

natural slope α, measured with a clinometer. The area of such a section (see Fig. 48) equals

FIG. 47.

FIG. 48.

$$\tfrac{1}{2}(a + d)(x_l + x_r) - \frac{ab}{2}.$$

But

$$x_l \tan \beta = a + d + x_l \tan \alpha,$$
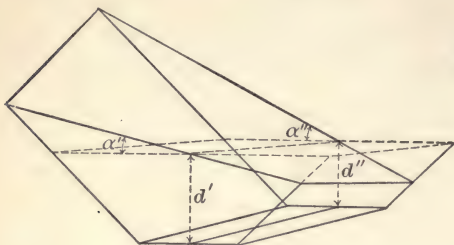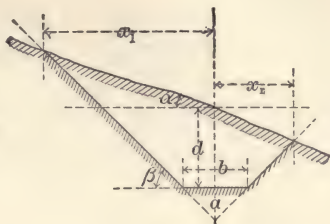
from which

$$x_l = \frac{a + d}{\tan \beta - \tan \alpha}.$$

Similarly,

$$x_r = \frac{a + d}{\tan \beta + \tan \alpha}.$$

Substituting,

$$\text{Area} = (a + d)^2 \frac{\tan \beta}{\tan^2 \beta - \tan^2 \alpha} - \frac{ab}{2}. \quad . \quad . \quad (48)$$

The values α, $\tan \beta$, $\tan^2 \beta$ are constant for all sections, so that it requires but little work to find the area of any section. As this method of cross-sectioning implies considerable approximation, it is generally a useless refinement to attempt to compute the volume with any greater accuracy than that obtained by averaging end areas. It may be noted that it may be easily proved that the correction to be applied is of the same form as that found in § 71 and equals

$$\frac{l}{12}[(x_l' + x_r') - (x_l'' + x_r'')][(d'' + a) - (d' + a)],$$

which reduces to

$$\text{Correction}=\frac{l}{6}\left\{\left[(a+d')\frac{\tan\beta}{\tan^2\beta-\tan^2\alpha'}-(a+d'')\frac{\tan\beta}{\tan^2\beta-\tan^2\alpha''}\right][d''-d']\right\}. \quad (49)$$

When $d'' = d'$ the correction vanishes. This shows that when the center heights are equal there is no correction—regardless of the slope. If the slope is uniform throughout, the form of the correction is simplified and is invariably *negative*. Under the usual conditions the correction is *negative*, i.e., the method *generally* gives *too large* results.

74. Level sections. When the country is very level or when only approximate preliminary results are required, it is sometimes assumed that the cross-sections are level. The method of level sections is capable of easy and rapid computation. The area may be written as

$$(a + d)^2 s - \frac{ab}{2}. \quad . \quad . \quad . \quad . \quad (50)$$
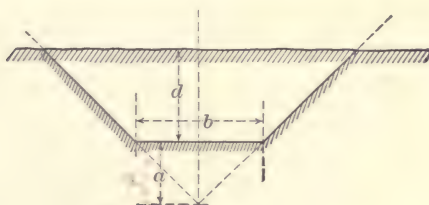
Fig. 49.

This also follows from Eq. (48) when $\alpha = 0$ and $\tan\beta = \frac{1}{s}$. s here represents the "slope ratio," *i.e.*, the ratio of the horizontal projection of the slope to the vertical. A table is very readily formed giving the area in square feet of a section of given depth and for any given width of roadbed and ratio of side-slopes. The area may also be readily determined (as illustrated in the following example) without the use of such a table; a table of squares will facilitate the work. Assuming

the cross-sections at equal distances ($= l$) apart, the total approximate volume for any distance will be

$$\frac{l}{2}[A_0 + 2(A_1 + A_2 + \ldots A_{n-1}) + A_n]. \quad . \quad (51)$$

The prismoidal correction may be directly derived from Eq. (46) as $\frac{l}{12}[2(a + d')s - 2(a + d'')s][(a + d'') - (a + d')]$, which reduces to

$$-\frac{ls}{6}(d' - d'')^2 \quad \text{or} \quad -\frac{l}{12}\frac{b}{a}(d' - d'')^2. \quad . \quad (52)$$

This may also be derived from Eq. (49), since $\alpha = 0$, $\tan \alpha = 0$, and $\tan \beta = 2a \div b$. This correction is *always* negative, showing that the method of averaging end areas, when the sections are level, always gives too large results. The prismoidal correction for any one prismoid is therefore a constant times the *square* of a difference. The squares are always positive whether the differences are positive or negative. The correction therefore becomes

$$-\frac{l}{12}\frac{b}{a}\Sigma(d' \sim d'')^2. \quad . \quad . \quad . \quad . \quad (53)$$

75. Numerical example: level sections. Given the following center heights for the same number of consecutive stations 100 feet apart; width of roadbed 18 feet; slope $1\frac{1}{2}$ to 1.

The products in the fifth column may be obtained very readily and with sufficient accuracy by the use of the slide-rule described in § 79. The products should be considered as $(a + d)(a + d) \div \frac{1}{s}$. In this problem $s = 1\frac{1}{2}, \frac{1}{s} = .6667$. To apply the rule to the first case above, place 6667 on scale B over 89 on scale A, then opposite 89 on scale B will be found

118.8 on scale A. The position of the decimal point will be evident from an approximate mental solution of the problem.

Sta.	Center Height.	$a + d$	$(a + d)^2$	$(a + d)^2s$	Areas.		$d' \sim d''$	$(d' \sim d'')^2$
17	2.9	8.9	79.21	118.81		118.81	1.8	3.24
18	4.7	10.7	114.49	171.74		343.48	2.1	4.41
19	6.8	12.8	163.84	245.76	$\times 2 =$	491.52	4.9	24.01
20	11.7	17.7	313.29	469.93		939.86	7.5	56.25
21	4.2	10.2	104.04	156.06		312.12	2.6	6.76
22	1.6	7.6	57.76	86.64		86.64		

$$\frac{ab}{2} = \frac{6 \times 18}{2} = 54 \qquad\qquad 10 \times 54 = \begin{array}{r} 2292.43 \\ 540 \\ \hline 1752.43 \end{array} \qquad 94.67$$

$$\frac{1752.43 \times 100}{2 \times 27} = 3245 \text{ cub. yards} \quad = \text{approx. vol.}$$

$$\text{Corr.} = -\frac{100 \times 18}{12 \times 6 \times 27} \times 94.67 \quad = -91 \text{ cub. yds.}$$

$$3245 - 91 = 3154 \text{ cub. yds.} \quad = \text{exact volume.}$$

The above demonstration of the correction to be applied to the approximate volume, found by averaging end areas, is introduced mainly to give an idea of the amount of that correction. Absolutely level sections are practically unknown, and the error involved in assuming any given sections as truly level will ordinarily be greater than the computed correction. If greater accuracy is required, more points should be obtained in the cross-sectioning, which will generally show that the sections are not truly level.

76. Equivalent sections. When sections are very irregular the following method may be used, especially if great accuracy is not required. The sections are plotted to scale and then a uniform slope line is obtained by stretching a thread so that the undulations are averaged and an *equivalent section* is obtained. The *center depth* (d) and the *slope angle* (α) of this line *can* be obtained from the drawing, but it is more convenient to measure the distances (x_l and x_r) from the center. The area

may then be obtained independent of the center depth as follows: Let s = the slope ratio of the side slopes = $\cot \beta = \dfrac{b}{2a}$. (See Fig. 48.)   Then the

$$\text{Area} = \frac{1}{2}\left(\frac{x_l + x_r}{s}\right)(x_l + x_r) - \frac{x_r}{s}\frac{x_r}{2} - \frac{x_l}{s}\frac{x_l}{2} - \frac{ab}{2}$$

$$= \frac{x_l x_r}{s} - \frac{ab}{2}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (54)$$

The true volume, according to the prismoidal formula, of a length of the road measured in this way will be

$$\frac{l}{6}\left[\frac{x_l' x_r'}{s} - \frac{ab}{2} + 4\left(\frac{x_l' + x_l''}{2}\frac{x_r' + x_r''}{2}\frac{1}{s} - \frac{ab}{2}\right) + \frac{x_l'' x_r''}{s} - \frac{ab}{2}\right].$$

If computed by averaging end areas, the approximate volume will be

$$\frac{l}{2}\left[\frac{x_l' x_r'}{s} - \frac{ab}{2} + \frac{x_l'' x_r''}{s} - \frac{ab}{2}\right].$$

Subtracting this result from the true volume, we obtain as the correction

$$\text{Correction} = \frac{l}{6s}(x_l'' - x_l')(x_r' - x_r''). \quad . \quad . \quad (55)$$

This shows that if the side distances to either the right or left are equal at adjacent stations the correction is *zero*, and also that if the difference is small the correction is also small and very probably within the limit of accuracy obtainable by that method of cross-sectioning. In fact, as has already been shown in the latter part of § 75, it will usually be a useless refinement to compute the prismoidal correction when the method of cross-sectioning is as rough and approximate as this method generally is.

77. Equivalent level sections. These sloping "two-level" sections are sometimes transformed into "level sections of equal area," and the volume computed by the method of level sections (§ 74). But the true volume of a prismoid with sloping ends does not agree with that of a prismoid with equivalent bases and level ends except under special conditions, and when this method is used a correction must be applied if accuracy is desired, although, as intimated before, the assumption that the sections have uniform slopes will frequently introduce greater inaccuracies than that of this method of computation. The following demonstration is therefore given to show the scope and limitations of the errors involved in this much used method.

In Fig. 50, let d_1 be the center height which gives an
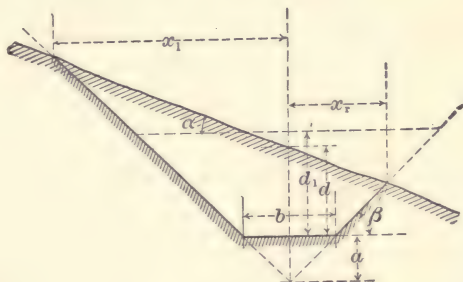
FIG. 50.

equivalent level section. The area will equal $(a + d_1)^2 s - \dfrac{ab}{2}$, which must equal the area given in § 76, $\dfrac{x_l x_r}{s} - \dfrac{ab}{2}$. $s = \dfrac{b}{2a}$.

$$\therefore (a + d_1)^2 s = \frac{x_l x_r}{s},$$

$$\text{or} \quad a + d_1 = \frac{\sqrt{x_l x_r}}{s}. \quad \cdot \quad \cdot \quad \cdot \quad \cdot \quad (56)$$

To obtain d_1 directly from notes, given in terms of d and α,

we may substitute the values of x_l and x_r given in § 73, which gives

$$a + d_1 = (a + d) \frac{\tan \beta}{\sqrt{\tan^2 \beta - \tan^2 \alpha}} = \frac{a + d}{\sqrt{1 - s^2 \tan^2 \alpha}}. \quad (57)$$

The *true* volume of the equivalent section may be represented by

$$\frac{ls}{6}\left[(a + d_1')^2 + 4\left(\frac{a + d_1'}{2} + \frac{a + d_1''}{2}\right)^2 + (a + d_1'')^2 \right].$$

From this there should be subtracted the volume of the " grade prism " under the roadbed to obtain the volume of the cut that would be actually excavated, but in the following comparison, as well as in other similar comparisons elsewhere made, the volume of the grade prism invariably cancels out, and so for the sake of simplicity it will be disregarded. This expression for volume may be transposed to

$$\frac{ls}{6}\left[\frac{x_l' x_r'}{s^2} + 4\left(\frac{\sqrt{x_l' x_r'}}{2s} + \frac{\sqrt{x_l'' x_r''}}{2s}\right)^2 + \frac{x_l'' x_r''}{s^2} \right].$$

The true volume of the prismoid with sloping ends is (see § 76)

$$\frac{l}{6}\left[\frac{x_l' x_r'}{s} + 4\left(\left(\frac{x_l' + x_l''}{2}\right)\left(\frac{x_r' + x_r''}{2}\right)\frac{1}{s}\right) + \frac{x_l'' x_r''}{s} \right].$$

The difference of the two volumes

$$= \frac{l}{6s}(x_l' x_r' + x_l'' x_r' + x_l' x_r'' + x_l'' x_r'' - x_l' x_r' - 2\sqrt{x_l' x_r' x_l'' x_r''} - x_l'' x_r'')$$

$$= \frac{l}{6s}(\sqrt{x_l' x_r''} - \sqrt{x_l'' x_r'})^2. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (58)$$

This shows that " equivalent level sections " do *not* in general give the true volume, there being an exception when

$x_l'x_r'' = x_l''x_r'$.   This condition is fulfilled when the slope is uniform, i.e., when $\alpha' = \alpha''$.   When this is nearly so the error is evidently not large.   On the other hand, if the slopes are inclined in opposite directions the error may be very considerable, particularly if the angles of slope are also large.

78. Three-level sections.   The next method of cross-sectioning in the order of complexity, and therefore in the order of
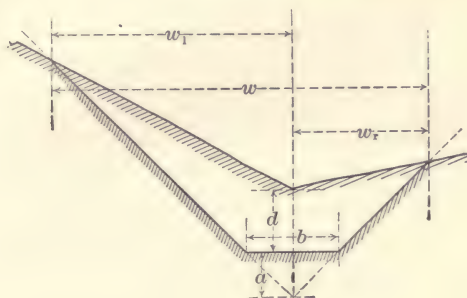
FIG. 51.

accuracy, is the method of three-level sections.   The area of the section is $\frac{1}{2}(a + d)(w_r + w_l) - \frac{ab}{2}$, which may be written $\frac{1}{2}(a + d)w - \frac{ab}{2}$, in which $w = w_r + w_l$.   If the volume is computed by averaging end areas, it will equal

$$\frac{l}{4}[(a + d')w' - ab + (a + d'')w'' - ab]. \quad . \quad . \quad (59)$$

If we divide by 27 to reduce to cubic yards, we have, when $l = 100$,

$$\text{Vol}_{(\ldots, \prime\prime)} = \tfrac{25}{27}(a + d')w' - \tfrac{25}{27}ab + \tfrac{25}{27}(a + d'')w'' - \tfrac{25}{27}ab.$$

For the next section

$$\text{Vol}_{(\prime\prime \ldots \prime\prime\prime)} = \tfrac{25}{27}(a + d'')w'' - \tfrac{25}{27}ab + \tfrac{25}{27}(a + d''')w''' - \tfrac{25}{27}ab.$$

For a partial station length compute as usual and multiply result by $\dfrac{\text{length in feet}}{100}$. The prismoidal correction may be obtained by applying Eq. (46) to each side in turn. For the left side we have

$$\frac{l}{12}[(a + d') - (a + d'')](w_l'' - w_l'), \quad \text{which equals}$$

$$\frac{l}{12}(d' - d'')(w_l'' - w_l').$$

For the right side we have, similarly,

$$\frac{l}{12}(d' - d'')(w_r'' - w_r').$$

The total correction therefore equals

$$\frac{l}{12}(d' - d'')[(w_l'' + w_r'') - (w_l' + w_r')]$$

$$= \frac{l}{12}(d' - d'')(w'' - w').$$

Reduced to cubic yards, and with $l = 100$,

$$\text{Pris. Corr.} = \tfrac{25}{81}(d' - d'')(w'' - w'). \quad . \quad . \quad (60)$$

When this result is compared with that given in Eq. (55) there is an apparent inconsistency. If two-level ground is considered as but a special case of three-level ground, it would seem as if the same laws should apply. If, in Eq. (55), $x_r' = x_r''$, and x_l'' is different from x_l', the equation reduces to zero; but in this case d' would also be different from d''; and since $x_l' + x_r'$ would $= w'$, and $x_l'' + x_r'' = w''$ in Eq. (60), $w'' - w'$ would not equal zero and the correction would be some finite quantity and not zero. The explanation lies in the difference in the form and volume of the prismoids, according to the method of the

formation of the warped surfaces. If the surface is supposed to be generated by the locus of a line moving parallel to the ends as plane directors and along two *straight* lines lying *in* the side-slopes, then $x_l^{mid.}$ will equal $\frac{1}{2}(x_l' + x_l'')$, and $x_r^{mid.}$ will equal $\frac{1}{2}(x_r' + x_r'')$, but the profile of the center line will *not* be straight and $d^{mid.}$ will *not* equal $\frac{1}{2}(d' + d'')$. On the other hand, if the surfaces be generated by *two* lines moving parallel to the ends as plane directors and along a *straight* center line and straight side lines lying in the slopes, a warped surface will be generated each side of the center line, which will have uniform slopes on each side of the center at the two ends and *nowhere else*. This shows that when the upper surface of earthwork is warped (as it generally is), two-level ground should not be considered as a special case of three-level ground. This discussion, however, is only valuable to explain an apparent inconsistency and error. The method of two-level ground should only be used when such refinements as are here discussed are of no importance as affecting the accuracy.

The following example is given to illustrate the method of three-level sections.

Station.	Center.	Left.	Right.	$a + d$	w	Yards.		$d' - d''$	$w'' - w'$	Pris. Corr.	$x_l \sim x_r$	$\dfrac{V(x_l \sim x_r)}{3R}$	Curv. Corr.*
17	2.6F	$\dfrac{10.6F}{22.9}$	$\dfrac{0.8F}{8.2}$	7.3	31.1	210					14.7	$+1$	
18	8.1F	$\dfrac{15.8F}{30.7}$	$\dfrac{3.4F}{12.1}$	12.8	42.8	507	595	-5.5	$+11.7$	-20	18.6	$+3$	$+4$
+40	10.7F	$\dfrac{20.2F}{37.3}$	$\dfrac{4.8F}{14.2}$	15.4	51.5	734	448	-2.6	$+8.7$	-3	23.1	$+6$	$+4$
19	6.4F	$\dfrac{14.0F}{28.0}$	$\dfrac{2.1F}{10.1}$	11.1	38.1	392	602	$+4.3$	-13.4	-11	17.9	$+2$	$+5$
20	3.7F	$\dfrac{5.8F}{15.7}$	$\dfrac{0.2F}{7.3}$	8.4	23.0	179	449	$+2.7$	-15.1	-13	8.4	$+1$	$+3$

Roadbed, 14' wide in fill. Approx. Vol. = 2094 -47 $+16$
Slope 1½ to 1. Pris. corr. $= 47$

$a = \dfrac{b}{2s} = \dfrac{14}{3} = 4.7$; True Vol. $= 2047$ (disregarding curv. corr.).*

$\dfrac{25}{27} ab = 61.$ * For the Derivation of the curvation correction, see § 93.

In the first column of yards

$$210 = \tfrac{25}{27}(a + d)w = \tfrac{25}{27} \times 7.3 \times 31.1;$$

$$507, \; 734, \quad \text{etc., are found similarly;}$$

$$595 = 210 - 61 + 507 - 61;$$

$$448 = \tfrac{40}{100}(507 - 61 + 734 - 61);$$

$$602 = \tfrac{60}{100}(734 - 61 + 392 - 61);$$

$$449 = 392 - 61 + 179 - 61.$$

For the prismoidal correction,

$$- 20 = \tfrac{25}{81}(d' - d'')(w'' - w') = \tfrac{25}{81}(2.6 - 8.1)(42.8 - 31.1)$$

$$= \tfrac{25}{81}(- 5.5)(+ 11.7).$$

For the next line, $- 3 = \tfrac{40}{100}[\tfrac{25}{81}(- 2.8)(+ 8.7)]$, and similarly for the rest. The " F " in the columns of center heights, as well as in the columns of " right " and " left," are inserted to indicate *fill* for all those pooints. Cut would be indicated by " C."

79. Computation of products. The quantities $\frac{25}{27}(a + d)w$ and $\frac{25}{27}ab$ represent in each case the product of two variable terms and a constant. These products are sometimes obtained from tables which are calculated for all ordinary ranges of the variable terms as arguments. A similar table computed for $\frac{25}{81}(d' - d'')(w'' - w')$ will assist similarly in computing the prismoidal correction. Prof. Charles L. Crandall, of Cornell University, is believed to be the first to prepare such a set of tables, which were first published in 1886 in "Tables for the Computation of Railway and Other Earthwork." Another

easy method of obtaining these products is by the use of a slide-
rule.   A slide-rule has been designed by the author to accom-
pany this volume.   It is designed particularly for this special
work, although it may be utilized for many other purposes for
which slide-rules are valuable.   To illustrate its use, suppose
$(a + d) = 28.2$, and $w = 62.4$; then

$$\frac{25}{27}(a + d)w = \frac{28.2 \times 62.4}{1.08}.$$

Set 108 (which, being a constant of frequent use, is specially
marked) on the sliding scale (B) opposite 282 on the other scale
(A), and then opposite 624 on scale B will be found 1629 on
scale A, the 162 being read directly and the 9 read by estima-
tion.   Although strict rules may be followed for pointing off
the final result, it only requires a very simple mental calculation
to know that the result must be 1629 rather than 162.9 or
16290.   For products less than 1000 cubic yards the result
may be read directly from the scale; for products between 1000
and 5000 the result may be read directly to the nearest 10
yards, and the tenths of a division estimated.   Between 5000 and
10,000 yards the result may be read directly to the nearest 20
yards, and the fraction estimated; but prisms of such volume
will never be found as simple triangular prisms—at least, an as-
sumption that any mass of ground was as regular as this would
probably involve more error than would occur from faulty esti-
mation of fractional parts.   Facilities for reading as high as
10,000 cubic yards would not have been put on the scale ex-
cept for the necessity of finding such products as $\frac{25}{27}(9.1 \times 9.5)$,
for example.   This product would be read off from the same
part of the rule as $\frac{25}{27}(91 \times 95)$.   In the first case the product
(80.0) could be read directly to the nearest .2 of a cubic yard,
which is unnecessarily accurate.   In the other case, the prod-
uct (8004) could only be obtained by estimating $\frac{4}{20}$ of a division.

The computation for the prismoidal correction may be made

similarly except that the divisor is 3.24 instead of 1.08. For example, $\frac{25}{81}(5.5 \times 11.7) = \dfrac{5.5 \times 11.7}{3.24}$. Set the 324 on scale *B* (also specially marked like 108) opposite 55 on scale *A*, and proceed as before.

80. Five-level sections. Sometimes the elevations over each edge of the roadbed are observed when cross-sectioning. These are distinctively termed "five-level sections." If the center, the slope-stakes, and *one* intermediate point on each side (*not* necessarily over the edge of the roadbed) are observed, it is termed an "irregular section." The field-work of cross-sectioning five-level sections is no less than for irregular sections with one intermediate point; the computations, although capable of peculiar treatment on account of the location of the intermediate point, are no easier, and in some respects more laborious; the cross-sections obtained will not in general represent the actual cross-sections as truly as when there is perfect freedom in locating the intermediate point; as it is generally inadvisable or unnecessary to employ five-level sections throughout the length of a road, the change from one method to another adds a possible element of inaccuracy and loses the advantage of uniformity of method, particularly in the notes and *form* of computations. On these accounts the method will not be further developed, except to note that this case, as well as any other, may be solved by dividing the whole prismoid into triangular prismoids, computing the volume by averaging end areas, and computing the prismoidal correction by adding the computed corrections for each elementary triangular prismoid.

81. Irregular sections. In cross-sectioning irregular sections, the distance from the center and the elevation above "grade" of every "break" in the cross-section must be observed. The area of the irregular section may be obtained by computing the area of the trapezoids (*five*, in Fig. 44) and subtracting the two external triangles. For Fig. 44 the area would be

$$\frac{h_l + k_l}{2}(x_l - y_l) + \frac{k_l + d}{2}y_l + \frac{d + j_r}{2}z_r + \frac{j_r + k_r}{2}(y_r - z_r)$$

$$+ \frac{k_r + h_r}{2}(x_r - y_r) - \frac{h_l}{2}\left(x_l - \frac{b}{2}\right) - \frac{h_r}{2}\left(x_r - \frac{b}{2}\right).$$
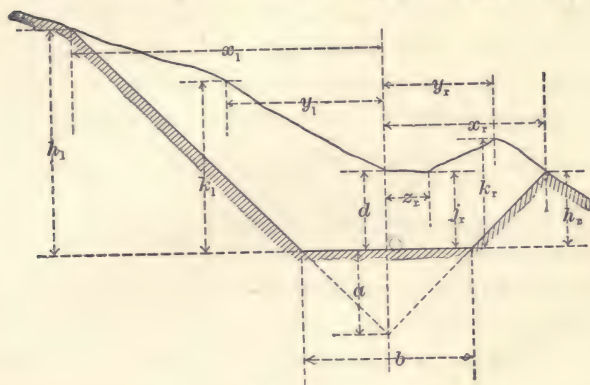
Fig. 44.

Expanding this and collecting terms, of which many will cancel, we obtain

$$\text{AREA} = \frac{1}{2}\left[x_l k_l + y_l(d - h_l) + x_r k_r + y_r(j_r - h_r) \right.$$

$$\left. + z_r(d - k_r) + \frac{b}{2}(h_l + h_r) \right]. \quad . \quad . \quad (61)$$

An examination of this formula will show a perfect regularity in its formation which will enable one to write out a similar formula for any section, no matter how irregular or how many points there are, without any of the preliminary work. The formula may be expressed in words as follows:

AREA equals one-half the sum of products obtained as follows:

the distance to each slope-stake times the height above grade of the point next inside the slope-stake;

the distance to each intermediate point in turn times the height of the point just inside minus the height of the point just outside;

finally, one-half the width of the roadbed times the sum of the slope-stake heights.

If one of the sides is perfectly regular from center to slope-stake, it is easy to show that the rule holds literally good. The " point next inside the slope-stake " in this case is the center; the intermediate terms for that side vanish. The *last term* must always be used. The rule holds good for three-level sections, in which case there are three terms, which may be reduced to two. Since these two terms are both variable quantities for each cross-section, the special method, given in § 78, in which one term $\left(\dfrac{ab}{2}\right)$ is a constant for all sections, is preferable. In the general method, each intermediate " break " adds another term.

82. Volume of an irregular prismoid. If there is a break at one cross-section which is not represented at the next, the ridge (or hollow) implied by that break is supposed to " vanish " at the next section. In fact, the volume will not be correctly
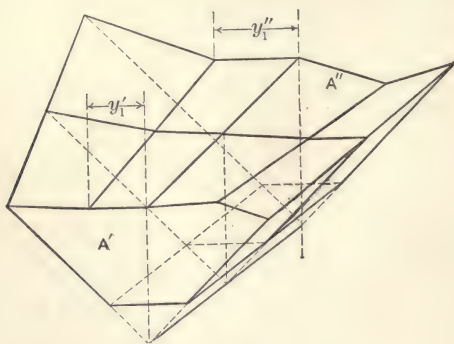
FIG. 52.

represented unless a cross-section is taken at the point where the ridge or hollow "vanishes" or "runs out." To obtain the true prismoidal correction it is necessary to observe on the ground the place where a break in an adjacent section, which is not represented in the section being taken, runs out. For example, in Fig. 52, the break on the left of section A'', at a

distance of y_l'' from the center, is observed to run out in section
A' at a distance of y_l' from the center. The volume of the
prismoid, computed by the prismoidal formula as in § 70, will
involve the midsection, to obtain the dimension of which will
require a laborious computation. A simpler process is to compute
the volume by averaging end areas as in § 81 and apply a
prismoidal correction. To do this write out an expression for
each end area similar to that given in Eq. 61. The sum of
these areas times $\frac{l}{2}$ gives the approximate volume. As before,

for partial station lengths, multiply the result by $\dfrac{\text{length in feet}}{100}$.

There will be no constant subtractive term, $\frac{25}{27}ab$, as in § 78.
The *true* prismoidal correction may be computed, as in § 83, or
the following approximate method may be used : Consider the
irregular section to be three-level ground *for the purpose of
computing the correction* only. This has the advantage of less
labor in computation than the use of the true prismoidal correc-
tion, and although the error involved may be considerable in
individual sections, the error is as likely to be positive as nega-
tive, and in the long run the error will not be large and generally
will be much less than would result by the neglect of any
prismoidal correction.

83. True prismoidal correction for irregular prismoids. As
intimated in § 82, each cross-section should be assumed to have
the same number of sides as the adjacent cross-section when
computing the prismoidal correction. This being done, it per-
mits the division of the whole prismoid into elementary triangu-
lar prismoids, the dimensions of the bases of which being given
in each case by a vertical distance above grade line and by the
horizontal distance between two adjacent breaks. The summa-
tion of the prismoidal corrections for each of the elementary
triangular prismoids will give the true prismoidal correction.
Assuming for an example the cross-section of Fig. 44, with a
cross-section of the same number of sides, and with dimensions

similarly indicated, for the other end, the prismoidal correction becomes (see Eq. 46)

$$
\frac{l}{12}\Bigg[(h_l' - h_l'')[(x_l'' - y_l'') - (x_l' - y_l')] + (k_l' - k_l'')[(x_l'' - y_l'') - (x_l' - y_l')]
$$

$$
+ (k_l' - k_l'')(y_l'' - y_l') + (d' - d'')(y_l'' - y_l') + (d' - d'')(z_r'' - z_r')
$$
$$
+ (j_{r'} - j_{r}'')(z_r'' - z_r') + (j_{r}' - j_{r}'')[(y_{r}'' - z_r'') - (y_{r}' - z_r')]
$$
$$
+ (k_{r}' - k_{r}'')[(y_{r}'' - z_{r}'') - (y_{r}' - z_r')]
$$
$$
+ (k_{r}' - k_{r}'')[(x_{r}'' - y_{r}'') - (x_{r}' - y_{r}')] + (h_{r}' - h_{r}'')[(x_{r}'' - y_{r}'') - (x_{r}' - y_{r}')]
$$
$$
- (h_l' - h_l'')\Big[\Big(x_{l}'' - \frac{b}{2}\Big) - \Big(x_l' - \frac{b}{2}\Big) \Big] - (h_{r}' - h_{r}'')\Big[\Big(x_{r}'' - \frac{b}{2}\Big) - \Big(x_{r}' - \frac{b}{2}\Big) \Big] \Bigg].
$$

Expanding this and collecting terms, of which many will cancel, we obtain

$$
\text{Pris. Corr.} = \frac{l}{12}\Big[(x_l'' - x_l')(k_l' - k_l'') + (y_l'' - y_l')[(d' - h_l') - (d'' - h_l'')]
$$
$$
+ (x_r'' - x_r')(k_r' - k_r'') + (y_r'' - y_r')[(j_r' - h_r') - (j_r'' - h_r'')]
$$
$$
+ (z_r'' - z_r')[(d' - k_r') - (d'' - k_r'')] \Big]. \quad \ldots \ldots \ldots \quad (62)
$$

By comparing this equation with Eq. 61 a remarkable coincidence in the law of formation may be seen, which enables this formula to be written by mere inspection and to be applied numerically with a minimum of labor from the computations for end areas, as will be shown (§ 84) by a numerical example. For each term in Eq. 61, as, for example, $y_r(j_r - h_r)$, there is a correction term in Eq. 62 of the form

$$
(y_r'' - y_r')[(j_r' - h_r') - (j_r'' - h_r'')].
$$

Each one of these terms (y_r'', y_r', $(j_r' - h_r')$, and $(j_r'' - h_r'')$) has been previously used in finding the end areas and has its place in the computation sheet. The summation of the products of these differences times a constant gives the total true prismoidal correction in cubic yards for the whole prismoid considered.

The *constant* is the same as that computed in § 78, i.e., $\frac{25}{81}$.

84. Numerical example; irregular sections; volume, with true prismoidal correction.

Sta.	Center { cut or fill.	Left.			Right.	
19	0.6c	$\dfrac{3.6c}{14.4}$	$\left(\dfrac{2.3c}{8.2}\right)$	$\left(\dfrac{1.8c}{6.0}\right)$	$\dfrac{0.1c}{4.2}$	$\dfrac{0.4c}{9.6}$
18	2.3c	$\dfrac{4.2c}{15.3}$	$\dfrac{6.8c}{8.4}$	$\dfrac{3.2c}{5.2}$	$\left(\dfrac{1.9c}{3.6}\right)$	$\dfrac{1.2c}{10.8}$
17	7.6c	$\dfrac{8.2c}{21.3}$	$\dfrac{10.2c}{17.4}$	$\dfrac{8.0c}{6.1}$	$\left(\dfrac{5.8c}{8.0}\right)$	$\dfrac{4.2c}{15.3}$
+ 42	10.2c	$\dfrac{12.2c}{27.3}$	$\left(\dfrac{12.3c}{22.0}\right)$	$\dfrac{12.6c}{8.2}$	$\dfrac{6.2c}{7.5}$	$\dfrac{8.4c}{21.6}$
16	6.8c	$\dfrac{8.9c}{22.4}$		$\dfrac{7.6c}{12.0}$	$\dfrac{3.2c}{4.1}$	$\dfrac{2.6c}{12.9}$

Roadbed 18 feet wide in cut; slope 1½ to 1.

The figures in the bracket $\left(\dfrac{12.3c}{22.0}\right)$ mean that it was noted in the field that the break, indicated at Sta. 17 as being 17.4 to the left, ran out at Sta. 16 + 42 at 22.0 to the left. By interpolation between 8.2 and 27.3 the height of this point is *computed* as 12.3. The quantities in the other brackets are obtained similarly. These quantities are only used when the computation of the true prismoidal correction is desired. They are not needed in computing the volume by averaging end areas, nor are they used at all if the prismoidal correction is to be obtained by assuming (*for this purpose*) the ground to be *three-level* ground.

In the tabular form on page 98 the figures within the braces ($\smile\smile$) are NOT used in computing the volume, but are only used to obtain the *differences* of widths or heights with which to compute the *true* prismoidal correction. It may be noted, as a check, that the volume, computed from these figures in the braces, is the same as that computed from the other figures.

VOLUME OF IRREGULAR PRISMOID, WITH TRUE PRISMOIDAL CORRECTION.

Sta.	Width.	Height.	Yards.		True pris. corr.		
					$w'' - w'$	$h' - h''$	Yards.
16	L⌈22.4 ⌊12.0 12.9⌉R 4.1 9.0	7.6 − 2.1 3.2 4.2 11.5	158 − 23 40 16 96				
+ 42	L⌈27.3 ⌊8.2 L{27.3 22.0 8.2 21.6⌉R 7.5 9.0	12.6 − 2.0 12.3 0.4 − 2.1 6.2 1.8 20.6	319 − 15 124 13 172	378	+ 4.9 − 3.8 + 8.7 + 3.4	− 5.0 − 0.1 − 3.0 + 2.4	− 7 0 − 8 + 3 (− 5)
17	L⌈21.3 ⌊17.4 6.1 15.3}R 8.0 15.3]R 9.0	10.2 − 0.2 − 2.6 5.8 3.4 7 6 12.4	201 − 3 − 14 107 103	584	− 6.0 − 4.6 − 2.1 − 6.3 + 0.5	+ 2.1 + 0.6 + 0.5 + 0.4 − 1.6	− 4 − 1 0 − 1 0 (− 3)
18	L⌈15.3 ⌊8.4 5.2 10.8]R 10.8}R 3.6 9.0	6.8 − 1.0 − 4.5 2.3 1.9 1.1 5.4	95 − 7 − 22 23 45	528	− 6.0 − 9.0 − 0.9 − 4.5	+ 3.4 + 0.8 + 1.9 + 5.3	− 6 − 2 − 1 − 7 (− 16)
19	L⌈14.4 14.4 L{8.2 6.0 9.6⌉R 4.2 9.0	0.6 2.3 − 1.8 − 1.7 0.1 0.2 4.0	8 1 1 33	177	− 0.9 − 0.2 + 0.8 − 1.2 + 0.6	+ 4.5 + 0.8 − 2.8 + 1.8 + 0.9	− 1 0 − 1 − 1 0 (− 3)

Approx. vol.   = 1667   − 27

True pris. corr. = − 27

True volume   = 1640 cubic yards

The figures within each brace (or bracket) constitute a group which must be used in connection with a group which has the same number of points, on the same side of the center, in the *next* cross-section, previous or succeeding. In the column of

" Yards " under "True pris. corr.," we have, for example,
$(-5) = \frac{42}{100}(-7 + 0 - 8 + 3)$.

85. Volume of irregular prismoid, with approximate prismoidal correction. If the prismoidal correction is obtained approximately, by the method outlined in § 82, the process will be as shown in the tabular form. Not only is the numerical work considerably less than the exact method, but the discrepancy in cubic yards is almost insignificant.

Sta.	Width.	Height.	Yards.		Cen. Height.	Total width.	$d'-d''$	$w''-w'$	Approx. pris. corr.
16	22.4 12.0 12.9 4.1 9.0	7.6 − 2.1 3.2 4.2 11.5	158 − 23 40 16 96		+ 6.8	35.3			
+ 42	27.3 8.2 21.6 7.5 9.0	12.6 − 2.0 6.2 1.8 20.6	319 − 15 124 13 172	378	+ 10.2	48.9	− 3.4	+ 13.6	− 14 (− 6)
17	21.3 17.4 6.1 15 3 9.0	10.2 − 0.2 − 2.6 7.6 12.4	201 − 3 − 14 107 103	584	+ 7.6	36.6	+ 2.6	− 12.3	− 10 (− 6)
18	15.3 8.4 5.2 10.8 9.0	6.8 − 1.0 − 4.5 2.3 5.4	95 − 7 − 22 23 45	528	+ 2.3	26.1	+ 5.3	− 10.5	− 17 (− 17)
19	14.4 9.6 4.2 9.0	0.6 0.1 0.2 4.0	8 1 1 33	177	+ 0.6	24.0	+ 1.7	− 2.1	− 1 (− 1)

Approx. volume   = 1667   − 30
Approx. pris. corr. = − 30

Corrected volume   = 1637 cubic yards

86. Illustration of value of approximate rules. The accompanying tabulation shows that when the volume of an irregular prismoid is computed by averaging end areas and is corrected by considering the ground as three-level ground (*for the pur-*

poses of the correction only), the error for the different sections is sometimes positive and sometimes negative, and in this case

Sections.	True volume.	Approx. vol. by averaging end areas.	Difference or *true* pris. corr.	Approx. pris. corr. on basis of three-level ground.	Error.	Approx. vol., computed from center and side heights *only.*	Error.
16........16 + 42	373	378	− 5	− 6	− 1	396	+ 23
16 + 42...17	581	584	− 3	− 6	− 3	577	− 4
17.........18	512	528	− 16	− 17	− 1	463	− 49
18 19	174	177	− 3	− 1	+ 2	147	− 27
	1640	1667	− 27	− 30	− 3	1583	− 57

amounts to only 3 yards in 1640—less than $\frac{1}{5}$ of 1%. If the prismoidal correction had been neglected, the error would have been 27 yards—nearly 2%. The approximate results are here *too large* for each section—as is usually the case. If points between the center and slope stakes are omitted and the volume computed as if the ground were *three-level* ground, the error is quite large in individual sections, but the errors are both positive and negative and therefore compensating.

87. Cross-sectioning irregular sections. The prismoids considered have *straight* lines joining corresponding points in the two cross-sections. The center line must be straight between two cross-sections. If a ridge or valley is found lying diagonally across the roadbed, a cross-section *must* be interpolated at the lowest (or highest) point of the profile. Therefore a "break" at any section cannot be said to run out at the other section on the *opposite* side of the center. It must run out on the *same* side of the center or possibly *at* the center. Very frequently complicated cross-sectioning may be avoided by computing the volume, by some special method, of a mound or hollow when the ground is comparatively regular except for the irregularity referred to.

88. Side-hill work. When the natural slope cuts the roadbed there is a necessity for both cut and fill at the same cross-section. When this occurs the cross-sections of both cut and fill are often so nearly triangular that they may be considered as such without

great error, and the volumes may be computed separately as triangular prismoids without adopting the more elaborate form of computation so necessary for complicated irregular sections. When the ground is too irregular for this the best plan is to follow the uniform system. In computing the cut, as in Fig. 53,
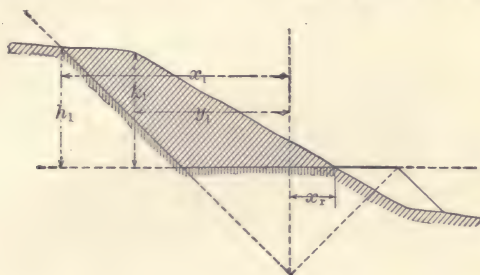
FIG. 53.

the left side would be as usual; there would be a small center cut and an ordinate of zero at a short distance to the right of the center. Then, *ignoring the fill*, and applying Eq. 61 strictly, we have two terms for the left side, one for the right, and the term involving $\frac{1}{2}b$, which will be $\frac{1}{2}bh_l$ in this case, since $h_r = 0$, and the equation becomes

$$\text{Area} = \frac{1}{2}[x_l k_l + y_l(d - h_l) + x_r d + \frac{1}{2}bh_l].$$

The area for fill may also be computed by a strict application
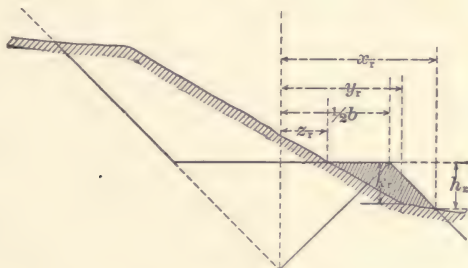
FIG. 54.

of Eq. 61, but for Fig. 54 all distances for the left side are zero and the elevation for the first point out is zero. d also must be

considered as zero. Following the rule, § 81, literally, the equation becomes

$$\text{Area}_{(\text{Fill})} = \tfrac{1}{2}[x_r k_r + y_r(o - h_r) + z_r(o - k_r) + \tfrac{1}{2}b(o + h_r)],$$

which reduces to

$$\tfrac{1}{2}[x_r k_r - y_r h_r - z_r k_r + \tfrac{1}{2}b h_r].$$

(*Note* that x_r, h_r, etc., have different significations and values in this and in the preceding paragraphs.) The "terminal pyramids" illustrated in Fig. 40 are instances of side-hill work for very short distances. Since side-hill work always implies *both* cut and fill at the same cross-section, whenever either the cut or fill disappears and the earthwork becomes wholly cut or wholly fill, that point marks the end of the "side-hill work," and a cross-section should be taken at this point.

89. Borrow-pits. The cross-sections of borrow-pits will vary not only on account of the undulations of the surface of the
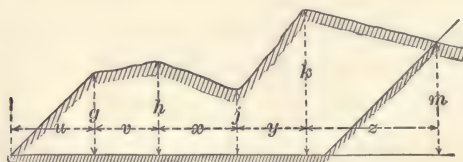
FIG. 55.

ground, but also on the sides, according to whether they are made by widening a convenient cut (as illustrated in Fig. 55) or simply by digging a pit. The sides should always be properly sloped and the cutting made cleanly, so as to avoid unsightly roughness. If the slope ratio on the right-hand side (Fig. 55) is s, the area of the triangle is $\tfrac{1}{2}sm^2$. The area of the section is $\tfrac{1}{2}[ug+(g+h)v+(h+j)x+(j+k)y+(k+m)z-sm^2]$. If all the horizontal measurements were referred to one side as an origin, a formula similar to Eq. 61 could readily be developed, but little or no advantage would be gained on account of any simplicity of computation. Since the *exact* volume of the earth borrowed is frequently necessary, the prismoidal correc-

tion should be computed; and since such a section as Fig. 55 does not even approximate to a three-level section, the method suggested in § 82 cannot be employed. It will then be necessary to employ the exact method, § 83, by dividing the volume into triangular prismoids and taking the summation of their corrections, found according to the general method of § 71.

90. Correction for curvature. The volume of a solid, generated by revolving a plane area about an axis lying in the plane but outside of the area, equals the product of the given area times the length of the path of the center of gravity of the area. If the centers of gravity of all cross-sections lie in the center of the road, where the length of the road is measured, there is absolutely no necessary correction for curvature. If all the cross-sections in any given length were exactly the same and therefore had the same eccentricity, the correction for curvature would be very readily computed according to the above principle. But when both the areas and the eccentricities vary from point to point, as is generally the case, a theoretically exact solution is quite complex, both in its derivation and application. Suppose, for simplicity, a curved section of the road, of uniform cross-sections and with the center of gravity of every cross-section at the same distance e from the center line of the road. The length of the path of the center of gravity will be to the length of the center line as $R \pm e : R$. Therefore we have *True vol.* : *nominal vol.* : : $R \pm e : R$.

$\therefore$ *True vol.* $= lA\dfrac{R \pm e}{R}$ for a volume of uniform area and eccentricity. For any other area and eccentricity we have, similarly, *True vol.'* $= lA'\dfrac{R \pm e'}{R}$. This shows that the effect of curvature is the same as increasing (or diminishing) the area by a quantity depending on the area and eccentricity, the increased (or diminished) area being found by multiplying the actual area by the ratio $\dfrac{R \pm e}{R}$. This being independent of the value of l, it is true for infinitesimal lengths. If the eccen-

tricity is assumed to vary uniformly between two sections, the *equivalent area* of a cross-section located midway between the two end cross-sections would be $A_m \dfrac{\left(R \pm \dfrac{e' + e''}{2}\right)}{R}$. Therefore the volume of a solid which, when straight, would be $\dfrac{l}{6}(A' + 4A_m + A'')$, would then become

$$True\ vol. = \frac{l}{6R}\left[A'(R \pm e') + 4A_m\left(R \pm \frac{e' + e''}{2}\right) + A''(R \pm e'')\right].$$

Subtracting the nominal volume (the true volume when the prismoid is straight), the

$$Correction = \pm \frac{l}{6R}\left[(A' + 2A_m)e' + (2A_m + A'')e''\right]. \quad (63)$$

Another demonstration of the same result is given by Prof. C. L. Crandall in his "Tables for the Computation of Railway and other Earthwork," in which is obtained by calculus methods the summation of elementary volumes having variable areas with variable eccentricities. The exact application of Eq. (63) requires that A_m be known, which requires laborious computations, but no error worth considering is involved if the equation is written approximately

$$Curv.\ corr. = \frac{l}{2R}(A'e' + A''e''), \quad . \quad . \quad . \quad (64)$$

which is the equation generally used. The approximation consists in assuming that the difference between A' and A_m equals the difference between A_m and A'' but with opposite sign. The error due to the approximation is always utterly insignificant.

91. Eccentricity of the center of gravity. The determination of the true positions of the centers of gravity of a long series of irregular cross-sections would be a very laborious operation, but fortunately it is generally sufficiently accurate to

consider the cross-sections as three-level ground, or, for side-hill work, to be triangular, *for the purpose of this correction*.   The
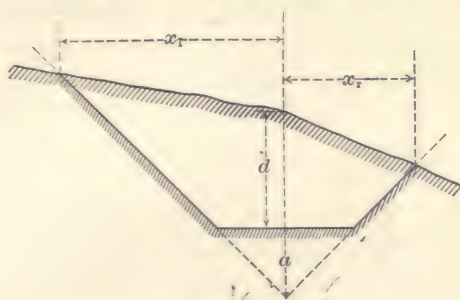
FIG. 56.

eccentricity of the cross-section of Fig. 56 (including the grade triangle) may be written

$$e = \frac{\dfrac{(a+d)x_l}{2}\dfrac{x_l}{3} - \dfrac{(a+d)x_r}{2}\dfrac{x_r}{3}}{\dfrac{(a+d)x_l}{2} + \dfrac{(a+d)x_r}{2}} = \frac{1}{3}\frac{x_l{}^2 - x_r{}^2}{x_l + x_r} = \frac{1}{3}(x_l - x_r). \quad (65)$$

The side toward x_r being considered positive in the above demonstration, if $x_r > x_l$, e would be negative, i.e., the center of gravity would be on the left side.   Therefore, for three-level ground, the correction for curvature (see Eq. 64) may be written

$$Correction = \frac{l}{6R}[A'(x_l' - x_r') + A''(x_l'' - x_r'')].$$

Since the approximate volume of the prismoid is

$$\frac{l}{2}(A + A') = \frac{l}{2}A' + \frac{l}{2}A'' = V' + V'',$$

in which V' and V'' represent the number of cubic yards corresponding to the area at each station, we may write

$$Corr.\ in\ cub.\ yds. = \frac{1}{3R}[V'(x_l' - x_r') + V''(x_l'' - x_r'')]. \quad (66)$$

It should be noted that the value of e, derived in Eq. 65, is the eccentricity of the whole area including the triangle under the roadbed. The eccentricity of the true area is greater than this and equals

$$e \times \frac{\text{true area} + \frac{1}{2}ab}{\text{true area}} = e_{1}.$$

The required quantity ($A'e'$ of Eq. 64) equals *true area* $\times e_{1}$, which equals (*true area* $+ \frac{1}{2}ab$) $\times e$. Since the value of e is very simple, while the value of e_{1} would, in general, be a complex quantity, it is easier to use the simple value of Eq. 65 and add $\frac{1}{2}ab$ to the area. Therefore, in the case of three-level ground the subtractive term $\frac{25}{27}ab$ (§ 78) should *not* be subtracted in computing this correction. For irregular ground, when computed by the method given in §§ 81 and 82, which does not involve the grade triangle, a term $\frac{25}{27}ab$ must be *added* at every station when computing the quantities V' and V'' for Eq. 66.

It should be noted that the factor $1 \div 3R$, which is constant for the length of the curve, may be computed with all necessary accuracy and without resorting to tables by remembering that

$$R = \frac{5730}{\text{degree of curve}}.$$

Since it is useless to attempt the computation of railroad earthwork closer than the nearest cubic yard, it will frequently be possible to write out all curvature corrections by a simple mental process upon a mere inspection of the computation sheet. Eq. 66 shows that the correction for each station is of the form $\frac{V(x_{l} - x_{r})}{3R}$. $3R$ is generally a large quantity—for a 6° curve it is 2865. ($x_{l} - x_{r}$) is generally small. It may frequently be seen by inspection that the product $V(x_{l} - x_{r})$ is roughly twice or three times $3R$, or perhaps less than half of $3R$, so that the corrective term for that station may be written 2, 3, or 0 cubic yards, the fraction being disregarded. For much larger absolute

amounts the correction must be computed with a correspondingly closer percentage of accuracy.

The algebraic sign of the curvature correction is best determined by noting that the center of gravity of the cross-section is on the right or left side of the center according as x_r is greater or less than x_l, and that the correction is *positive* if the center of gravity is on the *outside* of the curve, and *negative* if on the *inside*.

It is frequently found that x_l is uniformly greater (or uniformly less) than x_r throughout the length of the curve. Then the curvature correction for each station is uniformly positive or negative. But in irregular ground the center of gravity is apt to be irregularly on the outside or on the inside of the curve, and the curvature correction will be correspondingly positive or negative. If the curve is to the *right*, the correction will be positive or negative according as $(x_l - x_r)$ is positive or negative; if the curve is to the *left*, the correction will be positive or negative according as $(x_r - x_l)$ is positive or negative. Therefore when computing curves to the *right* use the form $(x_l - x_r)$ in Eqs. 66 and 68; when computing curves to the *left* use the form $(x_r - x_l)$ in these equations; the algebraic sign of the correction will then be strictly in accordance with the results thus obtained.

92. Center of gravity of side-hill sections. In computing the
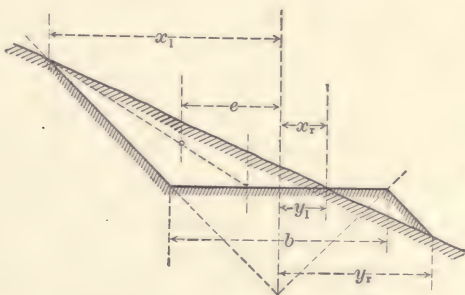
Fɪɢ. 57.

correction for side-hill work the cross section would be treated as triangular unless the error involved would evidently be too

great to be disregarded. The center of gravity of the triangle lies on the line joining the vertex with the middle of the base and at $\frac{1}{3}$ of the length of this line from the base. It is therefore equal to the distance from the center to the foot of this line plus $\frac{1}{3}$ of its horizontal projection. Therefore

$$
\begin{aligned}
e &= \left[\frac{b}{2} - \frac{1}{2}\left(\frac{b}{2} + x_r\right)\right] + \frac{1}{3}\left[x_l - \left(\frac{b}{2} - \frac{1}{2}\left(\frac{b}{2} + x_r\right)\right)\right] \\
&= \frac{b}{4} - \frac{x_r}{2} + \frac{x_l}{3} - \frac{b}{12} + \frac{x_r}{6} \\
&= \frac{b}{6} + \frac{x_l}{3} - \frac{x_r}{3} \\
&= \frac{1}{3}\left[\frac{b}{2} + (x_l - x_r)\right]. \quad \ldots \ldots \ldots \quad (67)
\end{aligned}
$$

By the same process as that used in § 91 the correction equation may be written

Corr. in cub. yds. $= \dfrac{1}{3R}\left[V'\left(\dfrac{b}{2} + (x_l' - x_r')\right) + V''\left(\dfrac{b}{2} + (x_l'' - x_r'')\right)\right].$ (68)

It should be noted that since the grade triangle is not used in this computation the volume of the grade prism is *not* involved in computing the quantities V' and V''.

The eccentricities of cross-sections in side-hill work are *never* zero, and are frequently quite large. The total volume is generally quite small. It follows that the correction for curvature is generally a vastly larger proportion of the total volume than in ordinary three-level or irregular sections.

If the triangle is wholly to one side of the center, Eq. 67 can still be used. For example, to compute the eccentricity of the triangle of fill, Fig. 57, denote the two distances to the slope-stakes by y_r and $-y_l$ (note the minus sign). Applying Eq. 67 literally (noting that $\dfrac{b}{2}$ must here be considered as negative in order to make the notation consistent) we obtain

$$
e = \frac{1}{3}\left[-\frac{b}{2} + (-y_l - y_r)\right].
$$

which reduces to

$$e = -\frac{1}{3}\left[\frac{b}{2} + y_l + y_r\right]. \quad . \quad . \quad . \quad . \quad (69)$$

As the algebraic signs tend to create confusion in these formulæ, it is more simple to remember that for a triangle lying on *both* sides of the center e is always numerically equal to $\frac{1}{3}\left[\frac{b}{2} + (x_l \sim x_r)\right]$, and for a triangle entirely on one side, e is numerically equal to $\frac{1}{3}\left[\frac{b}{2} + \text{the numerical } sum \text{ of the two dis-}\right.$ tances out]. The algebraic sign of e is readily determinable as in § 91.

93. Example of curvature correction. Assume that the fill in § 78 occurred on a 6° curve to the *right*. $\frac{1}{3R} = \frac{1}{2865}$. The quantities 210, 507, etc., represent the quantities V', V'', etc., since they include in each case the 61 cubic yards due to the grade prism. Then

$$\frac{V(x_l \sim x_r)}{3R} = \frac{210(22.9 - 8.2)}{2865} = \frac{3101.7}{2865} = +1.$$

The sign is plus since the center of gravity of the cross-section is on the left side of the center and the road curves to the right, thus making the true volume larger. For Sta. 18 the correction, computed similarly, is $+3$, and the correction for the whole section is $1 + 3 = 4$. For Sta. $18 + 40$ the correction is computed as 6 yards. Therefore, for the 40 feet, the correction is $\frac{40}{100}(3 + 6) = 3.6$, which is called 4. Computing the others similarly we obtain a total correction of $+16$ cubic yards.

94. Accuracy of earthwork computations. The preceding methods give the *precise volume* (except where approximations are distinctly admitted) of the prismoids which are *supposed* to represent the volume of the earthwork. To appreciate the accuracy necessary in cross-sectioning to obtain a given accuracy

in volume, consider that a fifteen-foot length of the cross-section, which is assumed to be straight, really sags 0.1 foot, so that the cross-section is in error by a triangle 15 feet wide and 0.1 foot high. This sag 0.1 foot high would hardly be detected by the eye, but in a length of 100 feet in each direction it would make an error of volume of 1.4 cubic yards in *each* of the two prismoids, assuming that the sections at the other ends were perfect. If the cross-sections at both ends of a prismoid were in error by this same amount, the volume of that prismoid would be in error by 2.8 cubic yards if the errors of area were both plus or both minus. If one were plus and one minus, the errors would neutralize each other, and it is the compensating character of these errors which permits any confidence in the results as obtained by the usual methods of cross-sectioning. It demonstrates the utter futility of attempting any closer accuracy than the nearest cubic yard. It will thus be seen that if an error really exists at *any* cross-section it involves the prismoids on *both* sides of the section, even though all the other cross-sections are perfect. As a further illustration, suppose that cross-sections were taken by the method of slope angle and center depth (§ 73), and that a cross-section, assumed as uniform, sags 0.4 foot in a width of 20 feet. Assume an equal error (of same sign) at the other end of a 100-foot section. The error of volume for that one prismoid is 38 cubic yards.

The computations further assume that the warped surface, passing through the end sections, coincides with the surface of the ground. Suppose that the cross-sectioning had been done with mathematical perfection; and, to assume a simple case, suppose a sag of 0.5 foot between the sections, which causes an error equal to the volume of a pyramid having a base of 20 feet (in each cross-section) times 100 feet (between the cross-sections) and a height of 0.5 foot. The volume of this pyramid is $\frac{1}{3}(20 \times 100) \times 0.5 = 333$ cub. ft. $= 12$ cub. yds. And yet this sag or hump of 6 inches would generally be utterly unnoticed, or at least disregarded.

When the ground is very rough and broken it is sometimes

practically impossible, even with frequent cross-sections, to locate warped surfaces which will closely coincide with all the sudden irregularities of the ground. In such cases the computations are necessarily more or less approximate and dependence must be placed on the compensating character of the errors.

95. Approximate computations from profiles. As a means of comparing the relative amounts of earthwork on two or more proposed routes which have been surveyed by preliminary surveys, it will usually be sufficiently accurate to compare the *areas* of cutting (assuming that the cut and fill are approximately balanced) as shown by the several profiles. The errors involved may be large in individual cases and for certain small sections, but fortunately the errors (in comparing two lines) will be largely compensated. The errors are much larger on side-hill work than when the cross-sections are comparatively level. The errors become large when the depth of cut or fill is very great. If the lines compared have the same general character as to the slope of the cross-sections, the proportion of side-hill work, and the average depth of cut or fill, the error involved in considering their relative volumes of cutting to be as the relative areas of cutting on the profiles (obtained perhaps by a planimeter) will probably be small. If the volume in each case is computed by assuming the sections as *level*, with a depth equal to the center cut, the error involved will depend only on the amount of side-hill work and the degree of the slope. If these features are about the same on the two lines compared, the error involved is still less.

FORMATION OF EMBANKMENTS.

96. Shrinkage of earthwork. The evidence on this subject as to the amount of shrinkage is very conflicting, a fact which is probably due to the following causes:

1. The various kinds of earthy material act very differently as respects shrinkage. There has been but little uniformity in the *classification of earths* in the tests and experiments that have been made.

2. Very much depends on the *method* of forming an embankment (as will be shown later).   Different reports have been based on different methods—often without mention of the method.

3. An embankment requires considerable *time* to shrink to its final volume, and therefore much depends on the time elapsed between construction and the measurement of what is supposed to be the settled volume.

P. J. Flynn quotes some experiments (*Eng. News*, May 1, 1886) made in India in which pits were dug, having volumes of 400 to 600 cubic feet.   The material, when piled into an embankment, measured largely in excess of the original measurement—as is the universal experience.   The pits were refilled with the same material.   As the rains, very heavy in India, settled the material in the pits, more was added to keep the pits full.   Even after the rainy season was over, there was in every case material in excess.   This would seem to indicate a permanent *expansion*, although it is possible that the observations were not continued for a sufficient time to determine the final settled volume.

On the contrary, notes made by Mr. Elwood Morris many years ago on the behavior of embankments of several thousand cubic yards, formed in layers by carts and scrapers, one winter intervening between commencement and completion, showed in each case a permanent *contraction* averaging about 10%.

All authorities agree that rockwork *expands* permanently when formed into an embankment, but the percentages of expansion given by different authorities differ even more than with earth—varying from 8 to 90%.   Of course this very large range in the coefficient is due to differences in the character of the rock.   The softer the rock and the closer its similarity to earth, the less will be its expansion.   On account of the conflicting statements made, and particularly on account of the influence of methods of work, but little confidence can be felt in any given coefficient, especially when given to a fraction of a per

cent, but the consensus of American practice seems to average
about as follows:

Permanent contraction of earth.......... about 10%
    "   expansion of rock............ 40 to 60%

These values for rock should be materially reduced, according
to judgment, when the rock is soft and liable to disintegrate.
The hardest rocks, loosely piled, may occasionally give even
higher results. The following is given by several authors as
the permanent contraction of several grades of earth:

Gravel or sand........................ about 8%
Clay................................   "   10%
Loam................................   "   12%
Loose vegetable surface soil............   "   15%

It may be noticed from the above table that the harder and
cleaner the material the less is the contraction. Perfectly clean
gravel or sand would not probably change volume appreciably.
The above coefficients of shrinkage and expansion may be used
to form the following convenient table.

Material.	To make 1000 cubic yards of embankment will require	1000 cubic yards measured in excavation will make
Gravel or sand.......... ...	1087 cubic yards	920 cubic yards
Clay.....................	1111 " "	900 " "
Loam....................	1136 " "	880 " "
Loose vegetable soil........	1176 " "	850 " "
Rock, large pieces..........	714 " "	1400 " "
" small "	625 " "	1600 " "
	measured in excavation	of embankment.

97. Allowance for shrinkage. On account of the initial
expansion and subsequent contraction of earth, it becomes
necessary to form embankments higher than their required
ultimate form in order to allow for the subsequent shrinkage.
As the shrinkage appears to be all vertical (practically), the
embankment must be formed as shown in Fig. 58. The effect

of shrinkage should not be confounded with that of slipping of the sides, which is especially apt to occur if the embankment is subjected to heavy rains very soon after being formed, and also when the embankments are originally steep.   It is often difficult
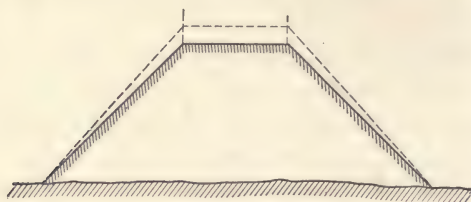
Fig. 58.

to form an embankment at a slope of 1 : 1 which will not slip more or less before it hardens.

Very high embankments shrink a greater percentage than lower ones.   Various rules giving the relation between shrink- age and height have been suggested, but they vary as badly as the suggested coefficients of contraction, probably for the same causes.   As the fact is unquestionable, however, the extra height of the embankment must be varied somewhat as in Fig. 59, which represents a longitudinal section of an embankment.
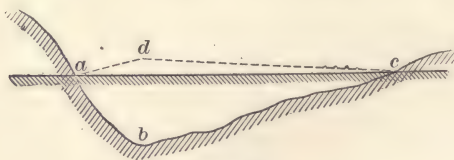
Fig. 59.

As considerable time generally elapses between the completion of the embankment and the actual running of trains, the grade *ad* will generally be nearly flattened down to its ultimate form before traffic commences, but such grades are occasionally objec- tionable if added to what is already a ruling grade.   With some kinds of soil the time required for complete settlement may be as much as two or three years, but, even in such cases, it is

probable that one-half of the settlement will take place during the first six months. The engineer should therefore require the contractor to make all fills about 8 to 15% (according to the material) higher than the profiles call for, in order that subsequent shrinkage may not reduce it to less than the required volume.

98. Methods of forming embankments. When the method is not otherwise objectionable, a high embankment can be formed very cheaply (assuming that carts or wheelbarrows are used) by dumping over the end and building to the full height (or even higher, to allow for shrinkage) as the embankment proceeds. This allows more time for shrinkage, saves nearly all the cost of spreading (see Item 4, § 111), and reduces the cost of roadways (Item 5). Of course this method is especially applicable when the material comes from a place as high as or higher than grade, so that no up-hill hauling is required.

Another method is to spread it in layers two or three feet thick (see Fig. 60), which are made concave upwards to avoid
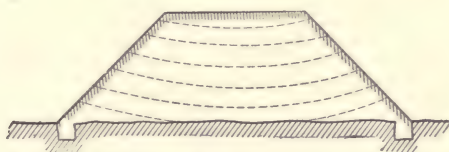
FIG. 60.

possible sliding on each other. Spreading in layers has the advantage of partially ramming each layer, so that the subsequent shrinkage is very small. Sometimes small trenches are dug along the lines of the toes of the embankment. This will frequently prevent the sliding of a large mass of the embankment, which will then require extensive and costly repairs, to say nothing of possible accidents if the sliding occurs after the road is in operation. Incidentally these trenches will be of value in draining the subsoil. When circumstances require an embankment on a hillside, it is advisable to cut out "steps" to prevent a possible sliding of the whole embankment. Merely

ploughing the side-hill will often be a cheaper and sufficiently effective method.
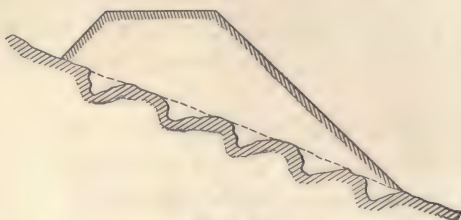
Fig. 61.

Occasionally the formation of a very high and long embankment may be most easily and cheaply accomplished by building a trestle to grade and opening the road. Earth can then be procured where most convenient, perhaps several miles away, loaded on cars with a steam-shovel, hauled by the trainload, and dumped from the cars with a patent unloader. On such a large scale, the cost per yard would be very much less than by ordinary methods—enough less sometimes to more than pay for the temporary trestle, besides allowing the road to be opened for traffic very much earlier, which is often a matter of prime financial importance. It may also obviate the necessity for extensive borrow-pits in the immediate neighborhood of the heavy fill and also utilize material which would otherwise be wasted.

<div align="center">COMPUTATION OF HAUL.</div>

99. Nature of subject. As will be shown later when analyzing the cost of earthwork, the most variable item of cost is that depending on the distance hauled. As it is manifestly impracticable to calculate the exact distance to which every individual cartload of earth has been moved, it becomes necessary to devise a means which will give at least an equivalent of the haulage of all the earth moved. Evidently the *average* haul for any mass of earth moved is equal to the distance from the center of gravity of the excavation to the center of gravity of the embank-

ment formed by the excavated material. As a rough approximation the center of gravity of a cut (or fill) may sometimes be considered to coincide with the center of gravity of that part of the profile representing it, but the error is frequently very large. The center of gravity may be determined by various methods, but the method of the "mass diagram" accomplishes the same ultimate purpose (the determination of the haul) with all-sufficient accuracy and also furnishes other valuable information.

100. Mass diagram. In Fig. 62 let $A'B' \ldots G'$ represent a profile and grade line drawn to the usual scales. Assume A'
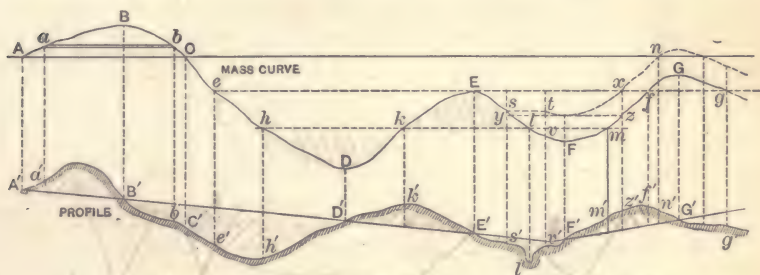
FIG. 62.—MASS DIAGRAM.

to be a point past which no earthwork will be hauled. Above every station point in the profile draw an ordinate which will represent the algebraic sum of the cubic yards of cut and fill (calling cut + and fill −) from the point A' to the point considered. In doing this shrinkage must be allowed for by considering how much embankment would actually be made by so many cubic yards of excavation of such material. For example, it will be found that 1000 cubic yards of sand or gravel, measured in place (see § 97), will make about 920 cubic yards of embankment; therefore all cuttings in sand or gravel should be discounted in about this proportion. Excavations in rock should be increased in the proper ratio. In short, all excavations should be valued according to the amount of *settled* embankment that could be made from them. The computations may be made systematically as shown in the tabular form. Place

in the first column a list of the stations; in the second column, the number of cubic yards of cut or fill between each station and the preceding station; in the third and fourth columns, the kind of material and the proper shrinkage factor; in the fifth column, a repetition of the quantities in cubic yards, except that the excavations are diminished (or increased, in the case of rock) to the number of cubic yards of settled embankment which may be made from them. In the sixth column, place the *algebraic sum* of the quantities in the fifth column (calling cuts + and fills −) from the starting-point to the station considered. These algebraic sums at each station will be the ordinates, drawn to some scale, of the mass curve. The scale to be used will depend somewhat on whether the work is heavy or light, but for ordinary cases a scale of 5000 cubic yards per inch may be used. Drawing these ordinates to scale, a curve A, B, . . . G may be obtained by joining the extremities of the ordinates.

Sta.		Yards $\begin{Bmatrix} \text{cut} + \\ \text{fill} - \end{Bmatrix}$	Material.	Shrinkage factor.	Yards, reduced for shrinkage.	Ordinate in mass curve.
46	+ 70	. . . .	. . . .	. . . .	. . . .	0
47		+ 195	Clayey soil	−10 per cent	+ 175	+ 175
48		+1792	" "	−10 "	+1613	+1788
	+ 60	+ 614	" "	−10 "	+ 553	+2341
49		− 143	. . . .	. . . .	− 143	+2198
50		− 906	. . . .	. . . .	− 906	+1292
51		−1985	. . . .	. . . .	−1985	− 693
52		−1721	. . . .	. . . .	−1721	−2414
	+ 30	− 112	. . . .	. . . .	− 112	−2526
53		+ 177	Hard rock	+60 per cent	+ 283	−2243
	+ 70	+ 180	" "	+60 "	+ 289	−1954
54		− 52	. . . .	. . . .	− 52	−2006
	+ 42	− 71	. . . .	. . . .	− 71	−2077
55		+ 276	Clayey soil	−10 per cent	+ 249	−1828
56		+1242	" "	−10 "	+1118	− 710
57		+1302	" "	−10 "	+1172	+ 462

101. Properties of the mass curve.

1. The curve will be rising while over cuts and falling while over fills.

2. A tangent to the curve will be horizontal (as at B, D, E, F, and G) when passing from cut to fill or from fill to cut.

3. When the curve is *below* the "zero line" it shows that material must be drawn *backward* (to the left); and *vice versa*, when the curve is *above* the zero line it shows that material must be drawn *forward* (to the right).

4. When the curve crosses the zero line (as at *A* and *C*) it shows (in this instance) that the cut between *A'* and *B'* will just provide the material required for the fill between *B'* and *C'*, and that no material should be hauled past *C'*, or, in general, past any intersection of the mass curve and the zero line.

5. If any horizontal line be drawn (as *ab*), it indicates that the cut and fill between *a'* and *b'* will just balance.

6. When the center of gravity of a given volume of material is to be moved a given distance, it makes no difference (at least theoretically) how far each individual load may be hauled or how any individual load may be disposed of. The summation of the products of each load times the distance hauled will be a constant, whatever the method, and will equal the total volume times the movement of the center of gravity. The *average haul*, which is the movement of the center of gravity, will therefore equal the summation of these products divided by the total volume. If we draw two horizontal parallel lines at an infinitesimal distance *dx* apart, as at *ab*, the small increment of cut *dx* at *a'* will fill the corresponding increment of fill at *b'*, and this material must be hauled the distance *ab*. Therefore the product of *ab* and *dx*, which is the product of distance times volume, is represented by the area of the infinitesimal rectangle at *ab*, and the total area *ABC* represents the summation of volume times distance for all the earth movement between *A'* and *C'*. This summation of products divided by the total volume gives the average haul.

7. The horizontal line, tangent at *E* and cutting the curve at *e*, *f*, and *g*, shows that the cut and fill between *e'* and *E'* will just balance, and that a *possible* method of hauling (whether desirable or not) would be to "borrow" earth for the fill between *C'* and *e'*, use the material between *D'* and *E'* for the

fill between e' and D', and similarly balance cut and fill between E' and f' and also between f' and g'.

8. Similarly the horizontal line *hklm* may be drawn cutting the curve, which will show another *possible* method of hauling. According to this plan, the fill between C' and h' would be made by borrowing; the cut and fill between h' and k' would balance; also that between k' and l' and between l' and m'. Since the area *ehDkE* represents the measure of haul for the earth between e' and E', and the other areas measure the corresponding hauls similarly, it is evident that the sum of the areas *ehDkE* and *ElFmf*, which is the measure of haul of all the material between e' and f', is largely in excess of the sum of the areas *hDk*, *kEl*, and *lFm*, plus the somewhat uncertain measures of haul due to borrowing material for $e'h'$ and wasting the material between m' and f'. Therefore to make the measure of haul a minimum a line should be drawn which will make the sum of the areas between it and the mass curve a minimum.   Of course this is not necessarily the cheapest plan, as it implies more or less borrowing and wasting of material, which *may* cost more than the amount saved in haul.   The comparison of the two methods is quite simple, however.   Since the amount of fill between e' and h' is represented by the *difference* of the ordinates at e and h, and similarly for m' and f', it follows that the amount to be borrowed between e' and h' will exactly equal the amount wasted between m' and f'.   By the first of the above methods the haul is excessive, but is definitely known from the mass diagram, and all of the material is utilized; by the second method the haul is reduced to about one-half, but there is a known quantity in cubic yards wasted at one place and the same quantity borrowed at another.   The length of haul necessary for the borrowed material would need to be ascertained; also the haul necessary to waste the other material at a place where it would be unobjectionable.   Frequently this is best done by widening an embankment beyond its necessary width.   The computation of the relative cost of the above methods will be discussed later (§ 116).

9. Suppose that it were deemed best, after drawing the mass curve, to introduce a trestle between s' and v', thus saving an amount in fill equal to tv. If such had been the original design, the mass curve would have been a straight horizontal line between s and t and would continue as a curve which would be at all points a distance tv above the curve $vFmzfGg$. If the line Ef is to be used as a zero line, its intersection with the new curve at x will show that the material between E' and z' will just balance if the trestle is used, and that the amount of haul will be measured by the area between the line Ex and the broken line $Estx$. The same computed result may be obtained without drawing the auxiliary curve txn . . . by drawing the horizontal line zy at a distance xz $(= tv)$ below Ex. The amount of the haul can then be obtained by adding the triangular area between Es and the horizontal line Ex, the rectangle between st and Ex, and the irregular area between vFz and y . . . z (which last is evidently equal to the area between tx and E . . . x). The disposal of the material at the right of z' would then be governed by the indications of the profile and mass diagram which would be found at the right of g'. In fact it is difficult to decide with the best of judgment as to the proper disposal of material without having a mass diagram extending to a considerable distance each side of that part of the road under immediate consideration.

102. Area of the mass curve. The area may be computed most readily by means of a planimeter, which is capable with reasonable care of measuring such areas with as great accuracy as is necessary for this work. If no such instrument is obtainable, the area may be obtained by an application of " Simpson's rule." The ordinates will usually be spaced 100 feet apart. Select an *even* number of such spaces, leaving, if necessary, one or more triangles or trapezoids at the ends for separate and independent computation. Let y_0 . . . . y_n be the ordinates, i.e., the number of cubic yards at each station of the mass curve, or the figures of " column six " referred to in § 100. Let the uniform distance between ordinates ($= 100$ feet) be called 1, i.e.,

one *station.* Then the units of the resulting area will be cubic yards hauled one station. Then the

$$\text{Area} = \tfrac{1}{3}[y_0 + 4(y_1 + y_3 + \ldots y_{(n-1)}) + 2(y_2 + y_4 + \ldots y_{(n-2)}) + y_n]. \quad (70)$$

When an ordinate occurs at a substation, the best plan is to ignore it at first and calculate the area as above. Then, if the difference involved is too great to be neglected, calculate the area of the triangle having the extremity of the ordinate at the substation as an apex, and the extremities of the ordinates at the adjacent stations as the ends of the base. This may be done by finding the ordinate at the substation that would be a proportional between the ordinates at the adjacent full stations. Subtract this from the real ordinate (or *vice versa*) and multiply the difference by $\frac{1}{2} \times 1$. An inspection will often show that the correction thus obtained would be too small to be worthy of consideration. If there is more than one substation between two full stations, the corrective area will consist of two triangles and one or more trapezoids which may be similarly computed, if necessary.

When the zero line (Fig. 62) is shifted to eE, the drop from AC (produced) to E is known in the same units, cubic yards. This constant may be subtracted from the numbers ("column 4," § 100) representing the ordinates, and will thus give, without any scaling from the diagram, the exact value of the modified ordinates.

103. Value of the mass diagram. The great value of the mass diagram lies in the readiness with which different plans for the disposal of material may be examined and compared. When the mass curve is once drawn, it will generally require only a shifting of the horizontal line to show the disposal of the material by any proposed method. The mass diagram also shows the extreme length of haul that will be required by any proposed method of disposal of material. This brings into consideration the "limit of profitable haul," which will be fully discussed in § 116. For the present it may be said that with each method of carrying material there is some limit beyond which the expense

of hauling will exceed the loss resulting from borrowing and
wasting. With wheelbarrows and scrapers the limit of profit-
able haul is comparatively short, with carts and tram-cars it is
much longer, while with locomotives and cars it may be several
miles. If, in Fig. 62, *eE* or *Ef* exceeds the limit of profitable
haul, it shows at once that some such line as *hklm* should be
drawn and the material disposed of accordingly.

104. Changing the grade line. The formation of the mass
curve and the resulting plans as to the disposal of material are
based on the mutual relations of the grade line and the surface
profile and the amounts of cut and fill which are thereby im-
plied. If the grade line is altered, every cross-section is
altered, the amount of cut and fill is altered, and the mass
curve is also changed. At the farther limit of the actual
change of the grade line the revised mass curve will have (in
general) a different ordinate from the previous ordinate at that
point. From that point on, the revised mass curve will be par-
allel to its former position, and the revised curve may be treated
similarly to the case previously mentioned in which a trestle was
introduced. Since it involves tedious calculations to determine
accurately how much the volume of earthwork is altered by a
change in grade line, especially through irregular country, the
effect on the mass curve of a change in the grade line cannot
therefore be readily determined except in an approximate way.
Raising the grade line will evidently increase the fills and
diminish the cuts, and *vice versa*. Therefore if the mass curve
indicated, for example, either an excessively long haul or the
necessity for borrowing material (implying a fill) and wasting
material farther on (implying a cut), it would be *possible* to
diminish the fill (and hence the amount of material to be bor-
rowed) by lowering the grade line near that place, and diminish
the cut (and hence the amount of material to be wasted) by
raising the grade line at or near the place farther on. Whether
the advantage thus gained would compensate for the possibly
injurious effect of these changes on the grade line would require
patient investigation. But the method outlined shows how the

mass curve might be used to indicate a possible change in grade line which might be demonstrated to be profitable.

105. Limit of free haul. It is sometimes specified in contracts for earthwork that *all* material shall be entitled to free haul up to some specified limit, say 500 or 1000 feet, and that all material drawn farther than that shall be entitled to an allowance on the *excess* of distance. It is manifestly impracticable to measure the excess for each load, as much so as to measure the actual haul of each load. The mass diagram also solves this problem very readily. Let Fig. 63 represent a pro-
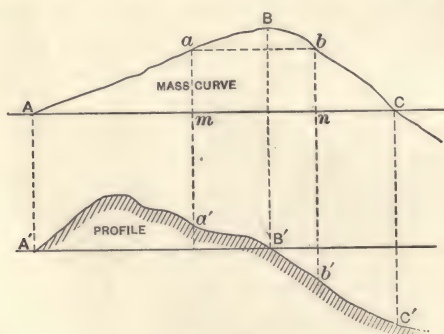
FIG. 63.

file and mass diagram of about 2000 feet of road, and suppose that 800 feet is taken as the limit of free haul. Find two points, *a* and *b*, in the mass curve *which are on the same horizontal line* and which are 800 feet apart. Project these points down to *a'* and *b'*. Then the cut and fill between *a'* and *b'* will just balance, and the cut between *A'* and *a'* will be needed for the fill between *b'* and *C'*. In the mass curve, the area between the horizontal line *ab* and the curve *aBb* represents the haulage of the material between *a'* and *b'*, which is all free. The rectangle *abmn* represents the haulage of the material in the cut *A'a'* across the 800 feet from *a'* to *b'*. This is also free. The sum of the two areas *Aam* and *bnC* represents the haulage entitled to an allowance, since it is the summation of the products of cubic yards times the *excess* of distance hauled.

If the amount of cut and fill was symmetrical about the point B', the mass curve would be a symmetrical curve about the vertical line through B, and the two limiting lines of free haul would be placed symmetrically about B and B'. In general there is no such symmetry, and frequently the difference is considerable. The area $aBbnm$ will be materially changed according as the two vertical lines am and bn, always 800 feet apart, are shifted to the right or left. It is easy to show that the area $aBbnm$ is a *maximum* when ab is horizontal. The minimum value would be obtained either when m reached A or n reached C, depending on the exact form of the curve. Since the position for the minimum value is manifestly unfair, the best definite value obtainable is the maximum, which must be obtained as above described. Since $aBbnm$ is made maximum, the remainder of the area, which is the allowance for overhaul, becomes a minimum. The areas Aam and bCn may be obtained as in § 102. If the whole area $AaBbCA$ has been previously computed, it may be more convenient to compute the area $aBbnm$ and subtract it from the total area.

Since the intersections of the mass curve and the " zero line " mark limits past which no material is drawn, it follows that there will be no allowance for overhaul except where the distance between consecutive intersections of the zero line and mass curve exceeds the limit of free haul.

Frequently all allowances for overhaul are disregarded; the profiles, estimates of quantities, and the required disposal of material are shown to bidding contractors, and they must then make their own allowances and bid accordingly. This method has the advantage of avoiding possible disputes as to the amount of the overhaul allowance, and is popular with railroad companies on this account. On the other hand the facility with which different plans for the disposal of material may be studied and compared by the mass-curve method facilitates the adoption of the most economical plan, and the elimination of uncertainty will frequently lead to a safe reduction of the bid, and so the method is valuable to both the railroad company and the contractor.

ELEMENTS OF THE COST OF EARTHWORK.

(The following analysis of the cost of earthwork follows the general method given in the well-known papers published by Ellwood Morris, C.E., in the Journal of the Franklin Institute in September and October, 1841. Numerous corroborative data have been obtained from various other sources, and also figures on methods not then in vogue.)

106. General divisions of the subject. The variations in the cost of earthwork are caused by the greatly varying conditions under which the work is done, chief among which is character of material, method of carriage, and length of haul. Any general system of computation must therefore differentiate the total cost into such elementary items that all differences due to variations in conditions may be allowed for. The variations due to character of material will be allowed for by an estimate on loose light sandy soil, and also an estimate on the heaviest soils, such as stiff clay and hard-pan. These represent the extremes (excluding rock, which will be treated separately), and the cost of intermediate grades must be estimated by interpolating between the extreme values. The general divisions of the subject will be : *

1. Loosening.
2. Loading.
3. Hauling.
4. Spreading.
5. Keeping roadways in order.
6. Repairs, wear, depreciation, and interest on cost of plant.
7. Superintendence and incidentals.
8. Contractor's profit.

By making the estimates on the basis of $1 per day for the cost of common labor, it is a simple matter to revise the estimates according to the local price of labor by multiplying the final estimate of cost by the price of labor in dollars per day.

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* Trautwine.

107. Item 1. LOOSENING. (a) Ploughs. Very light sandy soils can frequently be shovelled without any previous loosening, but it is generally economical, even with very light material, to use a plough. Morris quotes, as the results of experiments, that a three-horse plough would loosen from 250 to 800 cubic yards of earth per day, which at a valuation of $5 per day would make the cost per yard vary from 2 cents to 0.6 cent. Trautwine estimates the cost on the basis of two men handling a two-horse plough at a total cost of $3.87 per day, being $1 each for the men, 75 c. for each horse, and an allowance of 37 c. for the plough, harness, etc. From 200 to 600 cubic yards is estimated as a fair day's work, which makes a cost of 1.9 c. to 0.65 c. per yard, which is substantially the same estimate as above. Extremely heavy soils have sometimes been loosened by means of special ploughs operated by traction-engines.

(b) **Picks.** When picks are used for loosening the earth, as is frequently necessary and as is often done when ploughing would perhaps be really cheaper, an estimate [*] for a fair day's work is from 14 to 60 cubic yards, the 14 yards being the estimate for stiff clay or cemented gravel, and the 60 yards the estimate for the lightest soil that would require loosening. At $1 per day this means about 7 c. to 1.7 c. per cubic yard, which is about three times the cost of ploughing. Five feet of the face is estimated [†] as the least width along the face of a bank that should be allowed to enable each laborer to work with freedom and hence economically.

(c) **Blasting.** Although some of the softer shaly rocks may be loosened with a pick for about 15 to 20 c. per yard, yet rock in general, frozen earth, and sometimes even compact clay is most economically loosened by blasting. The subject of blasting will be taken up later, §§ 117–123.

(d) **Steam-shovels.** The items of loosening and loading merge together with this method, which will therefore be treated in the next section.

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[*] Trautwine.   [†] Hurst.

108. Item 2. LOADING. (a) Hand-shovelling. Much depends on proper management, so that the shovellers need not wait unduly either for material or carts. With the best of management considerable time is thus lost, and yet the intervals of rest need not be considered as entirely lost, as it enables the men to work, while actually loading, at a rate which it would be physically impossible for them to maintain for ten hours. Seven shovellers are sometimes allowed for each cart; otherwise there should be five, two on each side and one in the rear. Economy requires that the number of loads per cart per day should be made as large as possible, and it is therefore wise to employ as many shovellers as can work without mutual interference and without wasting time in waiting for material or carts. The figures obtainable for the cost of this item are unsatisfactory on account of their large disagreements. The following are quoted as the number of cubic yards that can be loaded into a cart by an average laborer in a working day of ten hours, the lower estimate referring to heavy soils, and the higher to light sandy soils: 10 to 14 cubic yards (Morris), 12 to 17 cubic yards (Haskoll), 18 to 22 cubic yards (Hurst), 17 to 24 cubic yards (Trautwine), 16 to 48 cubic yards (Ancelin). As these estimates are generally claimed to be based on actual experience, the discrepancies are probably due to differences of management. If the average of 15 to 25 cubic yards be accepted, it means, on the basis of $1 per day, 6.7 c. to 4 c. per cubic yard. These estimates apply only to earth. *Rockwork* costs more, not only because it is harder to handle, but because a cubic yard of solid rock, measured in place, occupies about 1.8 cubic yards when broken up, while a cubic yard of earth will occupy about 1.2 cubic yards. Rockwork will therefore require about 50% more loads to haul a given volume, *measured in place*, than will the same nominal volume of earthwork. The above authorities give estimates for loading rock varying from 6.9 c. to 10 c. per cubic yard. The above estimates apply only to the loading of carts or cars with shovels or by hand (loading masses of rock). The

cost of loading wheelbarrows and the cost of scraper work will be treated under the item of hauling.

(b) **Steam-shovels.*** Whenever the magnitude of the work will warrant it there is great economy in the use of steam-shovels. These have a " bucket " or " dipper " on the end of a long beam, the bucket having a capacity varying from $\frac{1}{2}$ to $2\frac{1}{2}$ cubic yards. Steam-shovels handle all kinds of material from the softest earth to shale rock, earthy material containing large boulders, tree-stumps, etc. The capacity of the larger sizes is about 3000 cubic yards in 10 hours. They perform all the work of loosening and loading. Their economical working requires that the material shall be hauled away as fast as it can be loaded, which usually means that cars on a track, hauled by horses or mules, or still better by a locomotive, shall be used. The expenses for a steam-shovel, costing about $5000, will average about $1000 per month. Of this the engineer will get $100 ; the fireman $50 ; the cranesman $90 ; repairs perhaps $250 to $300 ; coal, from 15 to 25 tons, cost very variable on account of expensive hauling ; water, a very uncertain amount, sometimes costing $100 per month ; about five laborers and a foreman, the laborers getting $1.25 per day and the foreman $2.50 per day, which will amount to $227.50 per month. This gang of laborers is employed in shifting the shovel when necessary, taking up and relaying tracks for the cars, shifting loaded and unloaded cars, etc. In shovelling through a deep cut, the shovel is operated so as to undermine the upper parts of the cut, which then fall down within reach of the shovel, thus increasing the amount of material handled for each new position of the shovel. If the material is too tough to fall down by its own weight, it is sometimes found economical to employ a gang of men to loosen it or even blast it rather than shift the shovel so frequently. Non-condensing engines of 50 horse-power use so much water that the cost of water-supply becomes a serious

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* For a thorough treatment of the capabilities, cost, and management of steam-shovels the reader is referred to "Steam-shovels and Steam-shovel Work," by E. A. Hermann. D. Van Nostrand Co., New York.

matter if water is not readily obtainable. The lack of water facilities will often justify the construction of a pipe line from some distant source and the installation of a steam-pump. Hence the seemingly large estimate of $100 per month for water-supply, although under favorable circumstances the cost may almost vanish. The larger steam-shovels will consume nearly a ton of coal per day of 10 hours. The expense of hauling this coal from the nearest railroad or canal to the location of the cut is often a very serious item of expense and may easily double the cost per ton. Some steam-shovels have been constructed to be operated by electricity obtained from a plant perhaps several miles away. Such a method is especially advantageous when fuel and water are difficult to obtain.

109. Item 3. HAULING. The cost of hauling depends on the number of round trips per day that can be made by each vehicle employed. As the cost of each vehicle is practically the same whether it makes many trips or few, it becomes important that the number of trips should be made a maximum, and to that end there should be as little delay as possible in loading and unloading. Therefore devices for facilitating the passage of the vehicles have a real money value.

(a) Carts. The average speed of a horse hauling a two-wheeled cart has been found to be 200 feet per minute, a little slower when hauling the load and a little faster when returning empty. This figure has been repeatedly verified. It means an allowance of one minute for each 100 feet (or " station ") of " lead—the lead being the distance the earth is hauled." The time lost in loading, dumping, waiting to load, etc., has been found to average 4 minutes per load. Representing the number of stations (100 feet) of lead by s, the number of loads handled in 10 hours (600 minutes) would be $600 \div (s + 4)$. The number of loads per cubic yard, measured in the bank, is differentiated by Morris into three classes, viz. :

> 3 loads per cubic yard in descending hauling;
> $3\frac{1}{2}$ "   "   "   "   " level hauling; and
> 4 "   "   "   "   " ascending hauling.

Attempts have been made to estimate the effect of the grade of the roadway by a theoretical consideration of its rate, and of the comparative strength of a horse on a level and on various grades. While such computations are always practicable on a railway (even on a temporary construction track), the traction on a temporary earth roadway is always very large and so very variable that any refinements are useless. On railroad earth-work the hauling is generally nearly level or it is *descending*—forming embankments on low ground with material from cuts in high ground. The only common exception occurs when an embankment is formed from borrow-pits on low ground. One method of allowing for ascending grade is to add to the horizontal distance 14 times the difference of elevation for work with carts and 24 times the difference of elevation for work with wheelbarrows, and use that as the lead. For example, using carts, if the lead is 300 feet and there is a difference of elevation of 20 feet, the lead would be considered equivalent to $300 + (14 \times 20) = 580$ feet on a level.

Trautwine assumes the average load for all classes of work to be $\frac{1}{5}$ cubic yard, which figure is justified by large experience. Using one figure for all classes of work simplifies the calculations and gives the number of cubic yards carried per day of 10 hours equal to $\dfrac{600}{3(s + 4)}$. Dividing the cost of a cart per day by the number of cubic yards carried gives the cost of hauling per yard. In computing the cost of a cart per day, Trautwine refers to the practice of having one driver manage four carts, thus making a charge of 25 c. per day for each cart for the driver. 75 c. is allowed for the horse, which is supposed to be the total cost, including that for Sundays and rainy days. 25 c. more is allowed for the cart, harness, repairs, etc., thus making a total cost of $1.25 per day. Some contractors employ a greater number of drivers and expect each to assist in loading. There is found to be no saving in total cost per yard, while the chances of loafing are perhaps greater. Morris instances five actual cases in which the cost of the cart (reduced to the basis of

$1 per day for labor) varied from $1.37 to $1.48. The items of these costs were not given.

Since the time required for loading loose rock is greater than for earthwork, less loads will be hauled per day. The time allowance for loading, etc., is estimated by Trautwine as 6 minutes instead of 4 as for earth. Considering the great expansion of rock when broken up (see § 97), one cubic yard of solid rock, measured in place, would furnish the equivalent of five loads of earthwork of $\frac{1}{5}$ cubic yard. Therefore, on the basis of five loads per cubic yard, the number of cubic yards handled per day per cart would be $\dfrac{600}{5(s+6)}$.

$$\text{Cost per yard in cents} = \frac{125 \times 5(s+6)}{600}. \quad . \quad (71)$$

(b) **Wagons.** For longer leads (i.e., from $\frac{1}{8}$ to $\frac{2}{8}$ of a mile) wagons drawn by two horses have been found most economical. The wagons have bottoms of loose thick narrow boards and are unloaded very easily and quickly by lifting the individual boards and breaking up the continuity of the bottom, thus depositing the load directly underneath the wagon. The capacity is about one cubic yard. The cost may be estimated on the same principles as that for carts.

(c) **Wheelbarrows.** According to Trautwine, the speed of moving wheelbarrows may be considered the same as for carts, 200 feet per minute; the time spent in loading and dumping is $1\frac{1}{4}$ minutes, and in addition about $\frac{1}{10}$ of the time is wasted in short rests, adjusting the wheeling plants, etc. On the basis of $1 per day for labor, an allowance of 5 c. for the barrow, and 14 loads per cubic yard, the cost of hauling per cubic yard (computed on the same principles as above) will be

$$\frac{105 \times 14(s+1.25)}{600 \times 0.9} \quad . \quad . \quad . \quad . \quad (72)$$

For rockwork the number of loads per cubic yard is estimated as 24, and the time spent in loading, etc., estimated at 1.6 minutes instead of 1.25 minutes, which makes the estimate

$$\text{Cost per cubic yard} = \frac{105 \times 24(s + 1.6)}{600 \times 0.9}. \quad . \quad (73)$$

(d) **Scrapers.** * Scrapers, or scoops, are especially useful in canal work, and also for railroad work when a low embankment is to be formed from borrow-pits at the sides, when the distance does not exceed 100 feet, nor the vertical height 15 feet. The slope should not exceed 1.5 to 1. Under these conditions scraper work is cheaper than any other method. Scooping may be done all in one direction, in which case two half-turns are made for each load moved; or it may be done in both directions (from both sides on to a bank, or, in canal work, from the center to each bank), in which case one load is hauled to each half-turn. The capacity of the scoops (the "drag" variety) is $\frac{1}{10}$ cubic yard; the time lost in loading, unloading, and all other ways per load (except in turning) will average $\frac{2}{3}$ minute; the time lost in each half-turn (semi-circle) is $\frac{1}{3}$ minute; the speed of the horses may be estimated as 70 feet of *lead* per minute, the lead being here considered as the *sum* of the vertical and horizontal distances, and the estimate including the time of going and returning. If a represents the sum of the horizontal and vertical distances, the number of cubic yards handled per day of 10 hours by "side-scooping" will be

$$0.1\left(\frac{600}{\frac{a}{70} + 1\frac{1}{3}}\right), \quad \text{which equals} \quad \frac{4200}{a + 93\frac{1}{3}}.$$

For "double-scooping" the formula becomes

$$0.1\left(\frac{600}{\frac{a}{70} + 1}\right), \quad \text{which equals} \quad \frac{4200}{a + 70}.$$

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* Condensed from Journ. Franklin Inst., Oct. 1841, by Morris.

Dividing the cost of a scraper per day (estimated at $2.75) by the number of yards handled per day gives the average cost per yard.

Except in very loose sandy soil it is best to plough the earth first, which will cost *about* 1 c. per yard. (See § 107.) Drag-scrapers are now made chiefly of steel, and their capacity is more nearly 0.15 cubic yard. Wheeled scrapers, having a capacity of about 0.5 cubic yard, are frequently used with even greater economy and for greater distances, as they are cheaper than carts up to 250 or 300 feet of lead. Both drag- and wheel-scrapers are best operated in gangs of perhaps 10, using extra or " snap " teams to help load, and a few extra men to help in loading and unloading. The average cost of one scraper per day may thus be easily calculated and the average number of cubic yards handled per day computed as above, from which the cost per yard may be estimated.

(e) **Cars and horses.** The items of cost by this method are (*a*) charge for horses employed, (*b*) charge for men employed strictly in hauling, (*c*) charge for shifting rails when necessary, (*d*) repairs, depreciation, and interest on cost of cars and track. Part of this cost should strictly be classified under items 5 and 6, mentioned in § 106, but it is perhaps more convenient to estimate them as follows.

The traction of a car on rails is so very small and constant that grade resistance constitutes a very large part of the total resistance if the grade is 1% or more. For all ordinary grades it is sufficiently accurate to say that the grade resistance is to the gross weight as the rise is to the distance. If the distance is supposed to be measured along the slope, the proportion is strictly true; i.e., on a 1% grade the grade resistance is 1 lb. per 100 of weight or 20 lbs. per ton. If the resistance on a level at the usual velocity is $\frac{1}{120}$, a grade of 1 : 120 (0.83%) will exactly double it. If the material is hauled *down* a grade of 1 : 120, the cars will run by gravity after being started. The work of hauling will then consist practically of hauling the empty cars up the grade. The grade resistance depends only

on the rate of grade and the weight, but the tractive resistance will be *greater per ton of weight* for the unloaded than for the loaded cars. The tractive power of a horse is less on a grade than on a level, not only because the horse raises his own weight in addition to the load, but is anatomically less capable of pulling on a grade than on a level. In general it will be possible to plan the work so that loaded cars need not be hauled *up* a grade, unless an embankment is to be formed from a low borrow-pit, in which case another method would probably be advisable. These computations are chiefly utilized in designing the method of work—the proportion of horses to cars. An example may be quoted from English practice (Hurst), in which the cars had a capacity of $3\frac{1}{3}$ cubic yards, weighing 30 cwt. empty. Two horses took five " wagons " $\frac{3}{4}$ of a mile on a level railroad and made 15 journeys per day of 10 hours, i.e., they handled 250 yards per day. In addition to those on the " straight road," another horse was employed to make up the train of loaded wagons. With a short lead the straight-road horses were employed for this purpose. In the above example the number of men required to handle these cars, shift the tracks, etc., is not given, and so the exact cost of the above work cannot be analyzed. It may be noticed that the two horses travelled $22\frac{1}{2}$ miles per day, drawing in one direction a load, including the weight of the cars, of about 57,300 lbs. or 28.65 net tons. Allowing $\frac{1}{120}$ as the necessary tractive force, it would require a pull of 477.5 lbs., or 239 lbs. for each horse. With a velocity of 220 feet per minute this would amount to $1\frac{1}{2}$ horse-power per horse, exerted for only a short time, however, and allowing considerable time for rest and for drawing only the empty cars. The cars generally used in this country have a capacity of $1\frac{1}{2}$ cubic yards and cost about \$65 apiece. Besides the shovellers and dumping-gang, several men and a foreman will be required to keep the track in order and to make the constant shifts that are necessary. Two trains are generally used, one of which is loaded while the other is run to the dump. Some passing-place is necessary, but this is generally

provided by having a switch at the cut and running the trains on each track alternately. This insures a train of cars always at the cut to keep the shovellers employed. The cost of hauling per cubic yard can only be computed when the number of laborers, cars, and horses employed are known, and these will depend on the lead, on the character of the excavation, on the grade, if any, etc., and must be so proportioned that the shovellers need not wait for cars to fill, nor the dumping-gang for material to handle, nor the horses and drivers for cars to haul. Much skill is necessary to keep a large force in smooth running order.

(f) **Cars and locomotives.** 30-lb. rails are the lightest that should be used for this work, and 35- or 40-lb. rails are better. One or two narrow-gauge locomotives (depending on the length of haul), costing about $2500 each, will be necessary to handle two trains of about 15 cars each, the cars having a capacity of about 2 cubic yards and costing about $100 each. Some cars can be obtained as low as $70. A force of about five men and a foreman will be required to shift the tracks. The track-shifters, except the foreman, may be common laborers. The dumping-gang will require about seven men. Even when the material is all taken *down* grade the grades may be too steep for the safe hauling of loaded cars down the grade, or for hauling empty cars up the grade. Under such circumstances temporary trestles are necessary to reduce the grade. When these are used, the uprights and bracing are left in the embankment—only the stringers being removed. This is largely a necessity, but is partially compensated by the fact that the trestle forms a core to the embankment which prevents lateral shifting during settlement. The average speed of the trains may be taken as 10 miles per hour or 5 miles of lead per hour. The time lost in loading and unloading is estimated (Trautwine) as 9 minutes or .15 of an hour. The number of trips per day of 10 hours will equal $\dfrac{10}{\frac{1}{5}\text{(miles of lead)} + .15}$ or $\dfrac{50}{\text{(miles of lead)} + .75}$. Of course this quotient *must* be a whole number. Knowing the

number of trains and their capacity, the total number of cubic yards handled is known, which, divided into the total daily cost of the trains, will give the cost of hauling per yard. The daily cost of a train will include

(*a*) Wages of engineer, who frequently fires his own engine ;

(*b*) Fuel, about ¼ to 1 ton of bituminous coal, depending on work done ;

(*c*) Water, a very variable item, frequently costing $3 to $5 per day ;

(*d*) Repairs, variable, frequently at rate of 50 to 60% per year ;

(*e*) Interest on cost and depreciation, 16 to 40%.

To these must be added, to obtain the total cost of the haul,

(*f*) Wages of the gang employed in shifting track.

110. Choice of method of haul dependent on distance. In light side-hill work in which material need not be moved more than 12 or 15 feet, i.e., moved *laterally* across the road-bed, the earth may be moved most cheaply by mere shovelling. Beyond 12 feet scrapers are more economical. At about 100 feet drag-scrapers and wheelbarrows are equally economical. Between 100 and 200 feet wheelbarrows are generally cheaper than either carts or drag-scrapers, but wheeled scrapers are always cheaper than wheelbarrows. Beyond 500 feet two-wheeled carts become the most economical up to about 1700 feet; then four-wheeled wagons become more economical up to 3500 feet. Beyond this cars on rails, drawn by horses or by locomotives, become cheaper. The economy of cars on rails becomes evident for distances as small as 300 feet provided the volume of the excavation will justify the outlay. Locomotives will always be cheaper than horses and mules providing the work to be done is of sufficient magnitude to justify the purchase of the necessary plant and risk the loss in selling the plant ultimately as second-hand equipment, or keeping the plant on hand and idle for an indefinite period waiting for other work. Horses will not be economical for distances much over a mile. For greater distances locomotives are more economical, but the

question of "limit of profitable haul" (§ 116) must be closely studied, as the circumstances are certainly not common when it is advisable to haul material much over a mile.

111. Item 4. SPREADING. The cost of spreading varies with the method employed in dumping the load. When the earth is tipped over the edge of an embankment there is little if any necessary work. Trautwine allows about $\frac{1}{4}$ c. per cubic yard for keeping the dumping-places clear and in order. This would represent the wages of one man at \$1 per day attending to the unloading of 1200 two-wheeled carts each carrying $\frac{1}{3}$ cubic yard. 1200 carts in 10 hours would mean an average of two per minute, which implies more rapid and efficient work than may be depended on. The allowance is probably too small. When the material is dumped in layers some levelling is required, for which Trautwine allows 50 to 100 cubic yards as a fair day's work, costing from 1 to 2 cents per cubic yard. The cost of spreading will not ordinarily exceed this and is frequently nothing—all depending on the method of unloading. It should be noted that Mr. Morris's examples and computations (Jour. Franklin Inst., Sept. 1841) disregard altogether any special charge for this item.

112. Item 5. KEEPING ROADWAYS IN ORDER. This feature is important as a measure of true economy, whatever the system of transportation, but it is often neglected. A petty saving in such matters will cost many times as much in increased labor in hauling and loss of time. With some methods of haul the cost is best combined with that of other items.

(a) **Wheelbarrows.** Wheelbarrows should generally be run on planks laid on the ground. The adjusting and shifting of these planks is done by the wheelers, and the time for it is allowed for in the 10% allowance for "short rests, adjusting the wheeling plank, etc." The actual cost of the planks must be added, but it would evidently be a very small addition per cubic yard in a large contract. When the wheelbarrows are run on planks placed on "horses" or on trestles the cost is very appreciable; but the method is frequently used with great economy. The

variations in the requirements render any general estimate of such cost impracticable.

(b) **Carts and wagons.** The cost of keeping roadways in order for carts and wagons is sometimes estimated merely as so much per cubic yard, but it is evidently a function of the *lead*. The work consists in draining off puddles, filling up ruts, picking up loose stones that may have fallen off the loads, and in general doing everything that will reduce the traction as much as possible. Temporary inclines, built to avoid excessive grade at some one point, are often measures of true economy. Trautwine suggests $\frac{1}{10}$ c. per cubic yard per 100 feet of lead for earthwork and $\frac{2}{10}$ c. for rockwork, as an estimate for this item when carts are used.

(c) **Cars.** When cars are used a shifting-gang, consisting of a foreman and several men (say five), are constantly employed in shifting the track so that the material may be loaded and unloaded where it is desired. The average cost of this item may be estimated by dividing the total daily cost of this gang by the number of cubic yards handled in one day.

113. Item 6. REPAIRS, WEAR, DEPRECIATION, AND INTEREST ON COST OF PLANT. The amount of this item evidently depends upon the character of the soil—the harder the soil the worse the wear and depreciation. The *interest on cost* depends on the current borrowing value of money. The estimate for this item has already been included in the allowances for horses, carts, ploughs, harness, wheelbarrows, steam-shovels, etc. Trautwine estimates $\frac{1}{4}$ c. per cubic yard for picks and shovels. Depreciation is generally a large percentage of the cost of earth-working tools, the life of all being limited to a few years, and of many tools to a few months.

114. Item 7. SUPERINTENDENCE AND INCIDENTALS. The incidentals include water-carriers, trimming cuts to grade, digging the side ditches, trimming up the sides of borrow-pits to prevent their becoming unsightly, etc. These last operations yield but little earth and cost far more than the price paid per cubic yard. Morris allows 1 c. per cubic yard for this item; Trautwine

allows $1\frac{3}{4}$ to 2 c. for it; while others combine items 6 and 7 and call them 5% of the total cost, which method has the merit of making the cost of items 6 and 7 a function of the character of soil and length of lead.

115. Item 8. Contractor's Profit. This is usually estimated at from 6 to 15%, according to the sharpness of the competition and the possible uncertainty as to true cost owing to unfavorable circumstances. The contractor's real profit may vary considerably from this. He often pays clerks, boards and lodges the laborers in shanties built for the purpose, or keeps a supply-store, and has various other items both of profit and expense. His profit is largely dependent on skill in so handling the men that all can work effectively without interference or delays in waiting for others. An unusual season of bad weather will often affect the cost very seriously. It is a common occurrence to find that two contractors may be working on the same kind of material and under precisely similar conditions and at the same price, and yet one may be making money and the other losing it—all on account of difference of management.

116. Limit of profitable haul. As intimated in §§ 103 and 110, there is with every method of haul a limit of distance beyond which the expense for excessive hauling will exceed the loss resulting from borrowing and wasting. This distance is somewhat dependent on local conditions, thus requiring an independent solution for each particular case, but the general principles involved will be about as follows: Assume that it has been determined, as in Fig. 62, that the cut and fill will exactly balance between two points, as between e and x, assuming that, as indicated in § 101 (9), a trestle has been introduced between s and t, thus altering the mass curve to $Estxn$ . . . Since there is a balance between A' and C', the material for the fill between C' and e' must be obtained either by " borrowing " in the immediate neighborhood or by transportation from the excavation between z' and n'. If cut and fill have been approximately balanced in the selection of grade line, as is ordinarily done, borrowing material for the fill $C'e'$ implies a wastage of material

at the cut $z'n'$. To compare the two methods, we may place against the plan of borrowing and wasting, (a) cost, if any, of extra right of way that may be needed from which to obtain earth for the fill $C'e'$; (b) cost of loosening, loading, hauling a distance equal to that between the centers of gravity of the borrow-pit and of the fill, and the other expenses incidental to borrowing M cubic yards for the fill $C'e'$; (c) cost of loosening, loading, hauling a distance equal to that between the centers of gravity of the cut $z'n'$ and of the spoil-bank, and the other expenses incidental to wasting M cubic yards at the cut $z'n'$; (d) cost, if any, of land needed for the spoil-bank. The cost of the other plan will be the cost of loosening, loading, hauling (the hauling being represented by the trapezoidal figure $Cexn$), and the other expenses incidental to making the fill $C'e'$ with the material from the cut $z'n'$, the amount of material being M cubic yards, which is represented in the figure by the vertical ordinate from e to the line Cn. The difference between these costs will be the cost, if any, of land for borrow-pit and spoil-bank *plus* the cost of loosening, loading, etc. (except hauling and roadways) of M cubic yards, *minus* the difference in cost of the excessive haul from Ce to xn and the comparatively short hauls from borrow-pit and to spoil-bank.

As an illustration, taking some of the estimates previously given for operating with average material, the cost of all items, except hauling and roadways, would be about as follows: loosening, with plough, 1.2 c., loading 5.0 c., spreading 1.5 c., wear, depreciation, etc., .25 c., superintendence, etc., 1.5 c.; total 8.95 c. Suppose that the haul for both borrowing and wasting averages 100 feet or 1 station. Then the cost of haul per yard, using carts, would be (§ 109, a) $[125 \times 3(1 + 4)] \div 600 = 3.125$ c. The cost of roadways would be about 0.1 c. per yard, making a total of 3.225 c. per cubic yard. Assume $M = 10000$ cubic yards and the area $Cexn = 180000$ yards-stations or the equivalent of 10000 yards hauled 1800 feet. This haul would cost $[125 \times 3(18 + 4)] \div 600 = 13.75$ c. per cubic yard. The cost of roadways will be $18 \times .1$ or 1.8 c.,

making a total of 15.55 c. for hauling and roadways. The difference of cost of hauling and roadways will be 15.55 — $(2 \times 3.225) = 9.10$ c. per yard or \$910 for the 10000 yards. Offsetting this is the cost of loosening, etc., 10000 yards, at 8.95 c., costing \$895. These figures may be better compared as follows:

LONG HAUL.	Loosening, etc., 10000 yards, @ 8.95 c.						\$ 895.	
	Hauling,	"	10000	"	@ 15.55 c.		1555.	
							\$2450.	

BORROWING AND WASTING.	Loosening, etc., 10000 yards (borrowed), @ 8.95	c.	\$895.
	"	"	10000	"	(wasted),	@ 8.95	c.	895.
	Hauling, etc.,	10000	"	(borrowed), @ 3.225 c.	322.50
	"	"	10000	"	(wasted),	@ 3.225 c.	322.50
							\$2435.00

These costs are practically balanced, but no allowance has been made for right of way. If any considerable amount had to be paid for that, it would decide this particular case in favor of the long haul. This shows that *under these conditions* 1800 feet is *about* the limit of profitable haul, the land costing nothing extra.

BLASTING.

117. Explosives. The effect of blasting is due to the extremely rapid expansion of a gas which is developed by the decomposition of a very small amount of solid matter. Blasting compounds may be divided into two general classes, (*a*) slow-burning and (*b*) detonating. Gunpowder is a type of the slow-burning compounds. These are generally ignited by heat; the ignition proceeds from grain to grain; the heat and pressure produced are comparatively low. Nitro-glycerine is a type of the detonating compounds. They are exploded by a shock which *instantaneously* explodes the whole mass. The heat and pressure developed are far in excess of that produced by the explosion of powder. Nitro-glycerine is so easily exploded that it is very dangerous to handle. It was discovered that if the nitro-glycerine was absorbed by a spongy material like infu-

sorial earth, it was much less liable to explode, while its power when actually exploded was practically equal to that of the amount of pure nitro-glycerine contained in the dynamite, which is the name given to the mixture of nitro-glycerine and infusorial earth. Nitro-glycerine is expensive; many other explosive chemical compounds which properly belong to the slow-burning class are comparatively cheap. It has been conclusively demonstrated that a mixture of nitro-glycerine and some of the cheaper chemicals has a greater explosive force than the sum of the strengths of the component parts when exploded separately. Whatever the reason, the fact seems established. The reason is possibly that the explosion of the nitro-glycerine is sufficiently powerful to produce a *detonation* of the other chemicals, which is impossible to produce by ordinary means, and that this explosion caused by detonation is more powerful than an ordinary explosion. The majority of the explosive compounds and "powders" on the market are of this character—a mixture of 20 to 60 per cent. of nitro-glycerine with variable proportions of one or more of a great variety of explosive chemicals.

The choice of the explosive depends on the character of the rock. A hard brittle rock is most effectively blasted by a detonating compound. The rapidity with which the full force of the explosive is developed has a shattering effect on a brittle substance. On the contrary, some of the softer tougher rocks and indurated clays are but little affected by dynamite. The result is but little more than an enlargement of the blast-hole. Quarrying must generally be done with blasting-powder, as the quicker explosives are too shattering. Although the results obtained by various experimenters are very variable, it may be said that pure nitro-glycerine is eight times as powerful as black powder, dynamite (75% nitro-glycerine) six times, and gun-cotton four to six times as powerful. For open work where time is not particularly valuable, black powder is by far the cheapest, but in tunnel-headings, whose progress determines the progress of the whole work, dynamite is so much more effective and so expedites the work that its use becomes economical.

118. Drilling. Although many very complicated forms of drill-bars have been devised, the best form (with slight modifications to suit circumstances) is as shown in Fig. 64, (*a*) and (*b*).
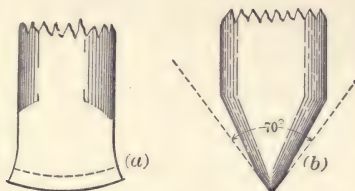
FIG. 64.

The width should flare at the bottom (*a*) about 15 to 30%. For hard rock the curve of the edge should be somewhat flatter and for soft rock somewhat more curved than shown, Fig. 64, (*a*). Sometimes the angle of the two faces is varied from that given, Fig. 64, (*b*), and occasionally the edge is purposely blunted so as to give a crushing rather than a cutting effect. The drills will require sharpening for each 6 to 18 inches depth of hole, and will require a new edge to be worked every 2 to 4 days. For drilling vertical holes the *churn-drill* is the most economical. The drill-bar is of iron, about 6 to 8 feet long, $1\frac{1}{4}''$ in diameter, weighs about 25 to 30 lbs., and is shod with a piece of steel welded on. The bar is lifted a few inches between each blow, turned partially around, and allowed to fall, the impact doing the work. From 5 to 15 feet of holes, depending on the character of the rock, is a fair day's work—10 hours. In very soft rocks even more than this may be done. This method is inapplicable for inclined holes or even for vertical holes in confined places, such as tunnel-headings. For such places the only practical *hand* method is to use hammers. This may be done by light drills and light hammers (one-man work), or by heavier drills held by one man and struck by one or two men with heavy hammers. The conclusion of an exhaustive investigation as to the relative economy of light or heavy hammers is that the light-hammer method is more economical for the softer rocks, the heavy-hammer method is more economical for the harder

rocks, but that the light-hammer method is always more ex-
peditious and hence to be preferred when time is important.

The subject of machine rock-drills is too vast to be treated
here. The method is only practicable when the amount of
work to be done is large, and especially when time is valuable.
The machines are generally operated by compressed air for tun-
nel-work, thus doing the additional service of supplying fresh
air to the tunnel-headings where it is most needed. The cost
per foot of hole drilled is quite variable, but is usually some-
what less than that of hand-drilling—sometimes but a small
fraction of it.

119. Position and direction of drill-holes. As the cost of
drilling holes is the largest single item in the total cost of blast-
ing, it is necessary that skill and judgment should be used in so
locating the holes that the blasts will be most effective. The
greatest effect of a blast will evidently be in the direction of the
" line of least resistance." In a strictly homogeneous material
this will be the shortest line from the center of the explosive to
the surface. The variations in homogeneity on account of
laminations and seams require that each case shall be judged
according to experience. In open-pit blasting it is generally
easy to obtain two and sometimes three exposed faces to the
rock, making it a simple matter to drill holes so that a blast will
do effective work. When a solid face of rock must be broken

into, as in a tunnel-heading, the
work is necessarily ineffectual and
expensive. A conical or wedge-
shaped mass will first be blown out
by simultaneous blasts in the holes
marked 1, Fig. 65; blasts in the
holes marked 2 and 3 will then com-
plete the cross-section of the head-
ing. A great saving in cost may
often be secured by skilfully taking

DRILL HOLES IN TUNNEL HEADING
FIG. 65.

advantage of seams, breaks, and irregularities. When the work
is economically done there is but little noise or throwing of rock,

a covering of old timbers and branches of trees generally sufficing to confine the smaller pieces which would otherwise fly up.

120. Amount of explosive. The amount of explosive required varies as the cube of the line of least resistance. The best results are obtained when the line of least resistance is $\frac{3}{4}$ of the depth of the hole; also when the powder fills about $\frac{1}{3}$ of the hole. For average rock the amount of powder required is as follows:

Line of least resistance...............	2 ft.	4 ft.	6 ft.	8 ft.
Weight of powder....................	$\frac{1}{4}$ lb.	2 lbs.	$6\frac{3}{4}$ lbs.	16 lbs.

Strict compliance with all of the above conditions would require that the diameter of the hole should vary for every case. While this is impracticable, there should evidently be some variation in the size of the hole, depending on the work to be done. For example, a 1″ hole, drilled 2′ 8″ deep, with its line of least resistance 2′, and loaded with $\frac{1}{4}$ lb. of powder, would be filled to a depth of $9\frac{1}{2}$″, which is nearly $\frac{1}{3}$ of the depth. A 2″ hole, drilled 8′ deep, with its line of least resistance 6′, and loaded with $6\frac{3}{4}$ lbs. of powder, would be filled to a depth of over 28″, which is also nearly $\frac{1}{3}$ of the depth. One pound of blasting-powder will occupy about 28 cubic inches. Quarrying necessitates the use of numerous and sometimes repeated light charges of powder, as a heavy blast or a powerful explosive like dynamite is apt to shatter the rock. This requires more powder to the cubic yard than blasting for mere excavation, which may usually be done by the use of $\frac{1}{4}$ to $\frac{1}{3}$ lb. of powder per cubic yard of easy open blasting. On account of the great resistance offered by rock when blasted in headings in tunnels, the powder used per cubic yard will run up to 2, 4, and even 6 lbs. per cubic yard. As before stated, nitroglycerine is about eight times (and dynamite about six times) as powerful as the same *weight* of powder.

121. Tamping. Blasting-powder and the slow-burning explosives require thorough tamping. Clay is probably the best,

but sand and fine powdered rock are also used. Wooden plugs, inverted expansive cones, etc., are periodically reinvented by enthusiastic inventors, only to be discarded for the simpler methods. Owing to the extreme rapidity of the development of the force of a nitro-glycerine or dynamite explosion, tamping is not so essential with these explosives, although it unquestionably adds to their effectiveness. Blasting under water has been effectively accomplished by merely pouring nitro-glycerine into the drilled holes through a tube and then exploding the charge without any tamping except that furnished by the superincumbent water. It has been found that air-spaces about a charge make a material reduction in the effectiveness of the explosion. It is therefore necessary to carefully ram the explosive into a solid mass. Of course the liquid nitro-glycerine needs no ramming, but dynamite should be rammed with a *wooden* rammer. Iron should be carefully avoided in ramming gunpowder. A copper bar is generally used.

122. Exploding the charge. Black powder is generally exploded by means of a fuse which is essentially a cord in which there is a thin vein of gunpowder, the cord being protected by tar, extra linings of hemp, cotton, or even gutta-percha. The fuse is inserted into the middle of the charge, and the tamping carefully packed around it so that it will not be injured. To produce the detonation required to explode nitro-glycerine and dynamite, there must be an initial explosion of some easily ignited explosive. This is generally accomplished by means of caps containing fulminating-powder which are exploded by electricity. The electricity (in one class of caps) heats a very fine platinum wire to redness, thereby igniting the sensitive powder, or (in another class) a spark is caused to jump through the powder between the ends of two wires suitably separated. Dynamite can also be exploded by using a small cartridge of gunpowder which is itself exploded by an ordinary fuse.

123. Cost. Trautwine estimates the cost of blasting (for mere excavation) as averaging 45 cents per cubic yard, falling as low as 30 cents for easy but *brittle* rock, and running up to

60 cents and even $1 when the cutting is shallow, the rock especially tough, and the strata unfavorably placed. Soft tough rock frequently requires more powder than harder brittle rock.

124. Classification of excavated material. The classification of excavated material is a fruitful source of dispute between con‑tractors and railroad companies, owing mainly to the fact that the variation between the softest earth and the hardest rock is so gradual that it is very difficult to describe distinctions between different classifications which are unmistakable and indisputable. The classification frequently used is (*a*) earth, (*b*) loose rock, and (*c*) solid rock. As blasting is frequently used to loosen "loose rock" and even "earth" (if it is frozen), the fact that blasting is employed cannot be used as a criterion, especially as this would (if allowed) lead to unnecessary blasting for the sake of classifying material as rock.

Earth. This includes clay, sand, gravel, loam, decomposed rock and slate, boulders or loose stones not greater than 1 cubic foot (3 cubic feet, P. R. R.), and sometimes even "hard-pan." In general it will signify material which *can* be loosened by a plough with two horses, or with which one picker can keep one shoveller busy.

Loose rock. This includes boulders and loose stones of more than one cubic foot and less than one cubic yard; stratified rock, not more than six inches thick, separated by a stratum of clay; also all material (not classified as earth) which may be loosened by pick or bar and which "*can* be quarried without blasting, although blasting may occasionally be resorted to."

Solid rock includes all rock found in masses of over one cubic yard which cannot be removed except by blasting.

It is generally specified that the engineer of the railroad company shall be the judge of the classification of the material, but frequently an appeal is taken from his decisions to the courts.

125. Specifications for earthwork. The following specifica‑tions, issued by the Norfolk and Western R. R., represent the average requirements. It should be remembered that very strict

specifications invariably increase the cost of the work, and frequently add to the cost more than is gained by improved quality of work.

1. The grading will be estimated and paid for by the cubic yard, and will include clearing and grubbing, and all open excavations, channels, and embankments required for the formation of the roadbed, and for turnouts and sidings; cutting all ditches or drains about or contiguous to the road; digging the foundation-pits of all culverts, bridges, or walls; reconstructing turnpikes or common roads in cases where they are destroyed or interfered with; changing the course or channel of streams; and all other excavations or embankments connected with or incident to the construction of said Railroad.

2. All grading, except where otherwise specified, whether for cuts or fills, will be measured in the excavations and will be classified under the following heads, viz.: Solid Rock, Loose Rock, Hard-pan, and Earth.

SOLID ROCK shall include all rock occurring in masses which, in the judgment of the said Engineer Maintenance of Way, may be best removed by blasting.

LOOSE ROCK shall include all kinds of shale, soapstone, and other rock which, in the judgment of the said Engineer Maintenance of Way, can be removed by pick and bar, and is soft and loose enough to be removed without blasting, although blasting may be occasionally resorted to; also, detached stone of less than one (1) cubic yard and more than one (1) cubic foot.

HARD-PAN shall consist of tough indurated clay or cemented gravel, which requires blasting or other equally expensive means for its removal, or which cannot be ploughed with less than four horses and a railroad plough, or which requires two pickers to a shoveller, the said Engineer Maintenance of Way to be the judge of these conditions.

EARTH shall include all material of an earthy nature, of whatever name or character, not unquestionably loose rock or hard-pan as above defined.

POWDER. The use of powder in cuts will not be considered

as a reason for any other classification than earth, unless the material in the cut is clearly other than earth under the above specifications.

3. Earth, gravel, and other materials taken from the excavations, except when otherwise directed by the said Engineer Maintenance of Way or his assistant, shall be deposited in the adjacent embankment; the cost of removing and depositing which, when the distance necessary to be hauled is not more than sixteen hundred (1600) feet, shall be included in the price paid for the excavation.

4. Extra Haul will be estimated and paid for as follows: whenever material from excavations is necessarily hauled a greater distance than sixteen hundred (1600) feet, there shall be paid in addition to the price of excavation the price of extra haul per 100 feet, or part thereof, after the first 1600 feet; the necessary haul to be determined in each case by the said Engineer Maintenance of Way or his assistant, from the profile and cross-sections, and the estimates to be in accordance therewith.

5. All embankments shall be made in layers of such thickness and carried on in such manner as the said Engineer Maintenance of Way or his assistant may prescribe, the stone and heavy materials being placed in slopes and top. And in completing the fills to the proper grade such additional heights and fulness of slope shall be given them, to provide for their settlement, as the said Engineer Maintenance of Way, or his assistant, may direct. Embankments about masonry shall be built at such times and in such manner and of such materials as the said Engineer Maintenance of Way or his assistant may direct.

6. In procuring materials for embankments from without the line of the road, and in wasting materials from cuttings, the place and manner of doing it shall in each case be indicated by the Engineer Maintenance of Way or his assistant; and care must be taken to injure or disfigure the land as little as possible. Borrow-pits and spoil-banks must be left by the Contractor in regular and sightly shape.

7. The lands of the said Railroad Company shall be cleared

to the extent required by the said Engineer Maintenance of
Way, or his assistant, of all trees, brushes, logs, and other
perishable materials, which shall be destroyed by burning or
deposited in heaps as the said Engineer Maintenance of Way,
or his assistant, may direct. Large trees must be cut not more
than two and one-half ($2\frac{1}{2}$) feet from the ground, and under
embankments less than four (4) feet high they shall be cut close
to the ground. All small trees and bushes shall be cut close to
the ground.

8. Clearing shall be estimated and paid for by the acre or
fraction of an acre.

9. All stumps, roots, logs, and other obstructions shall be
grubbed out, and removed from all places where embankments
occur less than two (2) feet in height; also, from all places
where excavations occur and from such other places as the said
Engineer Maintenance of Way or his assistant may direct.

10. Grubbing shall be estimated and paid for by the acre or
fraction of an acre.

11. Contractors, when directed by the said Engineer Main-
tenance of Way or his assistant in charge of the work, will
deposit on the side of the road, or at such convenient points as
may be designated, any stone, rock, or other materials that they
may excavate; and all materials excavated and deposited as
above, together with all timber removed from the line of the
road, will be considered the property of the Railroad Company,
and the Contractors upon the respective sections will be respon-
sible for its safe-keeping until removed by said Railroad Com-
pany, or until their work is finished.

12. Contractors will be accountable for the maintenance
of safe and convenient places wherever public or private roads
are in any way interfered with by them during the progress of
the work. They will also be responsible for fences thrown
down, and for gates and bars left open, and for all damages
occasioned thereby.

13. Temporary bridges and trestles, erected to facilitate the
progress of the work, in case of delays at masonry structures

from any cause, or for other reasons, will be at the expense of the Contractor.

14. The line of road or the gradients may be changed in any manner, and at any time, if the said Engineer Maintenance of Way or his assistant shall consider such a change necessary or expedient; but no claim for an increase in prices of excavation or embankment on the part of the Contractor will be allowed or considered unless made in writing before the work on that part of the section where the alteration has been made shall have been commenced. The said Engineer Maintenance of Way or his assistant may also, on the conditions last recited, increase or diminish the length of any section for the purpose of more nearly equalizing or balancing the excavations and embankments, or for any other reason.

15. The roadbed will be graded as directed by the said Engineer Maintenance of Way or his assistant, and in conformity with such breadths, depths, and slopes of cutting and filling as he may prescribe from time to time, and no part of the work will be finally accepted until it is properly completed and dressed off at the required grade.

CHAPTER IV.

TRESTLES.

126. Extent of use. Trestles constitute from 1 to 3% of the length of the average railroad. It was estimated in 1889 that there was then about 2400 miles of single-track railway trestle in the United States, divided among 150,000 structures and estimated to cost about $75,000,000. The annual charge for maintenance, estimated at ⅛ of the cost, therefore amounted to about $9,500,000 and necessitated the annual use of perhaps 300,000,000 ft. B.M. of timber. The corresponding figures at the present time must be somewhat in excess of this. The magnitude of this use, which is causing the rapid disappearance of forests, has resulted in endeavors to limit the use of timber for this purpose. Trestles may be considered as justifiable under the following conditions:

a. Permanent trestles.

1. Those of *extreme* height—then called viaducts and frequently constructed of iron or steel, as the Kinzua viaduct, 302 ft. high.

2. Those across waterways—*e.g.*, that across Lake Pontchartrain, near New Orleans, 22 miles long.

3. Those across swamps of soft deep mud, or across a river-bottom, liable to occasional overflow.

b. Temporary trestles.

1. To open the road for traffic as quickly as possible—often a reason of great financial importance.

2. To quickly replace a more elaborate structure, destroyed

by accident, on a road already in operation, so that the interruption to traffic shall be a minimum.

3. To form an earth embankment with earth brought from a distant point by the train-load, when such a measure would cost less than to borrow earth in the immediate neighborhood.

4. To bridge an opening temporarily and thus allow time to learn the regimen of a stream in order to better proportion the size of the waterway and also to facilitate bringing *suitable* stone for masonry from a distance.   In a new country there is always the double danger of either building a culvert too small, requiring expensive reconstruction, perhaps after a disastrous washout, or else wasting money by constructing the culvert unnecessarily large.   Much masonry has been built of a very poor quality of stone because it could be conveniently obtained and because good stone was unobtainable except at a prohibitive cost for transportation.   Opening the road for traffic by the use of temporary trestles obviates both of these difficulties.

127. Trestles vs. embankments.   Low embankments are very much cheaper than low trestles both in first cost and maintenance.   Very high embankments are very expensive to construct, but cost comparatively little to maintain.   A trestle of equal height may cost much less to construct, but will be expensive to maintain—perhaps $\frac{1}{8}$ of its cost per year.   To determine the height beyond which it will be cheaper to maintain a trestle rather than build an embankment, it will be necessary to allow for the cost of maintenance.   The height will also depend on the relative cost of timber, labor, and earthwork.   At the present average values, it will be found that for less heights than 25 feet the *first cost* of an embankment will *generally* be less than that of a trestle; this implies that a permanent trestle should never be constructed with a height less than 25 feet except for the reasons given in § 126.   The height at which a permanent trestle is certainly cheaper than earthwork is more uncertain.   A high grade line joining two hills will invariably imply at least a culvert if an embankment is used.   If the culvert is built of masonry, the cost of the embankment will be

so increased that the height at which a trestle becomes economical will be materially reduced. The cost of an embankment increases much more rapidly than the height—with very high embankments more nearly as the square of the height—while the cost of trestles does not increase as rapidly as the height. Although local circumstances may modify the application of any set rules, it is probably seldom that it will be cheaper to build an embankment 40 or 50 feet high than to permanently maintain a wooden trestle of that height. A steel viaduct would probably be the best solution of such a case. These are frequently used for permanent structures, especially when very high. The cost of maintenance is much less than that of wood, which makes the use of iron or steel preferable for permanent trestles unless wood is abnormally cheap. Neither the cost nor the construction of iron or steel trestles will be considered in this chapter.

128. Two principal types. There are two principal types of wooden trestles—pile trestles and framed trestles. The great objection to pile trestles is the rapid rotting of the portion of the pile which is underground, and the difficulty of renewal. The maximum height of pile trestles is about 30 feet, and even this height is seldom reached. Framed trestles have been constructed to a height of considerably over 100 feet. They are frequently built in such a manner that any injured piece may be readily taken out and renewed without interfering with traffic. Trestles consist of two parts—the supports called "bents," and the stringers and floor system. As the stringers and floor system are the same for both pile and framed trestles, the "bents" are all that need be considered separately.

PILE TRESTLES.

129. Pile bents. A pile bent consists generally of four piles driven into the ground deep enough to afford not only sufficient vertical resistance but also lateral resistance. On top of these piles is placed a horizontal "cap." The caps are fastened to the tops of the piles by methods illustrated in Fig. 66. The

method of fastening shown in each case should not be considered
as applicable only to the particular type of pile bent used to illus-
trate it.   Fig. 66 (a and d) illustrates a mortise-joint with a hard-
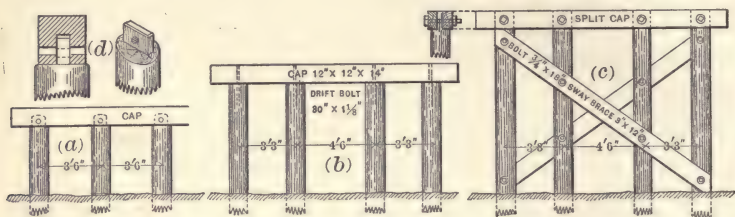
Fig. 66.

wood pin about $1\frac{1}{4}''$ in diameter.   The hole for the pin should
be bored separately through the cap and the mortise, and
the hole through the cap should be at a slightly higher
level than that through the mortise, so that the cap will be
drawn down tight when the pin is driven.   Occasionally an
iron dowel (an iron pin about $1\frac{1}{2}''$ in diameter and about $6''$
long) is inserted partly in the cap and partly in the pile.   The
use of drift-bolts, shown in Fig. 66 (b), is cheaper in first cost, but
renders repairs and renewals very troublesome and expensive.
"Split caps," shown in Fig. 66 (c), are formed by bolting two
half-size strips on each side of a tenon on top of the pile.
Repairs are very easily and cheaply made without interference
with the traffic and without injuring other pieces of the bent.
The smaller pieces are more easily obtainable in a sound con-
dition; the decay of one does not affect the other, and the first
cost is but little if any greater than the method of using a single
piece.   For further discussion, see § 136.

For very light traffic and for a height of about 5 feet three
vertical piles will suffice, as shown in Fig. 66 (a).   Up to a height
of 8 or 10 feet four piles may be used without sway-bracing, as
in Fig. 66 (b), if the piles have a good bearing.   For heights
greater than 10 feet sway-bracing is generally necessary.   The
outside piles are frequently driven with a batter varying from
1 : 12 to 1 : 4.

Piles are made, if possible, from timber obtained in the vicinity of the work. Durability is the great requisite rather than strength, for almost any timber is strong enough (except as noted below) and will be suitable if it will resist rapid decay. The following list is quoted as being in the order of preference on account of durability:

1. Red cedar	5. White pine	9. White oak	12. Black oak
2. Red cypress	6. Redwood	10. Post-oak	13. Hemlock
3. Pitch-pine	7. Elm	11. Red oak	14. Tamarac
4. Yellow pine	8. Spruce		

Red-cedar piles are said to have an average life of 27 years with a possible maximum of 50 years, but the timber is rather weak, and if exposed in a river to flowing ice or driftwood is apt to be injured. Under these circumstances oak is preferable, although its life may be only 13 to 18 years.

130. Methods of driving piles. The following are the principal methods of driving piles:

a. A hammer weighing 2000 to 3000 lbs. or more, sliding in guides, is drawn up by horse-power or a portable engine, and allowed to fall *freely*.

b. The same as above except that the hammer does not fall freely, but drags the rope and revolving drum as it falls and is thus quite materially retarded. The mechanism is a little more simple, but is less effective, and is sometimes made deliberately deceptive by a contractor by retarding the blow, in order to apparently indicate the requisite resistance on the part of the pile.

The above methods have the advantage that the mechanism is cheap and can be transported into a new country with comparative ease, but the work done is somewhat ineffective and costly compared with some of the more elaborate methods given below.

c. Gunpowder pile-drivers, which automatically explode a cartridge every time the hammer falls. The explosion not only forces the pile down, but throws up the hammer for the next blow. For a given height of fall the effect is therefore doubled. It has been shown by experience, however, that when it is at-

tempted to use such a pile-driver rapidly the mechanism becomes so heated that the cartridges explode prematurely, and the method has therefore been abandoned.

d. Steam pile-drivers, in which the hammer is operated directly by steam. The hammer falls freely a height of about 40 inches and is raised again by steam. The effectiveness is largely due to the rapidity of the blows, which does not allow time between the blows for the ground to settle around the pile and increase the resistance, which does happen when the blows are infrequent. '' The hammer-cylinder weighs 5500 lbs., and with 60 to 75 lbs. of steam gives 75 to 80 blows per minute. With 41 blows a large unpointed pile was driven 35 feet into a hard clay bottom in half a minute.'' Such a driver would cost about $800.

The above four methods are those usual for dry earth. In very soft wet or sandy soils, where an unlimited supply of water is available, the *water-jet* is sometimes employed. A pipe is fastened along the side of the pile and extends to the pile-point. If water is forced through the pipe, it loosens the sand around the point and, rising along the sides, decreases the side resistance so that the pile sinks by its own weight, aided perhaps by extra weights loaded on. This loading may be accomplished by connecting the top of the pile and the pile-driver by a block and tackle so that a portion of the weight of the pile-driver is continually thrown on the pile.

Excessive driving frequently fractures the pile below the surface and thereby greatly weakens its bearing power. To

prevent excessive '' brooming '' of the top of the pile, owing to the action of the hammer, the top should be protected by an iron ring fitted to the top of the pile. The '' brooming '' not only renders the driving ineffective and hence uneconomical, but vitiates the value of any test of the bearing power of the pile by noting the sinking due to a given

Fig. 67.   weight falling a given distance. If the pile is so soft that brooming is unavoidable, the top should be adzed off

frequently, and especially should it be done just before the final blows which are to test its bearing-power.

In a new country judgment and experience will be required to decide intelligently whether to employ a simple drop-hammer machine, operated by horse-power and easily transported but uneconomical in operation, or a more complicated machine working cheaply and effectively after being transported at greater expense.

131. Pile-driving formulæ. If $R =$ the resistance of a pile, and s the set of the pile during the last blow, w the weight of the pile-hammer, and h the fall during the last blow, then we may state the approximate relation that $Rs = wh$, or $R = \dfrac{wh}{s}$. This is the basic principle of all rational formulæ, but the maximum weight which a pile will sustain after it has been driven some time is by no means equal to the resistance of the pile during the last blow. There are also many other modifying elements which have been variously allowed for in the many proposed formulæ. The formulæ range from the extreme of empirical simplicity to very complicated attempts to allow properly for all modifying causes. As the simplest rule, specifications sometimes require that the piles shall be driven until the pile will not sink more than 5 inches under five consecutive blows of a 2000 lb., hammer falling 25 feet. The "*Engineering News* formula"[*] gives the safe load as $\dfrac{2wh}{s+1}$, in which $w =$ weight of hammer, $h =$ fall in *feet*, $s =$ set of pile in *inches* under the last blow. This formula is derived from the above basic formula by calling the safe load $\frac{1}{6}$ of the final resistance, and by adding (arbitrarily) 1 to the final set (s) as a compensation for the extra resistance caused by the settling of earth around the pile between each blow. This formula is used only for ordinary hammer-driving. When the piles are driven by a steam pile-driver the formula becomes

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[*] *Engineering News*, Nov. 17, 1892.

safe load $= \dfrac{2wh}{s + 0.1}$.   For the "gunpowder pile-driver," since the explosion of the cartridge drives the pile in with the same force with which it throws the hammer upward, the effect is *twice* that of the fall of the hammer, and the formula becomes safe load $= \dfrac{4wh}{s + 0.1}$.   In these last two formulæ the constant in the denominator is changed from $s + 1$ to $s + 0.1$.   The constant (1.0 or 0.1) is supposed to allow, as before stated, for the effect of the extra resistance caused by the earth settling around the pile between each blow.   The more rapid the blows the less the opportunity to settle and the less the proper value of the constant.

The above formulæ have been given on account of their simplicity and their practical agreement with experience.   Many other formulæ have been proposed, the majority of which are more complicated and attempt to take into account the weight of the pile, resistance of the guides, etc.   While these elements, as well as many others, have their influence, their effect is so overshadowed by the indeterminable effect of other elements—as, for example, the effect of the settlement of earth around the pile between blows—that it is useless to attempt to employ anything but a purely empirical formula.

132. Pile-points and pile-shoes. Piles are generally sharpened to a blunt point.   If the pile is liable to strike boulders, sunken logs, or other obstructions which are liable to turn the point, it
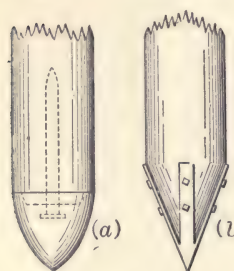
FIG. 68.

is necessary to protect the point by some form of shoe.   Several forms in cast iron have been used, also a wrought-iron shoe, having four "straps" radiating from the apex, the straps being nailed on to the pile, as shown in Fig. 68 (*b*).   The cast-iron form shown in Fig. 68 (*a*) has a base cast around a drift-bolt.   The recess on the top of the base receives the bottom of the pile and prevents a tendency to split the bottom of the pile or to force the shoe off laterally.

133. Details of design. No theoretical calculations of the strength of pile bents need be attempted on account of the extreme complication of the theoretical strains, the uncertainty as to the real strength of the timber used, the variability of that strength with time, and the insignificance of the economy that would be possible even if exact sizes could be computed. The piles are generally required to be not less than 10″ or 12″ in diameter at the large end. The P. R. R. requires that they shall be " not less than 14 and 7 inches in diameter at butt and small end respectively, exclusive of bark, which must be removed." The removal of the bark is generally required in good work. Soft *durable* woods, such as are mentioned in § 129, are best for the piles, but the caps are generally made of oak or yellow pine. The caps are generally 14 feet long (for single track) with a cross-section 12″ × 12″ or 12″ × 14″. "Split caps" would consist of two pieces 6″ × 12″. The sway-braces, never used for less heights than 6′, are made of 3″ × 12″ timber, and are spiked on with ⅝″ spikes 8″ long. The floor system will be the same as that described later for framed trestles.

134. Cost of pile trestles. The cost, per linear foot, of piling depends on the method of driving, the scarcity of suitable timber, the price of labor, the length of the piles, and the amount of shifting of the pile-driver required. The cost of soft-wood piles varies from 8 to 15 c. per lineal foot, and the cost of oak piles varies from 10 to 30 c. per foot according to the length, the longer piles costing more per foot. The cost of driving will average about $2.50 per pile, or 7.5 to 10 c. per lineal foot. Since the cost of shifting the pile-driver is quite an item in the total cost, the cost of driving a long pile would be *less* per foot than for a short pile, but on the other hand the cost of the pile is *greater* per foot, which tends to make the total cost per foot constant. Specifications generally say that the piling will be paid for per lineal foot of piling *left in the work.* The wastage of the tops of piles sawed off is always something, and is frequently very large. Sometimes a small amount per foot of piling sawed off is allowed the contractor as compensation for

his loss. This reduces the contractor's risk and possibly reduces
his bid by an equal or greater amount than the extra amount
actually paid him.

135. Typical Design. A typical design for a framed trestle
bent is given in Fig. 69. This represents, with slight variations
of detail, the plan according to which a large part of the framed
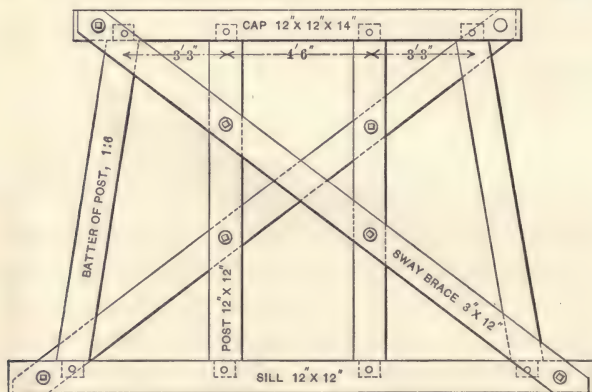
Fig. 69.

trestle bents of the country have been built—i.e., of those less
than 20 or 30 feet in height, not requiring multiple-story
construction.

136. Joints. (a) **The mortise-and-tenon joint** is illustrated in
Fig. 69 and also in Fig. 66 (*a*). The tenon should be about
3″ thick, 8″ wide, and 5½″ long. The mortise should be cut
a little deeper than the tenon. "Drip-holes" from the mortise

Fig. 70.

to the outside will assist in draining off water that
may accumulate in the joint and thus prevent the
rapid decay that would otherwise ensue. These
joints are very troublesome if a single post decays
and requires renewal. It is generally required that
the mortise and tenon should be thoroughly daubed
with paint before putting them together. This will tend to

make the joint water-tight and prevent decay from the accumulation and retention of water in the joint.

(b) **The plaster joint.** This joint is made by bolting and spiking a $3'' \times 12''$ plank on both sides of the joint. The cap and sill should be notched to receive the posts. Repairs are greatly facilitated by the use of these joints. This method has been used by the Delaware and Hudson Canal Co. [R. R.].
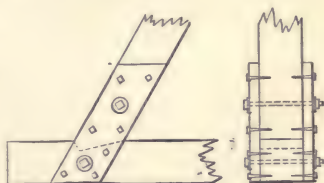
FIG. 71.

(c) **Iron plates.** An iron plate of the form shown in Fig. 72 (b) is bent and used as shown in Fig. 72 (a). Bolts passing through the bolt-holes shown secure the plates to the timbers and make a strong joint which may be readily loosened for repairs. By slight modifications in the design the method may be used for inclined posts and complicated joints.
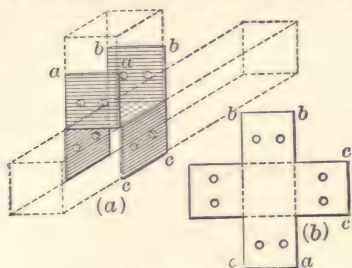
FIG. 72.

(d) **Split caps and sills.** These are described in § 129. Their advantages apply with even greater force to framed trestles.

(e) **Dowels and drift-bolts.** These joints facilitate cheap and rapid construction, but renewals and repairs are very difficult, it being almost impossible to extract a drift-bolt which has been driven its full length without splitting open the pieces containing it. Notwithstanding this objection they are extensively used, especially for temporary work which is not expected to be used long enough to need repairs.

137. Multiple-story construction. Single-story framed trestle bents are used for heights up to 18 or 20 feet and exceptionally up to 30 feet. For greater heights some such construction as is illustrated in a skeleton design in Fig. 73 is used. By using split sills between each story and separate vertical and batter posts in each story, any piece may readily be removed and

renewed if necessary.

Fig. 73.

.The height of these stories varies, in different designs, from 15 to 25 and even 30 feet. In some designs the structure of each story is independent of the stories above and below. This greatly facilitates both the original construction and subsequent repairs. In other designs the verticals and batter-posts are made continuous through two consecutive stories. The structure is somewhat stiffer, but is much more diffi-cult to repair.
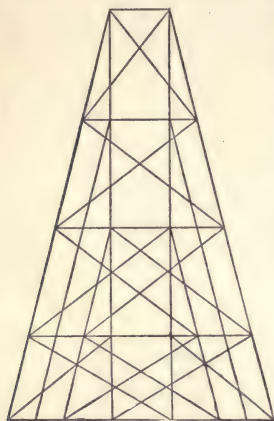
Since the bents of any trestle are usually of variable height and those heights are not always an even multiple of the uniform height desired for the stories, it becomes necessary to make the upper stories of uniform height and let
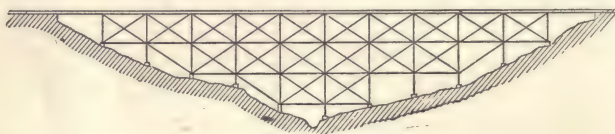
Fig. 74.

the odd amount go to the lowest story, as shown in Figs. 73 and 74.

138. Span. The shorter the span the greater the number of trestle bents; the longer the span the greater the required strength of the stringers supporting the floor. Economy de-mands the adoption of a span that shall make the sum of these requirements a minimum. The higher the trestle the greater the cost of each bent, and the greater the span that would be justifiable. Nearly all trestles have bents of variable height, but the advantage of employing uniform standard sizes is so great that many roads use the same span and sizes of timber not only for the panels of any given trestle, but also for all trestles

regardless of height. The spans generally used vary from 10 to 16 feet. The Norfolk and Western R. R. uses a span of 12' 6'' for all single-story trestles, and a span of 25' for all multiple-story trestles. The stringers are the same in both cases, but when the span is 25 feet, knee-braces are run
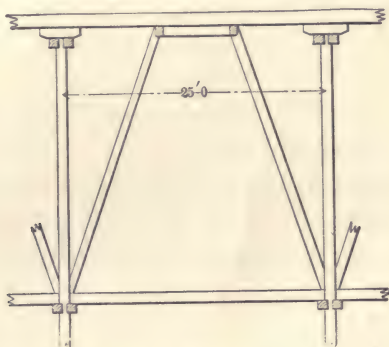
Fig. 75.

from the sill of the first story below to near the middle of each set of stringers. These knee-braces are connected at the top by a " straining-beam " on which the stringers rest, thus supporting the stringer in the center and virtually reducing the span about one-half.

139. Foundations. (a) Piles. Piles are frequently used as a foundation, as in Fig. 76, particularly in soft ground, and also for temporary structures. These foundations are cheap, quickly constructed, and are particularly valuable when it is financially necessary to open the road for traffic as soon as possible and with the least expenditure of money; but there is the disadvantage

Fig. 76.

of inevitable decay within a few years unless the piles are chemically treated, as will be discussed later. Chemical treatment, however, increases the cost so that such a foundation would often cost more than a foundation of stone. A pile should be driven under each post as shown in Fig. 76.
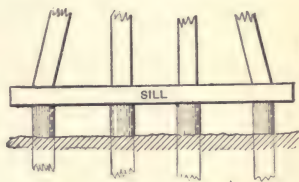
(b) **Mud-sills.** Fig. 77 illustrates the use of mud-sills as
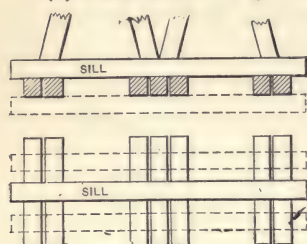
built by the Louisville and Nash-
ville R. R. Eight blocks $12'' \times 12''$
$\times 6'$ are used under each bent.
When the ground is very soft, two
additional timbers ($12'' \times 12'' \times$
length of bent-sill), as shown by the
dotted lines, are placed underneath.
The number required evidently de-

Fig. 77.

pends on the nature of the ground.

(c) **Stone foundations.** Stone foundations are the best and
the most expensive. For very high trestles the Norfolk and
Western R.R. employs foundations as shown in Fig. 78, the

SILL OF TRESTLE

Fig. 78.

walls being 4 feet thick. When the height of the trestle is 72
feet or less (the plans requiring for $72'$ in height a foundation-
wall $39'$ $6'$ long) the foundation is made continuous. The sill
of the trestle should rest on several short lengths of $3'' \times 12''$
plank, laid transverse to the sill on top of the wall.

140. Longitudinal bracing. This is required to give the
structure longitudinal stiffness and also to reduce the columnar
length of the posts. This bracing generally consists of hori-
zontal "waling-strips" and diagonal braces. Sometimes the
braces are placed wholly on the outside posts unless the trestle
is very high. For single-story trestles the P. R. R. employs
the "laced" system, i.e., a line of posts joining the cap of one
bent with the sill of the next, and the sill of that bent with the
cap of the next. Some plans employ braces forming an $\times$ in
alternate panels. Connecting these braces in the center more
than doubles their columnar strength. Diagonal braces, when
bolted to posts, should be fastened to them as near the ends of

the posts as possible.   The sizes employed vary largely, depend-
ing on the clear length and on whether they are expected to act
by tension or compression. $3'' \times 12''$ planks are often used
when the design would require tensile strength only, and
$8'' \times 8''$ posts are often used when compression may be
expected.

141. **Lateral bracing.**   Several of the more recent designs of
trestles employ diagonal lateral bracing between the caps of
adjacent bents.   It adds greatly to the stiffness of the trestle
and better maintains its alignment. $6'' \times 6''$ posts, forming
an $\times$ and connected at the center, will answer the purpose.

142. **Abutments.**   When suitable stone for masonry is at
hand and a suitable subsoil for a foundation is obtainable without
too much excavation, a masonry abutment will be the best.
Such an abutment would probably be used when masonry
footings for trestle bents were employed (§ 139, *c*).

Another method is to construct a " crib " of $10'' \times 12''$
timber, laid horizontally, drift-bolted together, securely braced
and embedded into the ground.   Except for temporary con-
struction such a method is generally objectionable on account of
rapid decay.

Another method, used most commonly for pile trestles, and
for framed trestles having pile
foundations (§ 139, *a*), is to use a
pile bent at such a place that the
natural surface on the *up-hill* side
is not far below the cap, and the
thrust of the material, filled in to
bring the surface to grade, is insig-
nificant. $3'' \times 12''$ planks are placed
behind the piles, cap, and stringers to retain the filled material.
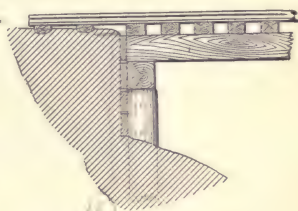
Fig. 79.

FLOOR SYSTEMS.

143. **Stringers.**   The general practice is to use two, three,
and even four stringers under each rail.   Sometimes a stringer

is placed under each guard-rail.   Generally the stringers are
made of two panel lengths and laid so that the joints alternate.
A few roads use stringers of only one panel length, but this
practice is strongly condemned by many engineers.   The
stringers should be separated to allow a circulation of air around
them and prevent the decay which would occur if they were
placed close together.   This is sometimes done by means of 2″
planks, 6′ to 8′ long, which are placed over each trestle bent.
Several bolts, passing through all the stringers forming a group
and through the separators, bind them all into one solid con-
struction.   Cast-iron " spools " or washers, varying from 4″ to
¾″ in length (or thickness), are sometimes strung on each bolt so
as to separate the stringers.   Sometimes washers are used
between the separating planks and the stringers, the object of
the separating planks then being to bind the stringers, especially
abutting stringers, and increase their stiffness.

The most common size for stringers is $8″ \times 16″$.   The
Pennsylvania Railroad varies the width, depth, and number of
stringers under each rail according to the clear span.   It may
be noticed that, assuming a uniform load per running foot, both

Clear span.	No. of pieces under each rail.	Width.	Depth.
10 feet	2	8 inches	15 inches
12 "	2	8 "	16 "
14 "	2	10 "	17 "
16 "	3	8 "	17 "

the pressure per square inch at the ends of the stringers (the
caps having a width of 12″) and also the stress due to trans-
verse strain are kept *approximately* constant for the variable gross
load on these varying spans.

144. Corbels.   A corbel (in trestle-work) is a stick of timber
(perhaps two placed side by side), about 3′ to 6′ long, placed
underneath and along the stringers and resting on the cap.
There are strong prejudices for and against their use, and a

corresponding diversity in practice. They are bolted to the stringers and thus stiffen the joint. They certainly reduce the objectionable crushing of the fibers at each end of the stringer, but if the corbel is no wider than the stringers, as is generally the case, the area of pressure between the corbels and the cap is

FIG. 80.

no greater and the pressure per square inch on the cap is no less than the pressure on the cap if no corbels were used. If the corbels and cap are made of hard wood, as is recommended by some, the danger of crushing is lessened, but the extra cost and the frequent scarcity of hard wood, and also the extra cost and labor of using corbels, may often neutralize the advantages obtained by their use.

145. Guard-rails. These are frequently made of $5'' \times 8''$ stuff, notched $1''$ for each tie. The sizes vary up to $8'' \times 8''$, and the depth of notch from $\frac{3}{4}''$ to $1\frac{1}{2}''$. They are generally bolted to every third or fourth tie. It is frequently specified that they shall be made of oak, white pine, or yellow pine. The joints are made over a tie, by halving each piece, as illustrated in Fig. 81. The joints on opposite sides of the trestle should be

FIG. 81.

"staggered." Some roads fasten every tie to the guard-rail, using a bolt, a spike, or a lag-screw.

Guard-rails were originally used with the idea of preventing the wheels of a derailed truck from running off the ends of the ties. But it has been found that an outer guard-rail alone (without an inner guard-rail) becomes an actual element of danger, since it has frequently happened that a derailed wheel has caught

on the outer guard-rail, thus causing the truck to slew around
and so produce a dangerous accident.   The true function of the
outside guard-rail is thus changed to that of a tie-spacer, which
keeps the ties from spreading when a derailment occurs.   The
inside guard-rail generally consists of an ordinary steel rail
spiked about 10 inches inside of the running rail.   These inner
guard-rails should be bent inward to a point in the center of the
track about 50 feet from the end of the bridge or trestle.   If
the inner guard-rails are placed with a clear space of 10 inches
inside the running rail, the outer guard-rails should be *at least*
6′ 10″ apart.   They are generally much farther apart than this.

146. Ties on trestles.   If a car is derailed on a bridge or
trestle, the heavily loaded wheels are apt to force their way be-
tween the ties by displacing them unless the ties are closely
spaced and fastened.   The clear space between ties is generally
equal to or less than their width.   Occasionally it is a little more
than their width.   6″ × 8″ ties, spaced 14″ to 16″ from cen-
ter to center, are most frequently used.   The length varies from
9′ to 12′ for single track.   They are generally notched $\frac{1}{2}$″ deep
on the under side where they rest on the stringers.   Oak ties
are generally required even when cheaper ties are used on the
other sections of the road.   Usually every third or fourth tie is
bolted to the stringers.   When stringers are placed underneath
the guard-rails, bolts are run from the top of the guard-rail to
the under side of the stringer.   The guard-rails thus hold down
the whole system of ties, and no direct fastening of the ties to
the stringers is needed.

147. Superelevation of the outer rail on curves.   The location
of curves on trestles should be avoided if possible, especially
when the trestle is high.   Serious additional strains are in-
troduced especially when the curvature is sharp or the
speed high.   Since such curves are sometimes practically un-
avoidable, it is necessary to design the trestle accordingly.
If a train is stopped on a curved trestle, the action of the train on
the trestle is evidently vertical.   If the train is moving with a
considerable velocity, the resultant of the weight and the cen-

trifugal action is a force somewhat inclined from the vertical. Both of these conditions may be expected to exist at times. If the *axis* of the system of posts is vertical (as illustrated in methods *a*, *b*, *c*, *d*, and *e*), any lateral force, such as would be produced by a moving train, will tend to rack the trestle bent. If the stringers are set vertically, a centrifugal force likewise tends to tip them sidewise. If the axis of the system of posts (or of the stringers) is inclined so as to coincide with the pressure of the train on the trestle when the train is moving at its normal velocity, there is no tendency to rack the trestle when the train is moving at that velocity, but there will be a tendency to rack the trestle or twist the stringers when the train is stationary. Since a moving train is usually the normal condition of affairs, as well as the condition which produces the maximum stress, an inclined axis is evidently preferable from a theoretical standpoint; but whatever design is adopted, the trestle should evidently be sufficiently cross-braced for either a moving or a stationary load, and any proposed design must be studied as to the effect of *both* of these conditions. Some of the various methods of securing the requisite superelevation may be described as follows:

(a) **Framing the outer posts longer than the inner posts, so that the cap is inclined at the proper angle; axis of posts vertical.** (Fig. 82.) The method requires more work in framing the trestle, but simplifies subsequent track-laying and maintenance, unless it should be found that the superelevation adopted is unsuitable, in which case it could be corrected by one of the other methods given below. The stringers tend to twist when the train is stationary.
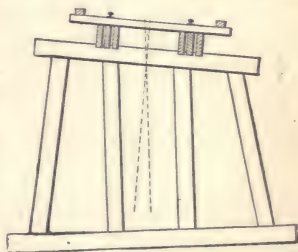
FIG. 82

(b) **Notching the cap so that the stringers are at a different elevation.** (Fig. 83.) This weakens the cap and requires that all ties shall be notched to a bevelled surface to fit the stringers,

which also weakens the ties. A centrifugal force will tend to twist the stringers and rack the trestle.
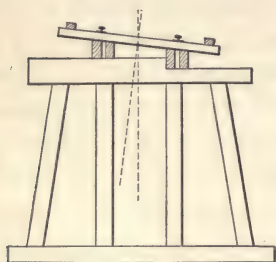
FIG. 83.

(c) **Placing wedges underneath the ties at each stringer.** These wedges are fastened with two bolts. Two or more wedges will be required for each tie. The additional number of pieces required for a long curve will be immense, and the work of inspection and keeping the nuts tight will greatly increase the cost of maintenance.

(d) **Placing a wedge under the outer rail at each tie.** This requires but one extra piece per tie. There is no need of a wedge under the inner tie in order to make the rail normal to the tread. The resulting inward inclination is substantially that produced by some forms of rail-chairs or tie-plates. The spikes (a little longer than usual) are driven through the wedge into the tie. Sometimes " lag-screws " are used instead of spikes. If experience proves that the superelevation is too much or too little, it may be changed by this method with less work than by any other.

(e) **Corbels of different heights.** When corbels are used (see § 144) the required inclination of the floor system may be obtained by varying the depth of the corbels.

(f) **Tipping the whole trestle.** This is done by placing the trestle on an inclined foundation. If very much inclined, the trestle bent must be secured against the possibility of slipping sidewise, for the slope would be considerable with a sharp curve, and the vibration of a moving train would reduce
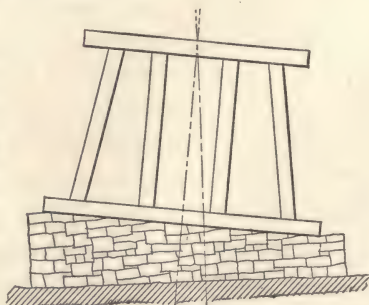
FIG. 84.

the coefficient of friction to a comparatively small quantity.

(g) **Framing the outer posts longer.** This case is identical

with case (*a*) except that the axis of the system of posts is inclined, as in case (*f*), but the sill is horizontal.

The above-described plans will suggest a great variety of methods which are possible and which differ from the above only in minor details.

148. Protection from fire. Trestles are peculiarly subject to fire, from passing locomotives, which may not only destroy the trestle, but perhaps cause a terrible disaster. This danger is sometimes reduced by placing a strip of galvanized iron along the top of each set of stringers and also along the tops of the caps. Still greater protection was given on a long trestle on the Louisville and Nashville R. R. by making a solid flooring of timber, covered with a layer of ballast on which the ties and rails were laid as usual.

Barrels of water should be provided and kept near all trestles, and on very long trestles barrels of water should be placed every two or three hundred feet along its length. A place for the barrels may be provided by using a few ties which have an extra length of about four feet, thus forming a small platform, which should be surrounded by a railing. The track-walkers should be held accountable for the maintenance of a supply of water in these barrels, renewals being frequently necessary on account of evaporation. Such platforms should also be provided as REFUGE-BAYS for track-walkers and trackmen working on the trestle. On very long trestles such a platform is sometimes provided with sufficient capacity for a hand-car.

149. Timber. Any strong durable timber may be used when the choice is limited, but oak, pine, or cypress are preferred when obtainable. When all of these are readily obtainable, the various parts of the trestle will be constructed of different kinds of wood—the stringers of long-leaf pine, the posts and braces of pine or red cypress, and the caps, sills, and corbels (if used) of white oak. The use of oak (or a similar hard wood) for caps, sills, and corbels is desirable because of its greater strength in resisting crushing across the grain, which is the critical test for these parts. There is no physiological basis to

the objection, sometimes made, that different species of timber, in contact with each other, will rot quicker than if only one kind of timber is used. When a very extensive trestle is to be built at a place where suitable growing timber is at hand but there is no convenient sawmill, it will pay to transport a portable sawmill and engine and cut up the timber as desired.

150. Cost of framed timber trestles. The cost varies widely on account of the great variation in the cost of timber. When a railroad is first penetrating a new and undeveloped region, the cost of timber is frequently small, and when it is obtainable from the company's right-of-way the only expense is felling and sawing. The work per M., B. M., is small, considering that a single stick $12'' \times 12'' \times 25'$ contains 300 feet, B. M., and that sometimes a few hours' work, worth less than $1, will finish all the work required on it. Smaller pieces will of course require more work per foot, B. M. Long-leaf pine can be purchased from the mills at from $8 to $12 per M. feet, B. M., according to the dimensions. To this must be added the freight and labor of erection. The cartage from the nearest railroad to the trestle may often be a considerable item. Wrought iron will cost about 3 c. per pound and cast iron 2 c., although the prices are often lower than these. The amount of iron used depends on the detailed design, but, as an average, will amount to $1.50 to $2 per 1000 feet, B. M., of timber. A large part of the trestling of the country has been built at a contract price of about $30 per 1000 feet, B. M., erected. While the cost will frequently rise to $40 and even $50 when timber is scarce, it will drop to $13 (cost quoted) when timber is cheap.

DESIGN OF WOODEN TRESTLES.

151. Common practice. A great deal of trestling has been constructed without any rational design except that custom and experience have shown that certain sizes and designs are *probably* safe. This method has resulted occasionally in failures but more frequently in a very large waste of timber. Many railroads

PLATE I.

NORFOLK & WESTERN R.R.
STANDARD ONE AND TWO STORY TRESTLES.
(SEPT. 10, 1891.)

ONE STORY BENT

$\left(\dfrac{H}{2} + 2.4\right) + \frac{1}{2}'' =$ LENGTH OF SILL.

H — 1'11" = LENGTH OF 12" x 12" TIMBER REQUIRED FOR PLUMB POSTS.
H — 21'11" = " " " " " IN LOWER STORY, DOUBLE BENTS,
H — 3'5" = " " " " " SINGLE "
H — 22'11" = " " " " " DOUBLE "

(LENGTH OF PLUMB POST X 1.031) + 3" = LENGTH OF BATTER POST, EXCEPT IN 6"x 12" INTERMEDIATE

BATTER POSTS WHERE, ADD 6" INSTEAD OF 3".
WITH ALTERNATIVE ARRANGEMENT OF STRINGERS, ADD 2" TO LENGTHS GIVEN BY ABOVE FORMULAS.

" — PLUMB POSTS BETWEEN SHOULDERS,

BATTER POST 10" x 12"
NOT OVER 36'0 LONG
SWAY BRACE 2 x 10
PLUMB POST
12" x 12" NOT OVER 35'3" LONG
22' LONG
3 x 12"
3 x 12"
2 x 10"
2 x 16"
SPIKED TO ALL PLATES
INTERMEDIATE STRIPS
2 SILLS
BOLTED AT ENDS ONLY

Not over 11'6

TWO STORY BENT

18'0 to bottom of cap

GUARD RAIL 6" x 8"
KNEE BRACE
CAP 10 x 12 x 9'0
KNEE BRACE
12" x 10"
CENTER LINE
12" x 10"
SUBGRADE
TIES DAPPED 1" FOR STRINGER

PACKING BLOCK 2" x 16" x 1'6"
STRINGERS 9 PIECES
8" x 16" x 26'0"
CORBEL 12" x 18" x 6'0"
KNEE BRACE 10" x 12" x 21'9"
STRAIGHTEN LEAF 10" x 12" x 6'0"
GUARD RAILS 6" x 8"
25'0" CENTER TO CENTER OF PLUMB POSTS FOR TWO STORY BENTS OR HIGHER.
15'6 CENTER TO CENTER OF PLUMB POSTS FOR ONE STORY BENTS OR HIGHER.

WALING STRIPS 3" x 12"
LONGITUDINAL BRACE 3" x 12"
28'0" LONG
OVERLAP 3'0"
KNEE BRACE 10" x 12" x 21'3"
2 x 10"
2 x 10"
MUD SILLS 5" x 14" x 3'0"

PACKING PIECE 2" x 16" x 6'0"
ALTERNATE ARRANGEMENT FOR 3 STRINGERS
7" x 16" x 1½ APART
2 CORBELS 12" x 18" x 6'0"
CENTER LINE

SWAY BRACE 10" x 12"
8" x 12
2 x 10"
12" x 12"
3" x 12"
3" x 12"
10 x 12"

BATTER POST 10" x 12" x 20'6"
BATTER, 2½" PER FOOT
SWAY BRACE 2"
PLUMB POST 12" x 12" x 20'0"
2 CAPS 6" x 12" x 16'7"
CENTER LINE
5" x 12" x 16'7"
2 SILLS
4" x 12"

CROSS TIE 8" x 10" x 14'0"
15'C. TO C.
SUBGRADE
2 CAPS, 6" x 12" x 10'0"

Not less than 15'

employ a uniform size for all posts, caps, and sills, and a uniform size for stringers, all regardless of the height or span of the trestle.   For repair work there are practical reasons favoring this.   "To attempt to run a large lot of sizes would be more wasteful in the end than to maintain a few stock sizes only. Lumber can be bought more cheaply by giving a general order for 'the run of the mill for the season,' or 'a cargo lot,' specifying approximate percentages of standard stringer size, of 12×12-inch stuff, 10×10-inch stuff, etc., and a liberal proportion of 3- or 4-inch plank, all lengths thrown in. The 12×12-inch stuff, etc., is ordered all lengths, from a certain specified length up.   In case of a wreck, washout, burn-out, or sudden call for a trestle to be completed in a stated time, it is much more economical and practical to order a certain number of carloads of 'trestle stuff' to the ground and there to select piece after piece as fast as needed, dependent only upon the length of stick required.   When there is time to make the necessary surveys of the ground and calculations of strength, and to wait for a special bill of timber to be cut and delivered, the use of different sizes for posts in a structure would be warranted to a certain extent." *   For new construction, when there is generally sufficient time to design and order the proper sizes, such wastefulness is less excusable, and under any conditions it is both safer and more economical to prepare *standard designs* which can be made applicable to varying conditions and which will at the same time utilize as much of the strength of the timber as can be depended on.   In the following sections will be given the elements of the preparation of such standard designs, which will utilize uniform sizes with as little waste of timber as possible. It is *not* to be understood that special designs should be made for each individual trestle.

152. Required elements of strength.   The *stringers* of trestles are subject to transverse strains, to crushing across the grain at the ends, and to shearing along the neutral axis.   The

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* From "Economical Designing of Timber Trestle Bridges."

strength of the timber must therefore be computed for all these kinds of stress. *Caps* and *sills* will fail, if at all, by crushing across the grain; although subject to other forms of stress, these could hardly cause failure in the sizes usually employed. There is an apparent exception to this: if piles are improperly driven and an uneven settlement subsequently occurs, it may have the effect of transferring practically all of the weight to two or three piles, while the *cap* is subjected to a severe transverse strain which may cause its failure. Since such action is caused generally by avoidable errors of construction it may be considered as abnormal, and since such a failure will generally occur by a *gradual* settlement, all danger may be avoided by reasonable care in inspection. *Posts* must be tested for their columnar strength. These parts form the bulk of the trestle and are the parts which can be definitely designed from known stresses. The stresses in the bracing are more indefinite, depending on indeterminate forces, since the inclined posts take up an unknown proportion of the lateral stresses, and the design of the bracing may be left to what experience has shown to be safe, without involving any large waste of timber.

153. Strength of timber. Until recently tests of the strength of timber have generally been made by testing small, selected, well-seasoned sticks of " clear stuff," free from knots or imperfections. Such tests would give results so much higher than the vaguely known strength of large unseasoned " commercial " timber that very large factors of safety were recommended—factors so large as to detract from any confidence in the whole theoretical design. Recently the U. S. Government has been making a thoroughly scientific test of the strength of full-size timber under various conditions as to seasoning, etc. The work has been so extensive and thorough as to render possible the economical designing of timber structures.

One important result of the investigation is the determination of the great influence of the moisture in the timber and the law of its effect on the strength. It has been also shown that timber soaked with water has substantially the same

strength as green timber, even though the timber had once been thoroughly seasoned. Since trestles are exposed to the weather they should be designed on the basis of using green timber. It has been shown that the strength of green timber is very regularly about 55 to 60% of the strength of timber in which the moisture is 12% of the dry weight, 12% being the proportion of moisture usually found in timber that is protected from the weather but not heated, as, e.g., the timber in a barn. Since the moduli of rupture have all been reduced to this standard of moisture (12%), if we take *one-eighth* of the rupture values, it still allows a factor of safety of about five, even on green timber.

Moduli of rupture for various timbers. [12% moisture.]

(Condensed from U. S. Forestry Circular, No. 15.)

No.	Species.	Weight per cubic foot.	Cross-bending.		Crushing end wise.	Crushing across grain.	Shearing along grain.
			Ultimate Strength.	Modulus of Elasticity.			
1	Long-leaf pine.....	38	12 600	2 070 000	8000	1180	700
2	Cuban "	39	13 600	2 370 000	8700	1220	700
3	Short-leaf "	32	10 100	1 680 000	6500	960	700
4	Loblolly "	33	11 300	2 050 000	7400	1150	700
5	White "	24	7 900	1 390 000	5400	700	400
6	Red "	31	9 100	1 620 000	6700	1000	500
7	Spruce "	39	10 000	1 640 000	7300	1200	800
8	Bald cypress.......	29	7 900	1 290 000	6000	800	500
9	White cedar.......	23	6 300	910 000	5200	700	400
10	Douglas spruce....	32	7 900	1 680 000	5700	800	500
11	White oak........	50	13 100	2 090 000	8500	2200	1000
12	Overcup "	46	11 300	1 620 000	7300	1900	1000
13	Post "	50	12 300	2 030 000	7100	3000	1100
14	Cow "	46	11 500	1 610 000	7400	1900	900
15	Red "	45	11 400	1 970000	7200	2300	1100
16	Texan "	46	13 100	1 860 000	8100	2000	900
19	Willow "	45	10 400	1 750 000	7200	1600	900
20	Spanish "	46	12 000	1 930000	7700	1800	900
21	Shagbark hickory..	51	16 000	2 390 000	9500	2700	1100
27	Pignut " ..	56	18 700	2 730000	10900	3200	1200
28	White elm........	34	10 300	1 540000	6500	1200	800
29	Cedar "	46	13 500	1 700000	8000	2100	1300
30	White ash........	39	10 800	1 640000	7200	1900	1100

AVERAGE SAFE ALLOWABLE WORKING UNIT STRESSES, IN POUNDS, PER SQUARE INCH. RECOMMENDED BY THE COMMITTEE ON "STRENGTH OF BRIDGE AND TRESTLE TIMBERS." (ASSOCIATION OF RAILWAY SUPERINTENDENTS OF BRIDGES AND BUILDINGS: FIFTH ANNUAL CONVENTION, NEW ORLEANS, OCTOBER, 1895.)

Kind of timber	Tension		Compression			Transverse		Shearing	
	With grain.	Across grain.	With grain.		Across grain.	Extreme fibre stress.	Modulus of elasticity.	With grain.	Across grain.
			End bearing.	Column under 15 diameters.					
Factor of safety....	Ten.	Ten.	Five.	Five.	Four.	Six.	Two.	Four.	Four.
White oak	1000	200	1400	900	500	1000	550 000	200	1000
White pine	700	50	1100	700	200	700	500 000	100	500
Southern, long-leaf, or Georgia yellow pine	1200	60	1600	1000	350	1200	850 000	150	1250
Douglas, Oregon, and Wash-} Yellow fir.	1200		1600	1200	300	1100	700 000	150	
ington fir or pine ; } Red fir.						800			
Northern or short-leaf yellow pine	1000		1200	800	250	1000	600 000	100	1000
Red pine	900	50	1200	800	200	800	600 000		
Norway pine	900	50	1200	800	200	700	600 000		
Canadian (Ottawa) white pine	800			1000				100	
Canadian (Ontario) red pine	1000			1000	200	800	700 000	100	750
Spruce and Eastern fir	1000	50	1200	800	150	700	600 000	100	600
Hemlock	800			800	200	600	450 000		
Cypress	600		1200	800	200	800	450 000		400
Cedar	600		1200	1000	250	800	350 000	150	400
Chestnut	800			800	200	800	500 000	100	
California redwood	900			800		750	350 000		
California spruce	700			800		800	600 000		

On page 177 there are quoted the values taken from the U. S. Government reports on the strength of timber, the tests probably being the most thorough and reliable that were ever made.

On page 178 are given the "average safe allowable working unit stresses in pounds per square inch," as recommended by the committee on "Strength of Bridge and Trestle Timbers," the work being done under the auspices of the Association of Railway Superintendents of Bridges and Buildings. The report was presented at their fifth annual convention, held in New Orleans, in October, 1895.

154. Loading. As shown in § 138, the span of trestles is always small, is generally 14 feet, and is never greater than 18' except when supported by knee-braces. The greatest load that will ever come on any one span will be the concentrated loading of the drivers of a consolidation locomotive. With spans of 14 feet or less it is impossible for even the four pairs of drivers to be on the same span at once. The weight of the rails, ties, and guard-rails should be added to obtain the total load on the stringers, and the weight of these, plus the weight of the stringers, should be added to obtain the pressure on the caps or corbels. This dead load is almost insignificant compared with the live load and may be included with it. The weight of rails, ties, etc., may be estimated at 200 pounds per foot. To obtain the weight on the caps the weight of the stringers must be added, which depends on the design and on the weight per cubic foot of the wood employed. But as the weight of the stringers is comparatively small, a considerable percentage of variation in weight will have but an insignificant effect on the result. Disregarding all refinements as to actual dimensions, the ordinary maximum loading for standard gauge railroads may be taken as that due to four pairs of driving-axles, spaced 5' 0" apart and giving a pressure of 25,000 pounds per axle. This should be increased to 40,000 pounds per axle (same spacing) for the heaviest traffic. On the basis of 25,000 pounds per axle the following results have been computed:

Span in feet.	Max. mom.— ft. lbs.	Max. shear.	Max load on one cap.
10	65 000	38 500	52 100
12	103 600	45 000	62 700
14	142 400	49 600	74 200
16	181 400	54 725	85 700
18	220 600	60 100	97 900

Although the dead load does not vary in proportion to the live load, yet, considering the very small influence of the dead load, there will be no appreciable error in assuming the corresponding values, for a load of 40,000 lbs. per axle, to be $\frac{40}{25}$ of those given in the above tabulation.

155. Factors of safety.—The most valuable result of the government tests is the knowledge that under given moisture conditions the strength of various species of sound timber is not the variable uncertain quantity it was once supposed to be, but that its strength can be relied on to a comparatively close percentage. This confidence in values permits the employment of lower factors of safety than have heretofore been permissible. Stresses, which when excessive would result in immediate destruction, such as cross-breaking and columnar stresses, should be allowed a higher factor of safety—say 6 or 8 for green timber. Other stresses, such as crushing across the grain and shearing along the neutral axis, which will be apparent to inspection before it is dangerous, may be allowed lower factors—say 3 to 5.

156. Design of stringers.—The strength of rectangular beams of equal width varies as the square of the depth; therefore deep beams are the strongest. On the other hand, when any cross-sectional dimension of timber much exceeds 12″ the cost is much higher per M., B.M., and it is correspondingly difficult to obtain thoroughly sound sticks, free from wind-shakes, etc. Wind-shakes especially affect the shearing strength. Also, if the required transverse strength is obtained by using high narrow stringers, the area of pressure between the stringers and the

cap may become so small as to induce crushing across the grain. This is a very common defect in trestle design. As already indicated in § 138, the span should vary roughly with the average height of the trestle, the longer spans being employed when the trestle bents are very high, although it is usual to employ the same span throughout any one trestle.

To illustrate, if we select a span of 14 feet, the load on one cap will be 74,200 lbs. If the stringers and cap are made of long-leaf yellow pine, which require the closely determined value of 1180 lbs. per square inch to produce a crushing amounting to 3% of the height on timber with 12% moisture, we may use 200 lbs. per square inch as a safe pressure even for green timber; this will require 371 square inches of surface. If the cap is 12″ wide, this will require a width of 31 inches, or say 2 stringers under each rail, each 8 inches wide. For rectangular beams

$$\text{Moment} = \tfrac{1}{6} R' b h^2.$$

Using for R' the safe value 1575 lbs. per square inch, we have

$$142400 \times 12 = \tfrac{1}{6} \times 1575 \times 32 \times h^2,$$

from which $h = 15''.9$. If desired, the width may be increased to 9″ and the depth correspondingly reduced, which will give similarly $h = 14''.8$, or say 15″. This shows that two beams, $9'' \times 15''$, under each rail will stand the transverse bending and have more than enough area for crushing.

The shear per square inch will equal

$$\frac{3}{2} \frac{\text{total shear}}{\text{cross section}} = \frac{3}{2} \frac{49600}{4 \times 9 \times 15} = 138 \text{ lbs. per sq. inch,}$$

which is a safe value, although it should preferably be less. Hence the above combination of dimensions will answer.

The deflection should be computed to see if it exceeds the

somewhat arbitrary standard of $\frac{1}{200}$ of the span.   The deflection for *uniform loading* is

$$\varDelta = \frac{5\,Wl^3}{32bh^3E},$$

in which $l =$ length in inches;
                $W =$ total load, assumed as uniform;
                $E =$ modulus of elasticity, given as 2,070,000 lbs.

per sq. in. for long-leaf pine, 12% dry, and assumed to be 1,200,000 for green timber.   Then

$$\varDelta = \frac{5 \times 72800 \times 168^3}{32 \times 36 \times 15^3 \times 1200000} = 0''.37$$

$$\tfrac{1}{200} \times 168'' = 0''.84,$$

so that the calculated deflection is well within the limit.   Of course the loading is not strictly uniform, but even with a liberal allowance the deflection is still safe.

For the heaviest practice (40000 lbs. per axle) these stringer dimensions must be correspondingly increased.

157. Design of posts.   Four posts are generally used for single-track work.   The inner posts are usually braced by the cross-braces, so that their columnar strength is largely increased; but as they are apt to get more than their share of work, the advantage is compensated and they should be treated as unsupported columns for the total distance between cap and sill in simple bents, or for the height of stories in multiple-story construction. The caps and sills are assumed to have a width of 12''.   It facilitates the application of bracing to have the columns of the same width and vary the other dimension as required.

Unfortunately the experimental work of the U. S. Government on timber testing has not yet progressed far enough to establish unquestionably a general relation between the strength of long columns and the crushing strength of short blocks.   The

following formula has been suggested, but it cannot be considered as established:

$$f = F \times \frac{700 + 15c}{700 + 15c + c^2}, \quad \text{in which}$$

$f =$ allowable working stress per sq. in. for long columns;
$F =$ "   "   "   "   "   "   " short blocks;

$c = \dfrac{l}{d}$;

$l =$ length of column in inches;
$d =$ least cross-sectional dimension in inches.

Enough work has been done to give great reliability to the two following formulæ for white pine and yellow pine, quoted from Johnson's "Materials of Construction," p. 684:

Working load per sq. in. $= p = 1000 - \dfrac{1}{4}\left(\dfrac{l}{h}\right)^2$, long-leaf pine;

"   "   "   "   " $= p = 600 - \dfrac{1}{8}\left(\dfrac{l}{h}\right)^2$, white pine;

in which $l =$ length of column in inches, and
             $h =$ least cross-sectional dimension in inches.

The frequent practice is to use $12'' \times 12''$ posts for all trestles. If we substitute in the above formula $l = 20' = 240''$ and $h = 12''$, we have $p = 1000 - \frac{1}{4}(\frac{240}{12})^2 = 900$ lbs.

$900 \times 144 = 129600$ lbs., the *working load* for *each* post. This is more than the total load on one trestle bent and illustrates the usual great waste of timber. Making the post $8'' \times 12''$ and calculating similarly, we have $p = 775$, and the working load per column is $775 \times 96 = 74400$ lbs. As considerable must be allowed for "weathering," which destroys the strength of the outer layers of the wood, and also for the dynamic effect of the live load, $8'' \times 12''$ may not be too great,

but it is certainly a safe dimension. $12'' \times 6''$ would possibly
prove amply safe in practice.   One method of allowing for
weathering is to disregard the outer half-inch on all sides of the
post, i.e., to calculate the strength of a post one inch smaller in
each dimension than the post actually employed.   On this basis
an $8'' \times 12'' \times 20'$ post, computed as a $7'' \times 11'$ post, would
have a *safe* columnar strength of 706 lbs. per square inch.   With
an area of 77 square inches, this gives a working load of 54362
lbs. for *each post*, or 217448 lbs. for the four posts.   Consider-
ing that 74200 lbs. is the maximum load on one cap (14 feet
span), the great excess of strength is apparent.

158. Design of caps and sills.   The stresses in caps and sills
are very indefinite, except as to crushing across the grain.   As
the stringers are placed almost directly over the inner posts, and
as the sills are supported just under the posts, the transverse
stresses are almost insignificant.   In the above case four posts
have an area of $4 \times 12'' \times 8'' = 384$ sq. in.   The total load,
74200 lbs., will then give a pressure of 193 pounds per square
inch, which is within the allowable limit.   This one feature
might require the use of $8'' \times 12''$ posts rather than $6'' \times 12''$
posts, for the smaller posts, although probably strong enough as
posts, would produce an objectionably high pressure.

159. Bracing.   Although some idea of the stresses in the
bracing could be found from certain assumptions as to wind-
pressure, etc., yet it would probably not be found wise to de-
crease, for the sake of economy, the dimensions which practice
has shown to be sufficient for the work.   The economy that
would be possible would be too insignificant to justify any risk.
Therefore the usual dimensions, given in §§ 139 and 140, should
be employed.

CHAPTER V.

TUNNELS.

SURVEYING.

160. Surface surveys. As tunnels are always dug from each end and frequently from one or more intermediate shafts, it is necessary that an accurate surface survey should be made between the two ends. As the natural surface in a locality where a tunnel is necessary is almost invariably very steep and rough, it requires the employment of unusually refined methods of work to avoid inaccuracies. It is usual to run a line on the surface that will be at every point vertically over the center line of the tunnel. Tunnels are generally made straight unless curves are absolutely necessary, as curves add greatly to the cost. Fig. 85 represents roughly a longitudinal section of the
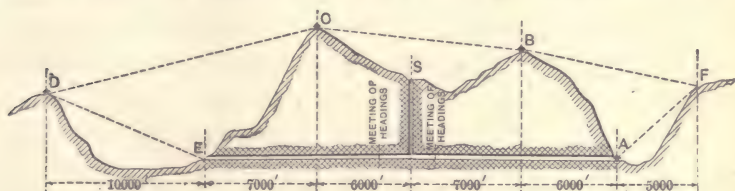
FIG. 85 —SKETCH OF SECTION OF THE HOOSAC TUNNEL.

Hoosac Tunnel. Permanent stations were located at A, B, C, D, E, and F, and stone houses were built at A, B, C, and D. These were located with ordinary field transits at first, and then all the points were placed as nearly as possible in one vertical plane by repeated trials and minute corrections, using a very large specially constructed transit. The stations D and F were necessary because E and A were invisible from C and B.

The alignment at A and E having been determined with great accuracy, the true alignment was easily carried into the tunnel.

The relative elevations of A and E were determined with great accuracy. Steep slopes render necessary many settings of the level per unit of horizontal distance and require that the work be unusually accurate to obtain even fair accuracy per unit of distance. The levels are usually re-run many times until the probable error is a very small quantity.

The exact horizontal distance between the two ends of the tunnel must also be known, especially if the tunnel is on a grade. The usual steep slopes and rough topography likewise render accurate horizontal measurements very difficult. Frequently when the slope is steep the measurement is best obtained by measuring along the slope and allowing for grade. This may be very accurately done by employing two tripods (level or transit tripods serve the purpose very well), setting them up slightly less than one tape-length apart and measuring between horizontal needles set in wooden blocks inserted in the top of each tripod. The elevation of each needle is also observed. The true horizontal distance between two successive positions of the needles then equals the square root of the difference of the squares of the inclined distance and the difference of elevation. Such measurements will probably be more accurate than those made by attempting to hold the tape horizontal and plumbing down with plumb-bobs, because (1) it is practically difficult to hold both ends of the tape truly horizontal; (2) on steep slopes it is impossible to hold the downhill end of a 100-foot tape (or even a 25-foot length) on a level with the other end, and the great increase in the number of applications of the unit of measurement very greatly increases the probable error of the whole measurement; (3) the vibrations of a plumb-bob introduce a large probability of error in transferring the measurement from the elevated end of the tape to the ground, and the increased number of such applications of the unit of measurement still further increases the probable error.

161. Surveying down a shaft. If a shaft is sunk, as at S,
Fig. 85, and it is desired to dig out the tunnel in both directions
from the foot of the shaft so as to meet the headings from the
outside, it is necessary to know, when at the bottom of the
shaft, the elevation, alignment, and horizontal distance from
each end of the tunnel.

The *elevation* is generally carried down a shaft by means of
a steel tape. This method involves the least number of appli-
cations of the unit of measurement and greatly increases the
accuracy of the final result.

The *horizontal distance from each end* may be easily trans-
ferred down the shaft by means of a plumb-bob, using some of
the precautions described in the next paragraph.

To transfer the *alignment* from the surface to the bottom of
a shaft requires the highest skill because the shaft is always
small, and to produce a line perhaps several thousand feet long
in a direction given by two points 6 or 8 feet apart requires
that the two points must be determined with extreme accuracy.
The eminently successful method adopted in the Hoosac Tunnel
will be briefly described : Two beams were securely fastened
across the top of the shaft (1030 feet deep), the beams being
placed transversely to the direction of the tunnel and as far
apart as possible and yet allow plumb-lines, hung from the
intersection of each beam with the tunnel center line, to swing
freely at the bottom of the shaft. These intersections of the
beams with the center line were determined by averaging the
results of a large number of careful observations for alignment.
Two fine parallel wires, spaced about $\frac{1}{16}''$ apart, were then
stretched between the beams so that the center line of the
tunnel bisected at all points the space between the wires.
Plumb-bobs, weighing 15 pounds, were suspended by fine wires
beside each cross-beam, the wires passing between the two
parallel alignment wires and bisecting the space. The plumb-
bobs were allowed to swing in pails of water at the bottom.
Drafts of air up the shaft required the construction of boxes
surrounding the wires. Even these precautions did not suffice

to absolutely prevent vibration of the wire at the bottom through a very small arc. The mean point of these vibrations in each case was then located on a rigid cross-beam suitably placed at the bottom of the shaft and at about the level of the roof of the tunnel. Short plumb-lines were then suspended from these points whenever desired; a transit was set (by trial) so that its line of collimation passed through both plumb lines and the line at the bottom could thus be prolonged.

162. Underground surveys. Survey marks are frequently placed on the timbering, but they are apt to prove unreliable on account of the shifting of the timbering due to settlement of the surrounding material. They should never be placed at the bottom of the tunnel on account of the danger of being disturbed or covered up. Frequently holes are drilled in the roof and filled with wooden plugs in which a hook is screwed exactly on line. Although this is probably the safest method, even these plugs are not always undisturbed, as the material, unless very hard, will often settle slightly as the excavation proceeds. When a tunnel is perfectly straight and not too long, alignment-points may be given as frequently as desired from permanent stations located *outside* the tunnel where they are not liable to disturbance. This has been accomplished by running the alignment through
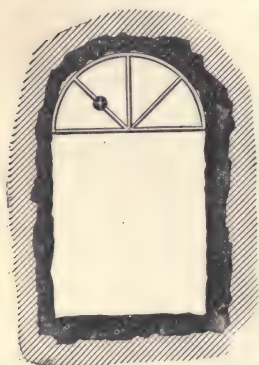
FIG. 86.

the upper part of the cross-section, at one side of the center, where it is out of the way of the piles of masonry material, débris, etc., which are so apt to choke up the lower part of the cross-section. The position of this line relative to the cross-section being fixed, the alignment of any required point of the cross-section is readily found by means of a light frame or template with a fixed target located where this line would intersect the frame when properly placed. A level-bubble on the frame will assist in setting the frame in its proper position.

In all tunnel surveying the cross-wires must be illuminated

by a lantern, and the object sighted at must also be illuminated. A powerful dark-lantern with the opening covered with *ground glass* has been found useful. This may be used to illuminate a plumb-bob string or a very fine rod, or to place behind a brass plate having a narrow slit in it, the axis of the slit and plate being coincident with the plumb-bob string by which it is hung.

On account of the interference to the surveying caused by the work of construction and also by the smoke and dust in the air resulting from the blasting, it is generally necessary to make the surveys at times when construction is temporarily suspended.

163. Accuracy of tunnel surveying. Apart from the very natural desire to do surveying which shall check well, there is an important financial side to accurate tunnel surveying. If the survey lines do not meet as desired when the headings come together, it may be found necessary, if the error is of appreciable size, to introduce a slight curve, perhaps even a reversed curve, into the alignment, and it is even conceivable that the tunnel section would need to be enlarged somewhat to allow for these curves. The cost of these changes and the perpetual annoyance due to an enforced and undesirable alteration of the original design will justify a considerable increase in the expenses of the survey. Considering that the cost of surveys is usually but a small fraction of the total cost of the work, an increase of 10 or even 20% in the cost of the surveys will mean an insignificant addition to the total cost and frequently, if not generally, it will result in a saving of many times the increased cost. The accuracy actually attained in two noted American tunnels is given as follows : The Musconetcong tunnel is about 5000 feet long, bored through a mountain 400 feet high. The error of alignment at the meeting of the headings was $0'.04$, error of levels $0'.015$, error of distance $0'.52$. The Hoosac tunnel is over 25,000 feet long. The heading from the east end met the heading from the central shaft at a point 11274 feet from the east end and 1563 feet from the shaft. The error in alignment was $\frac{5}{16}$ of an inch, that of levels " a few hundredths,"

error of distance "trifling." The alignment, corrected at the shaft, was carried on through and met the heading from the west end at a point 10138 feet from the west end and 2056 feet from the shaft. Here the error of alignment was $\frac{9}{16}''$ and that of levels 0.134 ft.

<div align="center">DESIGN.</div>

164. Cross-sections. Nearly all tunnels have cross-sections peculiar to themselves—all varying at least in the details. The *general* form of a great many tunnels is that of a rectangle surmounted by a semi-circle or semi-ellipse. In very soft material an inverted arch is necessary along the bottom. In such cases the sides will generally be arched instead of vertical. The sides are frequently battered. With very long tunnels, several forms of cross-section will often be used in the same tunnel, owing to differences in the material encountered. In solid rock, which will not disintegrate upon exposure, no lining is required, and the cross-section will be the irregular section left by the blasting, the only requirement being that no rock shall be left within the required cross-sectional figure. Farther on, in the same tunnel, when passing through some very soft treacherous material, it may be necessary to put in a full arch lining—top, sides, and bottom—which will be nearly circular in cross-section. For an illustration of this see Figs. 87 and 88.

The width of tunnels varies as greatly as the designs. Single-track tunnels generally have a width of 15 to 16 feet. Occasionally they have been built 14 feet wide, and even less, and also up to 18 feet, especially when on curves. 24 to 26 feet is the most common width for double track. Many double-track tunnels are only 22 feet wide, and some are 28 feet wide. The heights are generally 19 feet for single track and 20 to 22 feet for double track. The variations from these figures are considerable. The lower limits depend on the cross-section of the rolling stock, with an indefinite allowance for clearance and ventilation. Cross-sections which coincide too closely with what is
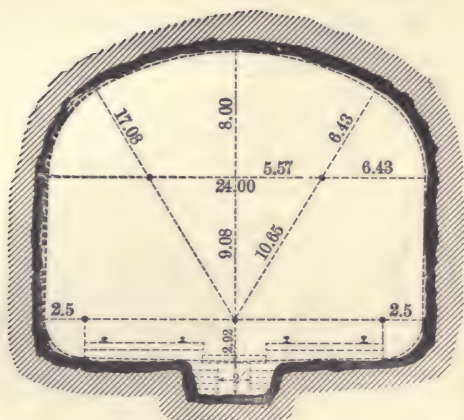
FIG. 87.—HOOSAC TUNNEL. SECTION THROUGH SOLID ROCK.
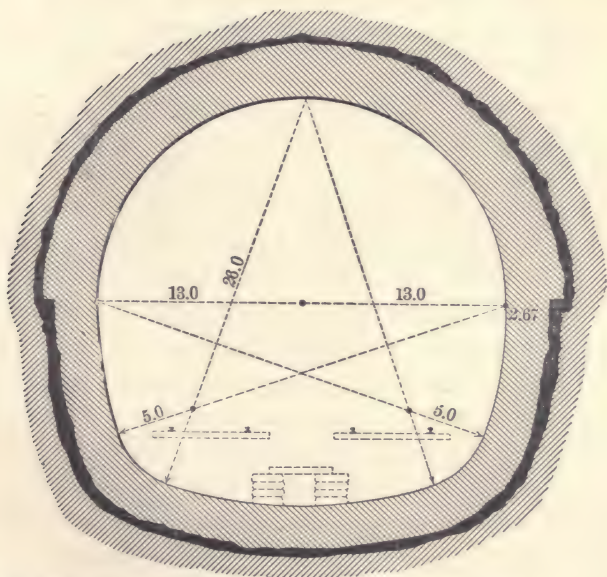
FIG. 88.—HOOSAC TUNNEL. SECTION THROUGH SOFT GROUND.

absolutely required for clearance are objectionable, because any slight settlement of the lining which would otherwise be harmless would then become troublesome and even dangerous. Figs. 87, 88, and 89 * show some typical cross-sections.
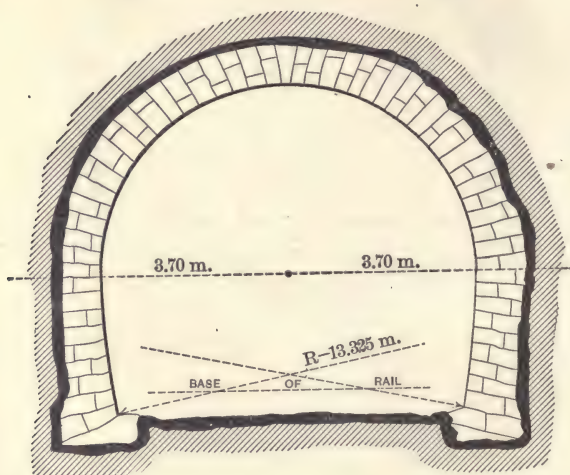
Fig. 89.—St. Cloud Tunnel.

165. Grade. A grade of at least 0.2% is needed for drainage. If the tunnel is at the summit of two grades, the tunnel grade should be practically level, with an allowance for drainage, the actual summit being perhaps in the center so as to drain both ways. When the tunnel forms part of a long ascending grade, it is advisable to reduce the grade through the tunnel unless the tunnel is very short. The additional atmospheric resistance and the decreased adhesion of the driver wheels on the damp rails in a tunnel will cause an engine to work very hard and still more rapidly vitiate the atmosphere until the accumulation of poisonous gases becomes a source of actual danger to the engineer and fireman of the locomotive and of extreme discomfort to the passengers. If the nominal ruling grade of the road were maintained through a tunnel, the maximum resistance would be

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* Drinker's "Tunneling."

PLATE II.
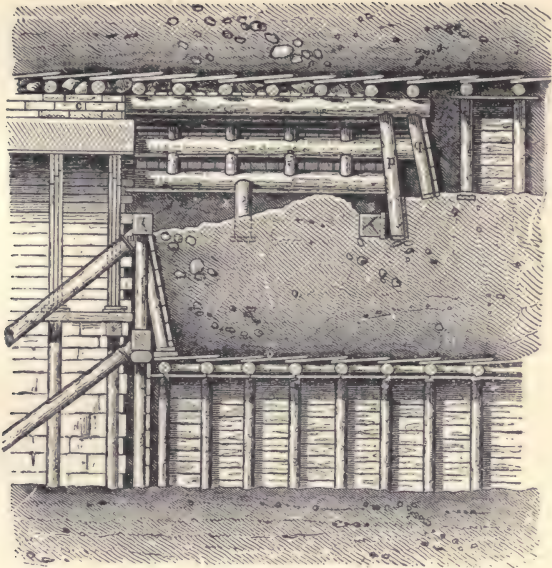
Tunnel-timbering—English System (a).
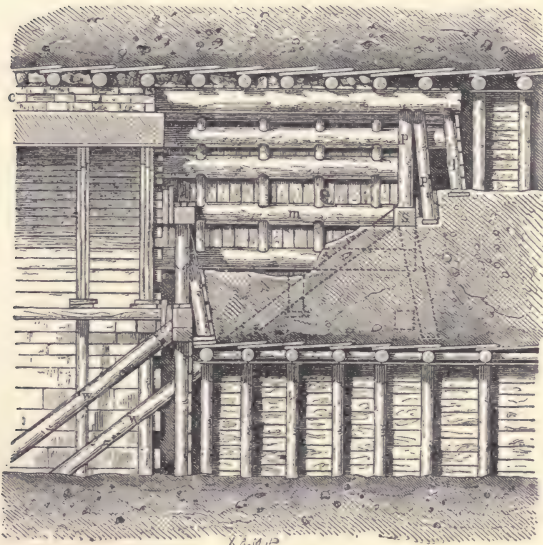
Tunnel-timbering—English System (b).

PLATE III.

TUNNEL-TIMBERING—ENGLISH SYSTEM (c).
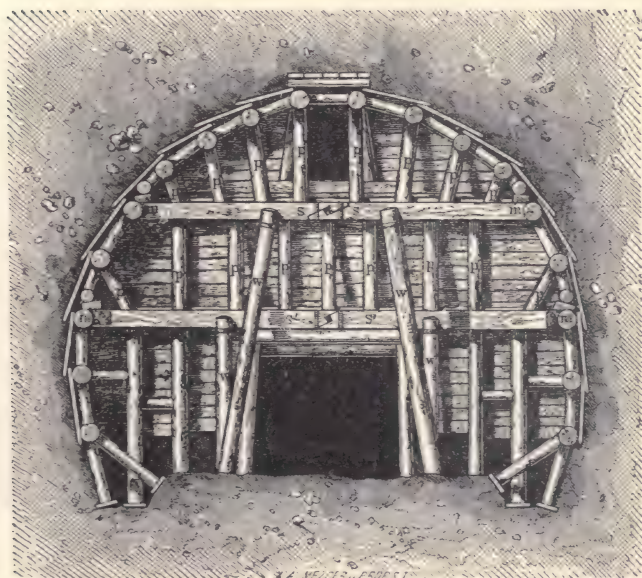
TUNNEL-TIMBERING—ENGLISH SYSTEM (d).

(To face page 192.)

found in the tunnel. This would probably cause trains to stall there, which would be objectionable and perhaps dangerous.

166. Lining. It is a characteristic of many kinds of rock and of all earthy material that, although they may be self-sustaining when first exposed to the atmosphere, they rapidly disintegrate and require that the top and perhaps the sides and even the bottom shall be lined to prevent caving in. In this country, when timber is cheap, it is occasionally framed as an arch and used as the *permanent* lining, but masonry is always to be preferred. Frequently the cross-section is made extra large so that a masonry lining may subsequently be placed inside the wooden lining and thus postpone a large expense until the road is better able to pay for the work. In very soft unstable material, like quicksand, an arch of cut stone voussoirs may be necessary to withstand the pressure. A good quality of brick is occasionally used for lining, as they are easily handled and make good masonry if the pressure is not excessive. Only the best of cement mortar should be used, economy in this feature being the worst of folly. Of course the excavation must include the outside line of the lining. Any excavation which is made outside of this line (by the fall of earth or loose rock or by excessive blasting) must be refilled with stone well packed in. Occasionally it is necessary to fill these spaces with concrete. Of course it is not necessary that the lining be uniform throughout the tunnel.

167. Shafts. Shafts are variously made with square, rectangular, elliptical, and circular cross-sections. The rectangular cross-section, with the longer axis parallel with the tunnel, is most usually employed. Generally the shaft is directly over the center of the tunnel, but that always implies a complicated connection between the linings of the tunnel and shaft, provided such linings are necessary. It is easier to sink a shaft near to one side of the tunnel and make an opening through the nearly vertical side of the tunnel. Such a method was employed in the Church Hill Tunnel, illustrated in Fig. 90.* Fig. 91 † shows

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* Drinker's "Tunneling."

† Ržiha, "Lehrbuch der Gesammten Tunnelbaukunst."

a cross-section for a large main shaft. Many shafts have been
built with the idea of being left open permanently for ventila-
tion and have therefore been elaborately lined with masonry.
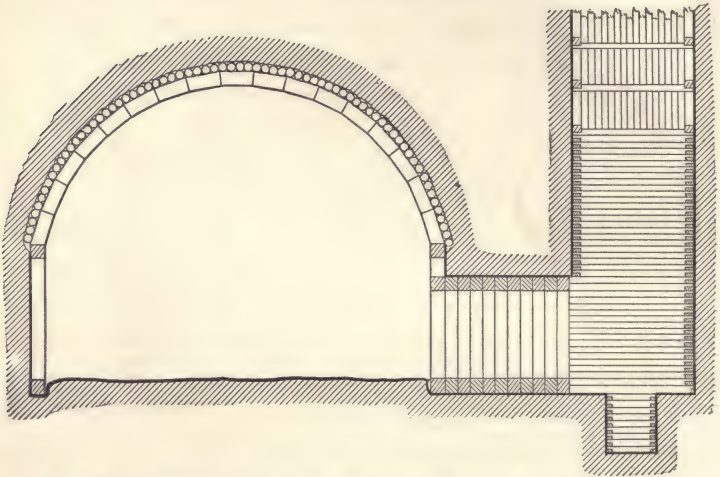
Fig. 90.—Connection with Shaft, Church Hill Tunnel.
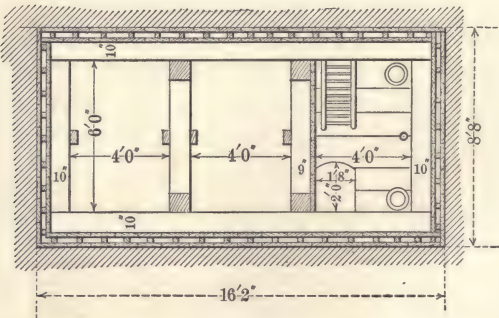
Fig. 91.—Cross-section. Large Main Shaft.

The general consensus of opinion now appears to be that shafts
are worse than useless for ventilation; that the quick passage of
a train through the tunnel is the most effective ventilator; and
that shafts only tend to produce cross-currents and are ineffective
to clear the air. In consequence, many of these elaborately
lined shafts have been permanently closed, and the more recent

PLATE IV.
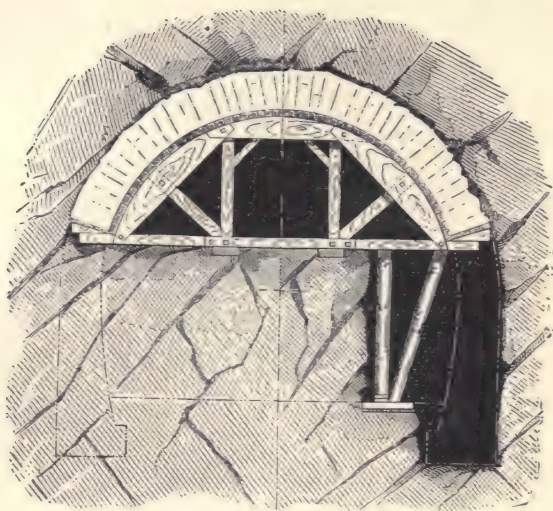
Tunnel-timbering—French System (a).

Tunnel-timbering — French System (b).

PLATE V.
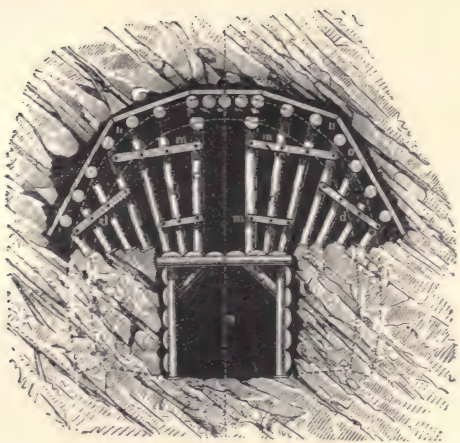
TUNNEL TIMBERING—BELGIAN SYSTEM (*a*).
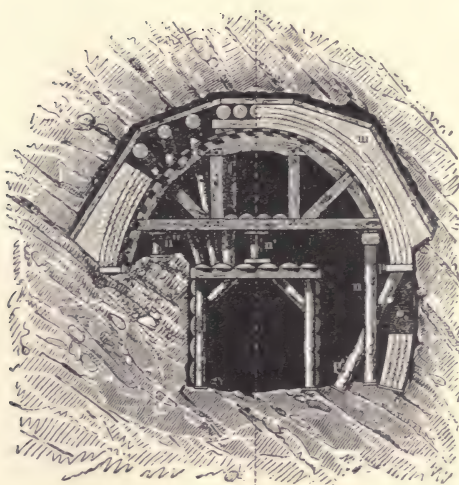
TUNNEL-TIMBERING—BELGIAN SYSTEM (*b*).

practice is to close up a shaft as soon as the tunnel is completed. Shafts always form drainage-wells for the material they pass through, and sometimes to such an extent that it is a serious matter to dispose of the water that collects at the bottom, requiring the construction of large and expensive drains.

168. Drains. A tunnel will almost invariably strike veins of water which will promptly begin to drain into the tunnel and not only cause considerable trouble and expense during construction, but necessitate the provision of permanent drains for its perpetual disposal. These drains must frequently be so large as to appreciably increase the required cross-section of the tunnel. Generally a small open gutter on each side will suffice for this purpose, but in double-track tunnels a large covered drain is often built between the tracks. It is sometimes necessary to thoroughly grout the outside of the lining so that water will not force its way through the masonry and perhaps injure it, but may freely drain down the sides and pass through openings in the side walls near their base into the gutters.

<div align="center">CONSTRUCTION.</div>

169. Headings. The methods of all tunnel excavation depend on the general principle that all earthy material, except the softest of liquid mud and quicksand, will be self-sustaining over a greater or less area and for a greater or less time after excavation is made, and the work consists in excavating some material and immediately propping up the exposed surface by timbering and poling-boards. The excavation of the cross-section begins with cutting out a "heading," which is a small horizontal drift whose breast is constantly kept 15 feet or more in advance of the full cross-sectional excavation. In solid self-sustaining rock, which will not decompose upon exposure to air, it becomes simply a matter of excavating the rock with the least possible expenditure of time and energy. In soft ground the heading must be heavily timbered, and as the heading is gradually enlarged the timbering must be gradually extended

and perhaps replaced, according to some regular system, so that
when the full cross-section has been excavated it is supported
by such timbering as is intended for it.   The heading is some-
times made on the center line near the top; with other plans,
on the center line near the bottom; and
sometimes two simultaneous headings are run
in the two lower corners.   Headings near the
bottom serve the purpose of draining the
material above it and facilitating the excava-
tion.   The simplest case of heading timber-
ing is that shown in Fig. 92, in which cross-
timbers are placed at intervals just under the
roof, set in notches cut in the side walls and
supporting poling-boards which sustain what-
ever pressure may come on them. Cross-timbers near the bottom
support a flooring on which vehicles for transporting material
may be run and under which the drainage may freely escape.
As the necessity for timbering becomes greater, side timbers and
even bottom timbers must be added, these timbers supporting
poling-boards, and even the breast of the heading must be pro-
tected by boards suitably braced, as shown in Fig. 93.   The
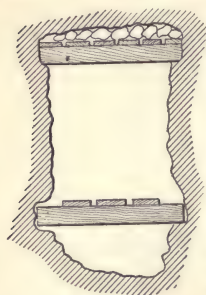
FIG. 92.
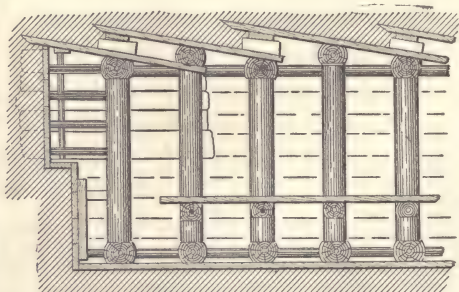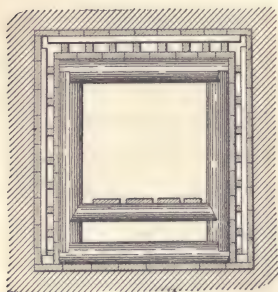
FIG 93 —TIMBERING FOR TUNNEL HEADING.

supporting timbers are framed into collars in such a manner that
added pressure only increases their rigidity.

170. **Enlargement.**   Enlargement is accomplished by remov-
ing the poling-boards, one at a time, excavating a greater or less

PLATE VI.
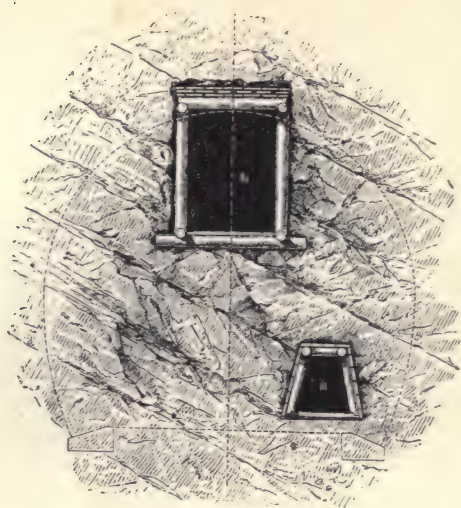
TUNNEL TIMBERING—GERMAN SYSTEM (a).
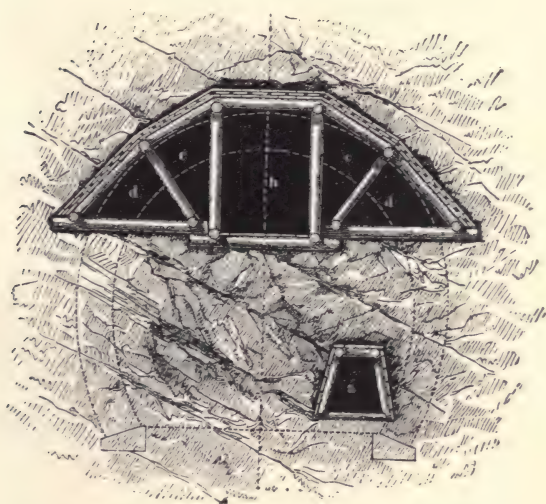
TUNNEL-TIMBERING—GERMAN SYSTEM (b).

PLATE VII.
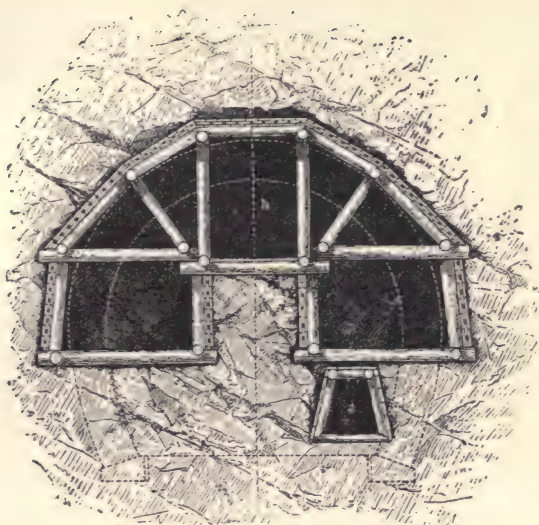
TUNNEL-TIMBERING—GERMAN SYSTEM (c).
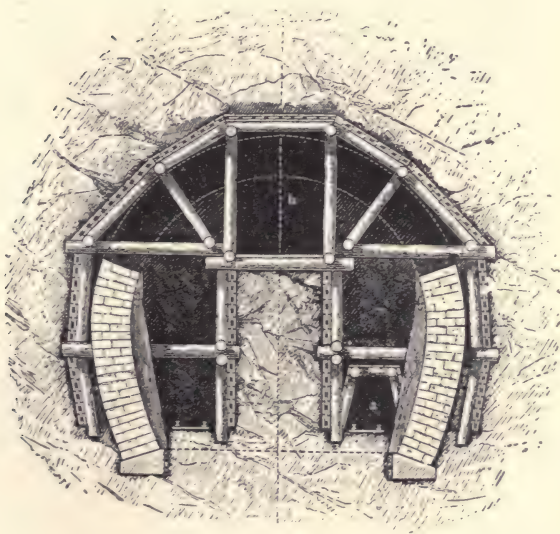
TUNNEL-TIMBERING—GERMAN SYSTEM (d)

(*To face page* 196.)

amount of material, and immediately supporting the exposed
material with poling-boards suitably braced. (See Figs. 93 and
94.) This work being systematically done, space is thereby
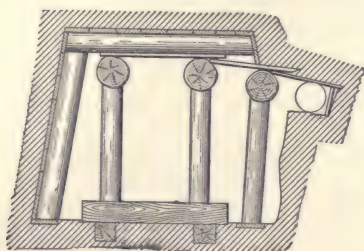
FIG. 94.

obtained in which the framing for the full cross-section may be
gradually introduced. The framing is constructed with a cross-
section so large that the masonry lining may be constructed
within it.

171. Distinctive features of various methods of construction.
There are six general systems, known as the English, German,
Belgian, French, Austrian, and American. They are so named
from the origin of the methods, although their use is not confined
to the countries named. Fig. 95 shows by numbers (1 to 5)
the order of the excavation within the cross-sections. The Eng-
lish, Austrian, and American systems are alike in excavating the
entire cross-section before beginning the construction of the
masonry lining. The German method leaves a solid core (5)
until practically the whole of the lining is complete. This has
the disadvantage of extremely cramped quarters for work, poor
ventilation, etc. The Belgian and French methods agree in
excavating the upper part of the section, building the arch at
once, and supporting it temporarily until the side walls are
built. The Belgian method then takes out the core (3), removes
very short sections of the sides (4), immediately underpinning
the arch with short sections of the side walls and thus gradually
constructing the whole side wall. The French method digs out
the sides (3), supporting the arch temporarily with timbers and

then replacing the timbers with masonry; the core (4) is taken
out last.   The French method has the same disadvantage as the
German—working in a cramped space. The Belgian and French
systems have the disadvantage that the arch, supported tempo-
rarily on timber, is very apt to be strained and cracked by the
slight settlement that so frequently occurs in soft material. The
English, Austrian, and American methods differ mainly in the
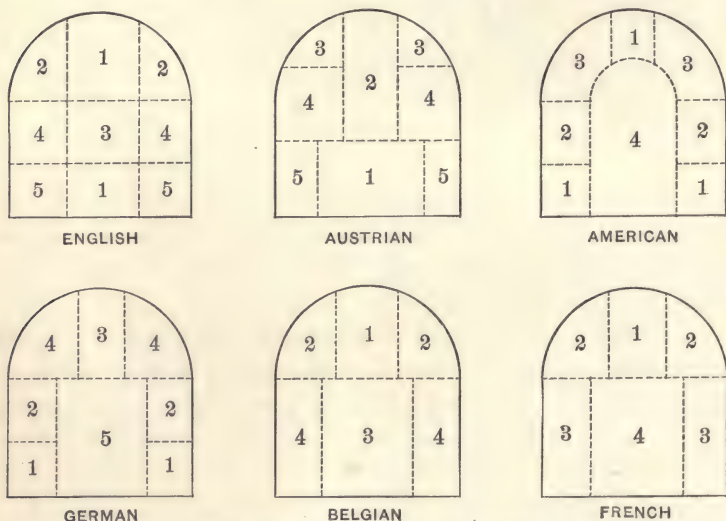
FIG. 95.—ORDER OF WORKING BY THE VARIOUS SYSTEMS.

design of the timbering.   The English support the roof by lines
of very heavy *longitudinal* timbers which are supported at com-
paratively wide intervals by a heavy framework occupying the
whole cross-section.   The Austrian system uses such frequent
cross-frames of timber-work that poling-boards will suffice to
support the material between the frames.   The American sys-
tem agrees with the Austrian in using frequent cross-frames
supporting poling-boards, but differs from it in that the " cross-
frames " consist simply of arches of 3 to 15 wooden voussoirs,
the voussoirs being blocks of $12'' \times 12''$ timber about 2 to 8 feet
long and cut with joints normal to the arch.   These arches are
put together on a centering which is removed as soon as the arch

PLATE VIII.

TUNNEL-TIMBERING—AUSTRIAN SYSTEM (a).
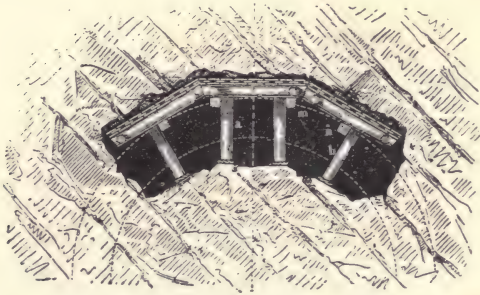
TUNNEL-TIMBERING—AUSTRIAN SYSTEM (b).

TUNNEL-TIMBERING—AUSTRIAN SYSTEM (c).

PLATE IX.
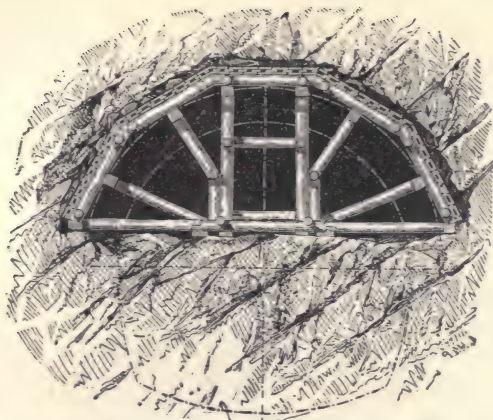
TUNNEL-TIMBERING—AUSTRIAN SYSTEM (d).
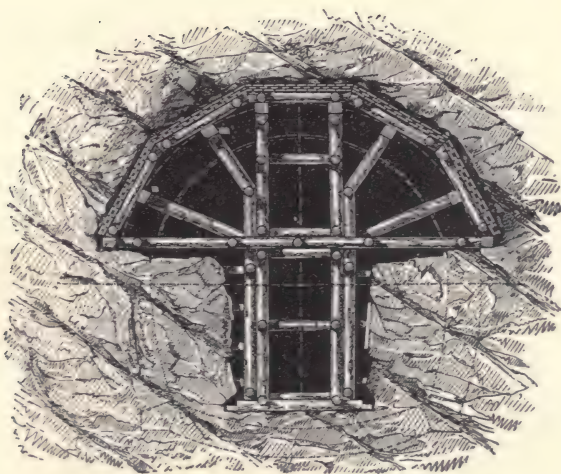
TUNNEL-TIMBERING—AUSTRIAN SYSTEM (e).

PLATE X.
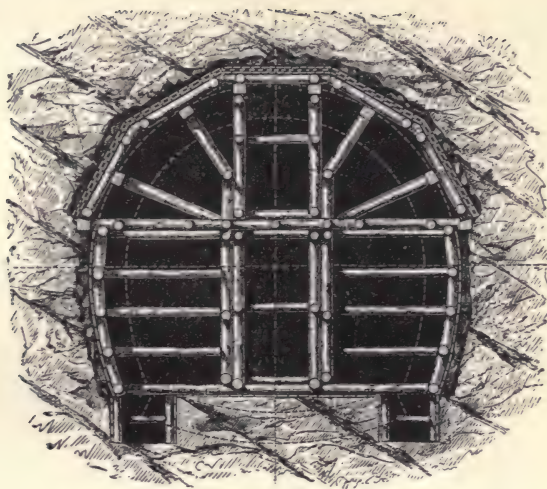
TUNNEL-TIMBERING—AUSTRIAN SYSTEM (*f*).
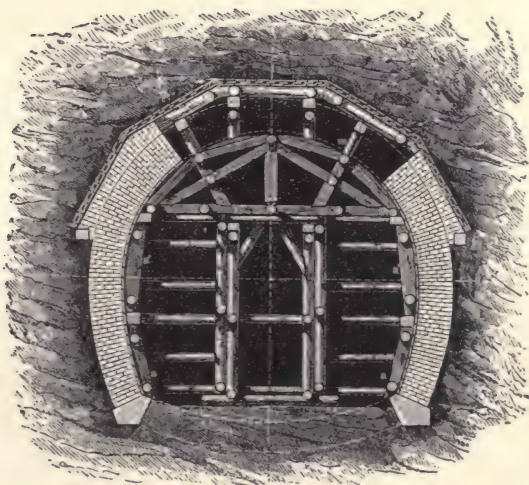
TUNNEL-TIMBERING—AUSTRIAN SYSTEM (*g*).

is keyed up and thus immediately opens up the full cross-section, so that the center core (4) may be immediately dug out and the masonry constructed in a large open space. The American system has been used successfully in very soft ground, but its advantages are greater in loose rock, when it is much cheaper than the other methods which employ more timber. Fig. 90 illustrates the use of the American system. The figure shows the wooden arch in place. The masonry arch may be placed when convenient, since it is *possible* to lay the track and commence traffic as soon as the wooden arch is in place. Plates II to XIV illustrate the methods of excavating and timbering by these various systems.

172. Ventilation during construction. Tunnels of any great length must be artificially ventilated during construction. If the excavated material is rock so that blasting is necessary, the need for ventilation becomes still more imperative. The invention of compressed-air drills simultaneously solved two difficulties. It introduced a motive power which is unobjectionable in its application (as gas would be), and it also furnished at the same time a supply of just what is needed—pure air. If no blasting is done (and sometimes even when there is blasting), air must be supplied by direct pumping. The cooling effect of the sudden expansion of compressed air only reduces the otherwise objectionably high temperature sometimes found in tunnels. Since pure air is being continually pumped in, the foul air is thereby forced out.

173. Excavation for the portals. Under normal conditions there is always a greater or less amount of open cut preceding and following a tunnel. Since all tunnel methods depend (to some slight degree at least) on the capacity of the exposed material to act as an arch, there is implied a considerable thickness of material above the tunnel. This thickness is reduced to nearly zero over the tunnel portals and therefore requires special treatment, particularly when the material is very soft. Fig. 96 *

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* Ržiha, " Lehrbuch der Gersammten Tunnelbaukunst."

illustrates one method of breaking into the ground at a portal. The loose stones are piled on the framing to give stability to the framing by their weight and also to retain the earth on the slope above.   Another method is to sink a temporary shaft to the tunnel near the portal; immediately enlarge to the full size and build the masonry lining; then work back to the portal.
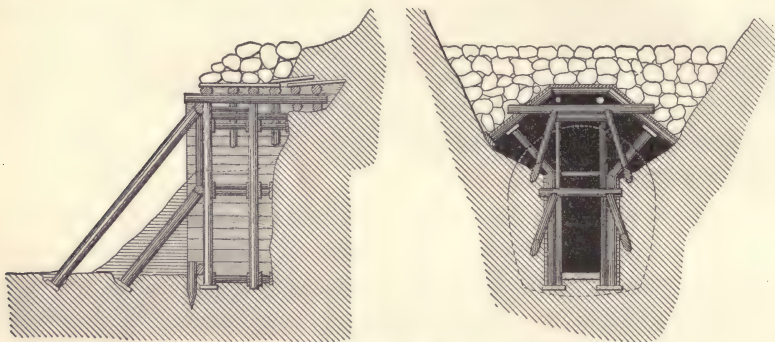
FIG. 96.—TIMBERING FOR TUNNEL PORTAL.

This method is more costly, but is preferable in very treacherous ground, it being less liable to cause landslides of the surface material.

174. Tunnels vs. open cuts.   In cases in which an open cut rather than a tunnel is a possibility the ultimate consideration is generally that of first cost combined with other financial considerations and annual maintenance charges directly or indirectly connected with it. Even when an open cut may be constructed at the same cost as a tunnel (or perhaps a little cheaper) the tunnel may be preferable under the following conditions:

1. When the soil indicates that the open cut would be liable to landslides.

2. When the open cut would be subject to excessive snow-drifts or avalanches.

3. When land is especially costly or it is desired to run under existing costly or valuable buildings or monuments.   When running through cities, tunnels are sometimes constructed as open cuts and then arched over.

PLATE XI.

TEMPORARY TIMBERING OF HEADING.

PERMANENT TIMBERING OF HEADING.

PHŒNIXVILLE TUNNEL. P. S. V. R.R.

PLATE XII.

TIMBERING OF FULL TUNNEL SECTION.

1′ 10½″

13′ 6″

13′ 6″ 1′ 6″ 13′ 6″

6′ 4″

LINING AND CENTERING.
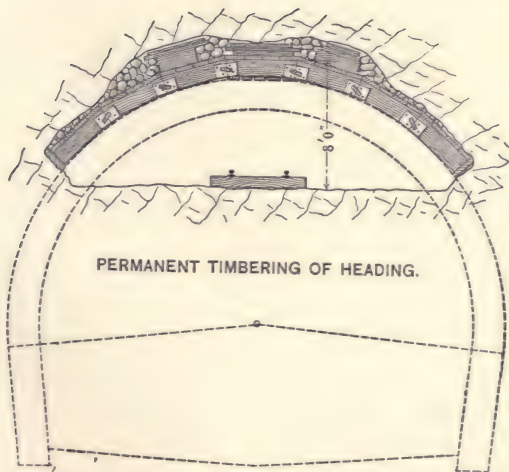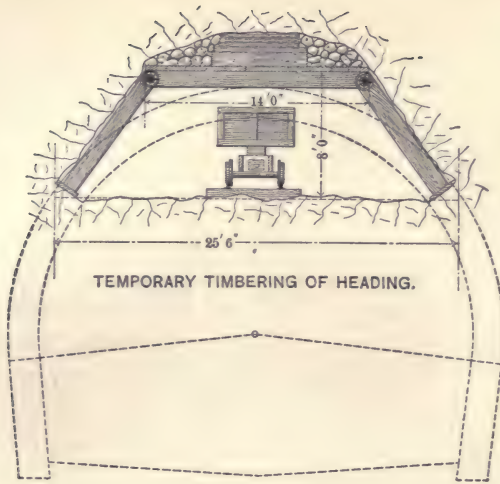
PHŒNIXVILLE TUNNEL. P. S. V. R.R.

(To face page 200.)

PLATE XIII.

PHŒNIXVILLE TUNNEL.   P. S. V. R.R.

PLATE XIV.

1884

13′-6″

13′-6″   1′6″   13′-6″

5″-3″

12′-2″
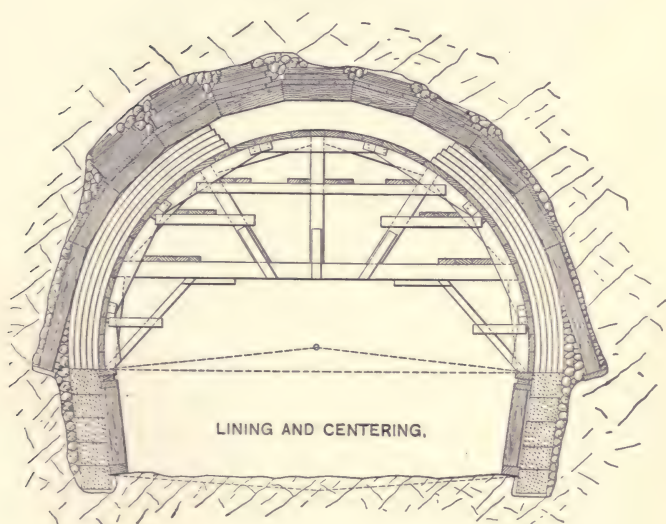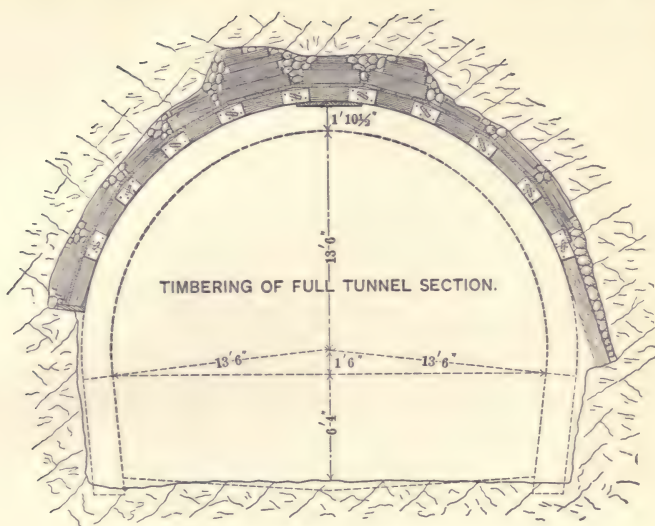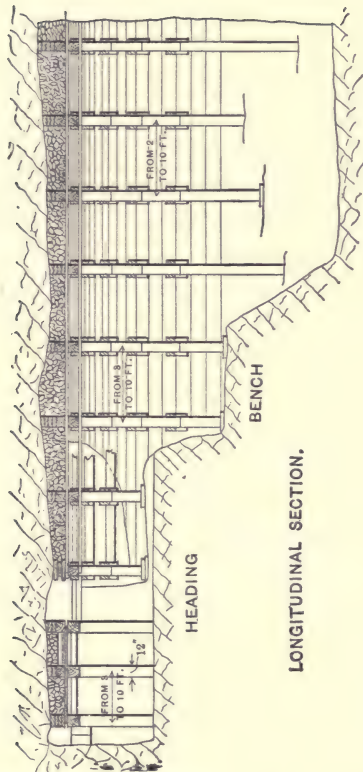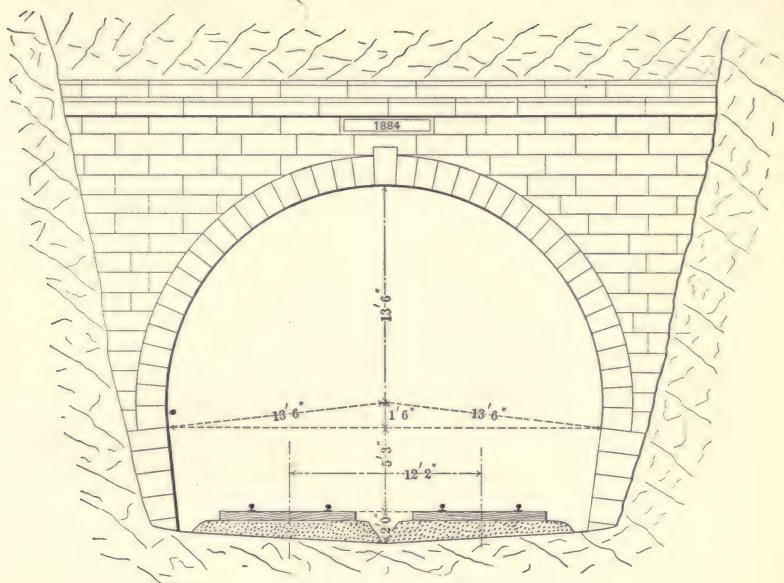
ELEVATION OF PORTAL.

BATTER: 2″ PER FOOT
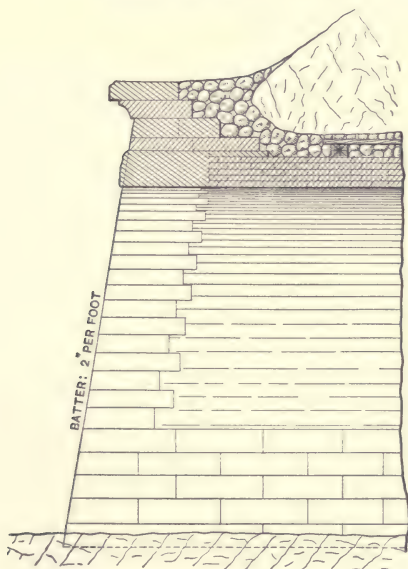
LONGITUDINAL SECTION OF PORTAL.
PHŒNIXVILLE TUNNEL. P. S. V. R.R.
(To face page 200.)

These cases apply to tunnels *vs.* open cuts when the alignment is fixed by other considerations than the mere topography. The broader question of excavating tunnels to avoid excessive grades or to save distance or curvature, and similar problems, are hardly susceptible of general analysis except as questions of railway economics and must be treated individually.

175. Cost of tunneling. The cost of any construction which involves such uncertainties as tunneling is very variable. It depends on the material encountered, the amount and kind of timbering required, on the size of the cross-section, on the price of labor, and especially on the *reconstruction* that *may* be necessary on account of mishaps.

Headings generally cost $4 to $5 per cubic yard for excavation, while the remainder of the cross-section in the same tunnel may cost about half as much. The average cost of a large number of tunnels in this country may be seen from the following table : *

| Material. | Cost per cubic yard. | | | | Cost per lineal foot. | |
| | Excavation. | | Masonry. | | | |
	Single.	Double.	Single.	Double.	Single.	Double.
Hard rock.....	$5.89	$5.45	$12.00	$ 8.25	$ 69.76	$142.82
Loose rock....	3.12	3.48	9.07	10.41	80.61	119.26
Soft ground...	3.62	4.64	15.00	10.50	135.31	174.42

A considerable variation from these figures may be found in individual cases, due sometimes to unusual skill (or the lack of it) in prosecuting the work, but the figures will generally be sufficiently accurate for preliminary estimates or for the comparison of two proposed routes.

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* Figures derived from Drinker's "Tunneling."

CHAPTER VI.

CULVERTS AND MINOR BRIDGES.

176. Definition and object. Although a variable percentage of the rain falling on any section of country soaks into the ground and does not immediately reappear, yet a very large percentage flows over the surface, always seeking and following the lowest channels. The roadbed of a railroad is constantly intersecting these channels, which frequently are normally dry. In order to prevent injury to railroad embankments by the impounding of such rainfall, it is necessary to construct waterways through the embankment through which such rainflow may freely pass. Such waterways, called culverts, are also applicable for the bridging of very small although perennial streams, and therefore in this work the term culvert will be applied to all water-channels passing through a railroad embankment which are not of sufficient magnitude to require a special structural design, such as is necessary for a large masonry arch or a truss bridge.

177. Elements of the design. A well-designed culvert must afford such free passage to the water that it will not "back up" over the adjoining land nor cause any injury to the embankment or culvert. The ability of the culvert to discharge freely all the water that comes to it evidently depends chiefly on the area of the waterway, but also on the form, length, slope, and materials of construction of the culvert and the nature of the approach and outfall. When the embankment is very low and the amount of water to be discharged very great, it sometimes becomes necessary to allow the water to discharge "under a head," i.e.,

with the surface of the water above the top of the culvert. Safety then requires a much stronger construction than would otherwise be necessary to avoid injury to the culvert or embankment by washing. The necessity for such construction should be avoided if possible.

AREA OF THE WATERWAY.

178. Elements involved. The determination of the required area of the waterway involves such a multiplicity of indeterminate elements that any close determination of its value from purely theoretical considerations is a practical impossibility. The principal elements involved are:

a. Rainfall. The real test of the culvert is its capacity to discharge without injury the flow resulting from the extraordinary rainfalls and "cloud bursts" that may occur once in many years. Therefore, while a knowledge of the average annual rainfall is of very little value, a record of the maximum rainfall during heavy storms for a long term of years may give a relative idea of the maximum demand on the culvert.

b. Area of watershed. This signifies the total area of country draining into the channel considered. When the drainage area is very small it is sometimes included within the area surveyed by the preliminary survey. When larger it is frequently possible to obtain its area from other maps with a percentage of accuracy sufficient for the purpose. Sometimes a special survey for the purpose is considered justifiable.

c. Character of soil and vegetation. This has a large influence on the rapidity with which the rainflow from a given area will reach the culvert. If the soil is hard and impermeable and the vegetation scant, a heavy rain will run off suddenly, taxing the capacity of the culvert for a short time, while a spongy soil and dense vegetation will retard the flow, making it more nearly uniform and the maximum flow at any one time much less.

d. Shape and slope of watershed. If the watershed is very long and narrow (other things being equal), the water from the

remoter parts will require so much longer time to reach the culvert that the flow will be comparatively uniform, especially when the slope of the whole watershed is very low. When the slope of the remoter portions is quite steep it may result in the nearly simultaneous arrival of a storm-flow from all parts of the watershed, thus taxing the capacity of the culvert.

e. **Effect of design of culvert.** The principles of hydraulics show that the slope of the culvert, its length, the form of the cross-section, the nature of the surface, and the form of the approach and discharge all have a considerable influence on the area of cross-section required to discharge a given volume of water in a given time, but unfortunately the combined hydraulic effect of these various details is still a very uncertain quantity.

179. Methods of computation of area. There are three possible methods of computation.

(a) **Theoretical.** As shown above it is a practical impossibility to estimate correctly the combined effect of the great multiplicity of elements which influence the final result. The nearest approach to it is to estimate by the use of empirical formulæ the amount of water which will be presented at the upper end of the culvert in a given time and then to compute, from the principles of hydraulics, the rate of flow through a culvert of given construction, but (as shown in § 178, *e*) such methods are still very unreliable, owing to lack of experimental knowledge. This method has *apparently* greater scientific accuracy than other methods, but a little study will show that the elements of uncertainty are as great and the final result no more reliable. The method is most reliable for streams of uniform flow, but it is under these conditions that method (*c*) is most useful. The theoretical method will not therefore be considered further.

(b) **Empirical.** As illustrated in § 180, some formulæ make the area of waterway a function of the drainage area, the formula being affected by a coefficient the value of which is estimated between limits according to the judgment. Assuming that the formulæ are sound, their use only narrows the limits of

error, the final determination depending on experience and judgment.

(c) **From observation.** This method, considered by far the best for permanent work, consists in observing the high-water marks on contracted channel-openings which are on the same stream and as near as possible to the proposed culvert. If the country is new and there are no such openings, the wisest plan is to bridge the opening by a temporary structure in wood which has an ample waterway (see § 126, *b*, 4) and carefully observe all high-water marks on that opening during the 6 to 10 years which is ordinarily the minimum life of such a structure. As shown later, such observations may be utilized for a close computation of the required waterway. Method (*b*) may be utilized for an approximate calculation for the required area for the temporary structure, using a value which is intentionally excessive, so that a permanent structure of sufficient capacity may subsequently be constructed *within* the temporary structure.

180. Empirical formulæ. Two of the best known empirical formulæ for area of the waterway are the following:

(a) **Myer's formula:**
Area of waterway in square feet $= C \times \sqrt{\text{drainage area in acres}}$,
where C is a coefficient varying from 1 for flat country to 4 for mountainous country and rocky ground. As an illustration, if the drainage area is 100 acres, the waterway area should be from 10 to 40 square feet, according to the value of the coefficient chosen. It should be noted that this formula does not regard the great variations in rainfall in various parts of the world nor the design of the culvert, and also that the final result depends largely on the choice of the coefficient.

(b) **Talbot's formula:**
Area of waterway in square feet $= C \times \sqrt[4]{(\text{drainage area in acres})^3}$.
" For steep and rocky ground C varies from $\frac{2}{3}$ to 1. For rolling agricultural country subject to floods at times of melting snow, and with the length of the valley three or four times its width, C is about $\frac{1}{3}$; and if the stream is longer in proportion to the area, decrease C. In districts not affected by accumulated snow, and

where the length of the valley is several times the width, $\frac{1}{5}$ or $\frac{1}{6}$, or even less, may be used. C should be increased for steep side slopes, especially if the upper part of the valley has a much greater fall than the channel at the culvert." *   As an illustration, if the drainage area is 100 acres the area of waterway should be $C \times 31.6$.   The area should then vary from 5 to 31 square feet, according to the character of the country.   Like the previous estimate, the result depends on the choice of a coefficient and disregards local variations in rainfall, except as they may be arbitrarily allowed for in choosing the coefficient.

181. Value of empirical formulæ.   The fact that these formulæ, as well as many others of similar nature that have been suggested, depend so largely upon the choice of the coefficient shows that they are valuable "more as a guide to the judgment than as a working rule," as Prof. Talbot explicitly declares in commenting on his own formula.   In short, they are chiefly valuable in indicating a probable maximum and minimum between which the true result probably lies.

182. Results based on Observation.   As already indicated in § 179, observation of the stream in question gives the most reliable results.   If the country is new and no records of the flow of the stream during heavy storms has been taken, even the life of a temporary wooden structure may not be long enough to include one of the unusually severe storms which must be allowed for, but there will usually be some high-water mark which will indicate how much opening will be required. The following quotation illustrates this: " A tidal estuary may generally be safely narrowed considerably from the extreme water lines if stone revetments are used to protect the bank from wash.   Above the true estuary, where the stream cuts through the marsh, we generally find nearly vertical banks, and we are safe if the faces of abutments are placed even with the banks.   In level sections of the country, where the current is sluggish, it is usually safe to encroach somewhat on the

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* Prof. A. N. Talbot, "Selected Papers of the Civil Engineers' Club of the Univ. of Illinois."

general width of the stream, but in rapid streams among the
hills the width that the stream has cut for itself through the
soil should not be lessened, and in ravines carrying mountain
torrents the openings must be left very much larger than the
ordinary appearance of the banks of the stream would seem to
make necessary." *

As an illustration of an observation of a storm-flow through
a temporary trestle, the following is quoted: "Having the
flood height and velocity, it is an easy matter to determine the
volume of water to be taken care of. I have one ten-bent pile
trestle 135 feet long and 24 feet high over a spring branch that
ordinarily runs about six cubic inches per second. Last sum-
mer during one of our heavy rainstorms (four inches in less
than three hours) I visited this place and found by float observa-
tions the surface velocity at the highest stage to be 1.9 feet per
second. I made a high-water mark, and after the flood-water
receded found the width of stream to be 12 feet and an average
depth of $2\frac{3}{4}$ feet. This, with a surface velocity of 1.9 feet per
second, would give approximately a discharge of 50 cubic feet,
or 375 gallons, per second. Having this information it is easy
to determine size of opening required." †

183. Degree of accuracy required. The advantages result-
ing from the use of standard designs for culverts (as well as
other structures) have led to the adoption of a comparatively
small number of designs. The practical use made of a compu-
tation of required waterway area is to determine which one of
several standard designs will most nearly fulfill the require-
ments. For example, if a 24-inch iron pipe, having an area of
3.14 square feet, is considered to be a little small, the next size
(30-inch) would be adopted; but a 30-inch pipe has an area of
4.92 square feet, which is 56% larger. A similar result, except
that the percentage of difference might not be quite so marked,

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* J. P. Snow, Boston & Maine Railway. From Report to Association of
Railway Superintendents of Bridges and Buildings. 1897.

† A. J. Kelley, Kansas City Belt Railway. From Report to Association
of Railway Superintendents of Bridges and Buildings. 1897.

will be found by comparing the areas of consecutive standard designs for stone box culverts.

The advisability of designing a culvert to withstand any storm-flow that may *ever* occur is considered doubtful. Several years ago a record-breaking storm in New England carried away a very large number of bridges, etc., hitherto supposed to be safe. It was not afterward considered that the design of those bridges was faulty, because the extra cost of constructing bridges capable of withstanding such a flood, added to interest for a long period of years, would be enormously greater than the cost of repairing the damages of such a storm once or twice in a century. Of course the element of danger has some weight, but not enough to justify a great additional expenditure, for common prudence would prompt unusual precautions during or immediately after such an extraordinary storm.

PIPE CULVERTS.

184. Advantages. Pipe culverts, made of cast iron or earthenware, are very durable, readily constructed, moderately cheap, will pass a larger volume of water in proportion to the area than many other designs on account of the smoothness of the surface, and (when using iron pipe) may be used very close to the track when a low opening of large capacity is required. Another advantage lies in the ease with which they may be inserted through a somewhat larger opening that has been temporarily lined with wood, without disturbing the roadbed or track.

185. Construction. Permanency requires that the foundation shall be firm and secure against being washed out. To accomplish this, the soil of the trench should be hollowed out to fit the lower half of the pipe, making suitable recesses for the bells. In very soft treacherous soil a foundation-block of concrete is sometimes placed under each joint, or even throughout the whole length. When pipes are laid through a slightly larger timber culvert great care should be taken that the pipes are properly supported, so that there will be no settling nor

development of unusual strains when the timber finally decays and gives way. To prevent the washing away of material around the pipe the ends should be protected by a bulkhead. This is best constructed of masonry (see Fig. 97), although wood is sometimes used for cheap and minor constructions. The joints should be calked, especially when the culvert is liable to run full or when the outflow is impeded and the culvert is liable to be partly or wholly filled during freezing weather. The cost of a calking of clay or even hydraulic cement is insignificant compared with the value of the additional safety afforded. When the grade of the pipe is perfectly uniform, a very low rate of grade will suffice to drain a pipe culvert, but since some unevenness of grade is inevitable through uneven settlement or imperfect construction, a grade of 1 in 20 should preferably be required, although much less is often used. The length of a pipe culvert is approximately determined as follows :

Length $= 2s$ (*depth of embankment to top of pipe*) $+$ (*width of roadbed*),

in which s is the slope ratio (horizontal to vertical) of the banks. In practice an even number of lengths will be used which will most nearly agree with this formula.

186. Iron-pipe culverts. Simple cast-iron pipes are used in sizes from 12″ to 48″ diameter. These are usually made in lengths of 12 feet with a few lengths of 6 feet, so that any required length may be more nearly obtained. The lightest pipes made are sufficiently strong for the purpose, and even those which would be rejected because of incapacity to withstand pressure may be utilized for this work. In Fig. 97 are shown the standard plans used on the C. C. C. & St. L. Ry., which may be considered as typical plans.

Pipes formed of cast-iron segments have been used up to 12 feet diameter. The shell is then made comparatively thin, but is stiffened by ribs and flanges on the outside. The segments break joints and are bolted together through the flanges. The joints are made tight by the use of a tarred rope, together with neat cement.
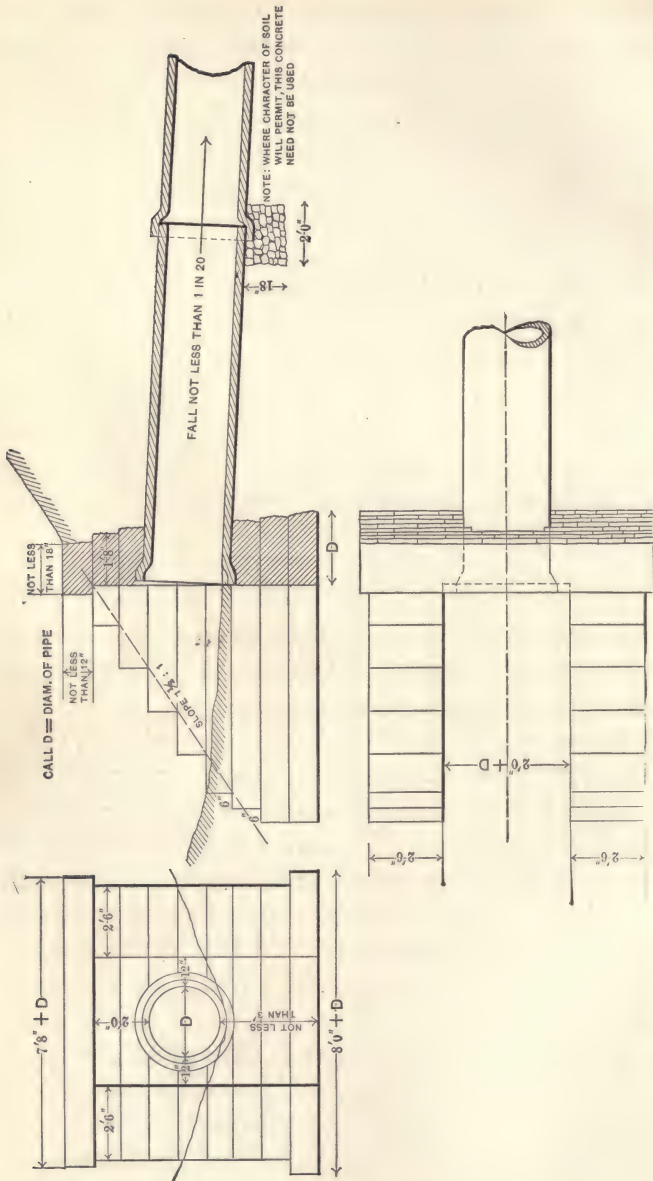
FIG. 97.—STANDARD CAST-IRON PIPE CULVERT. C. C. C. & ST. L. RY. (May 1893.)

187. Tile-pipe culverts. The pipes used for this purpose vary from 12″ to 24″ in diameter. When a larger capacity is required two or more pipes may be laid side by side, but in such a case another design might be preferable. It is frequently specified that " double-strength " or " extra-heavy " pipe shall be used, evidently with the idea that the stresses on a culvert-pipe are greater than on a sewer-pipe. But it has been conclusively demonstrated that, no matter how deep the embankment, the pressure cannot exceed a somewhat uncertain maximum, also that the greatest danger consists in placing the pipe so near the ties that shocks may be directly transferred to the pipe without the cushioning effect of the earth and ballast. When the pipes are well bedded in *clear* earth and there is a sufficient depth of earth over them to avoid direct impact (at least three feet) the ordinary sewer-pipe will be sufficiently strong. " Double-strength " pipe is frequently less perfectly burned, and
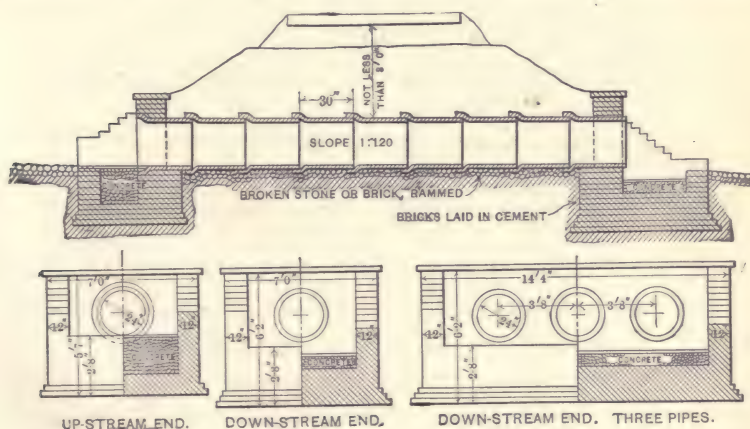
FIG. 98.—STANDARD VITRIFIED-PIPE CULVERT. PLANT SYSTEM. (1891.)

the supposed extra strength is not therefore obtained. In Fig. 98 are shown the standard plans for vitrified-pipe culverts as used on the " Plant system." Tile pipe is much cheaper than iron pipe, but is made in much shorter lengths and requires much more work in laying and especially to obtain a uniform grade.

BOX CULVERTS.

188. Wooden box culverts. This form serves the purpose of
a cheap temporary construction which allows the use of a bal-
lasted roadbed. As in all temporary constructions, the area
should be made considerably larger than the calculated area
(§§ 179–182), not only for safety but also in order that, if the
smaller area is demonstrated to be sufficiently large, the per-
manent construction (probably pipe) may be placed inside with-
out disturbing the embankment. All designs agree in using
heavy timbers ($12'' \times 12''$, $10'' \times 12''$, or $8'' \times 12''$) for the
side walls, cross-timbers for the roof, every fifth or sixth timber
being notched down so as to take up the thrust of the side walls,
and planks for the flooring. Fig. 99 shows some of the standard
designs as used by the C., M. & St. P. Ry.
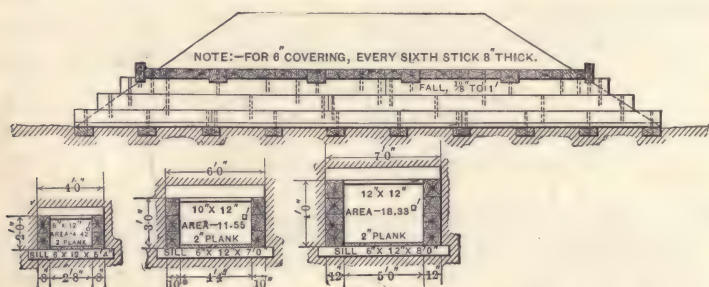
FIG. 99.—STANDARD TIMBER BOX CULVERT. C., M. & ST. P. RY. (Feb. 1889.)

189. Stone box culverts. In localities where a *good* quality
of stone is cheap, stone box culverts are the cheapest form of
permanent construction for culverts of medium capacity, but
their use is decreasing owing to the frequent difficulty in obtain-
ing really suitable stone within a reasonable distance of the
culvert. The clear span of the cover-stones varies from 2 to 4
feet. The required thickness of the cover-stones is sometimes
calculated by the theory of transverse strains on the basis of cer-
tain assumptions of loading—as a function of the height of the
embankment and the unit strength of the stone used. Such a
method is simply another illustration of a class of calculations

which look very precise and beautiful, but which are worse than
useless (because misleading) on account of the hopeless uncertainty
as to the true value of certain quantities which must be used in
the computations.   In the first place the true value of the unit
tensile strength of stone is such an uncertain and variable
quantity that calculations based on any assumed value for it are
of small reliability.   In the second place the weight of the prism
of earth lying directly above the stone, plus an allowance for live
load, is by no means a measure of the load on the stone nor of
the forces that tend to fracture it.   All earthwork will tend to
form an arch above any cavity and thus relieve an uncertain and
probably variable proportion of the pressure that might other-
wise exist.   The higher the embankment the *less* the *propor-
tionate* loading, until at some uncertain height an increase in
height will not increase the load on the cover-stones.   The effect
of frost is likewise large, but uncertain and not computable.   The
usual practice is therefore to make the thickness such as experi-
ence has shown to be safe with a good quality of stone, i.e.,
about 10 or 12 inches for 2 feet span and up to 16 or 18 inches
for 4 feet span.   The side walls should be carried down deep
enough to prevent their being undermined by scour or heaved
by frost.   The use of cement mortar is also an important feature
of first-class work, especially when there is a rapid scouring cur-
rent or a liability that the culvert will run under a head.   In
Fig. 100 are shown standard plans for single and double stone box
culverts as used on the Norfolk and Western R.R.

190. Old-rail culverts.   It sometimes happens (although very
rarely) that it is necessary to bring the grade line within 3 or 4
feet of the bottom of a stream and yet allow an area of 10 or 12
square feet.   A single large pipe of sufficient area could not be
used in this case.   The use of several smaller pipes side by side
would be both expensive and inefficient.   For similar reasons
neither wooden nor stone box culverts could be used.   In such
cases, as well as in many others where the head-room is not so
limited, the plan illustrated in Fig. 101 is a very satisfactory
solution of the problem.   The old rails, having a length of 8 or
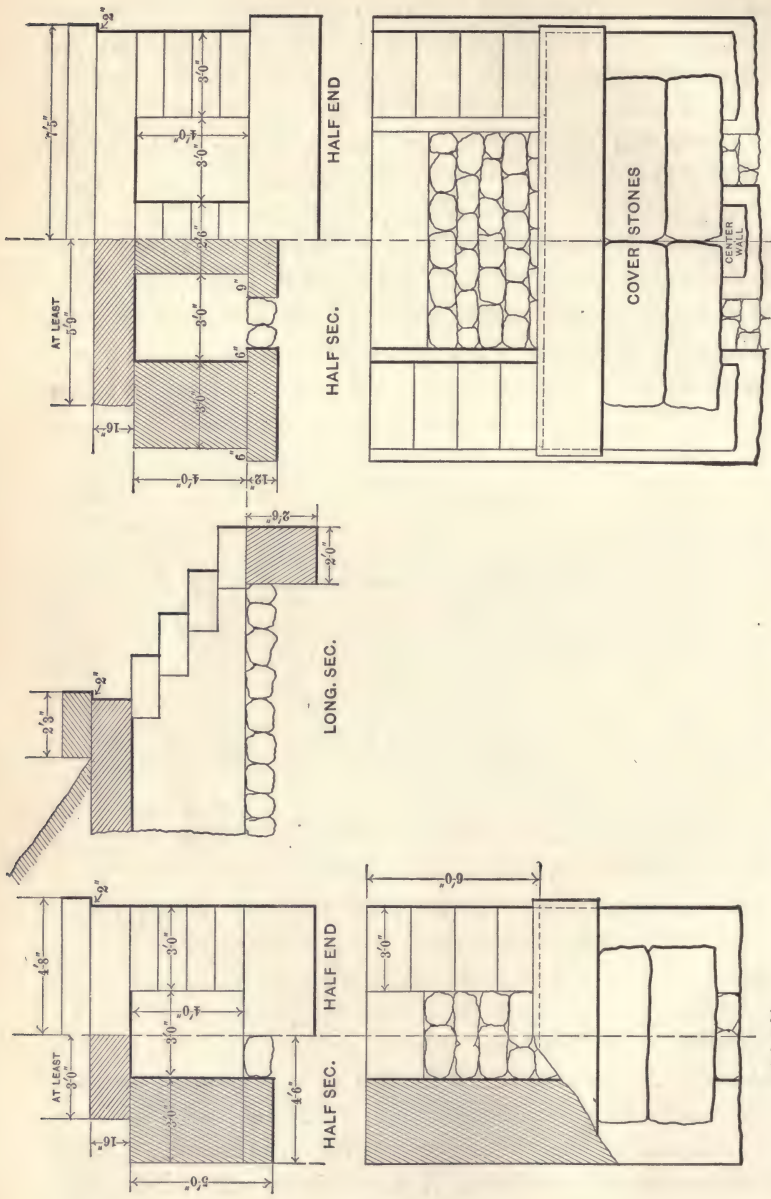
FIG. 100.—STANDARD SINGLE AND DOUBLE STONE CULVERTS (3′×4′). N. & W. R.R. (1890.)

9 feet, are laid close together across a 6-foot opening. Some-
times the rails are held together by long bolts passing through
the webs of the rails. In the plan shown the rails are confined
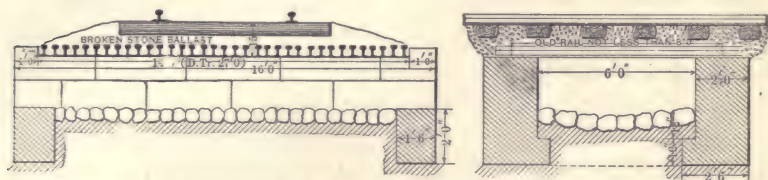
FIG. 101.—STANDARD OLD-RAIL CULVERT. N. & W. R.R. (1895.)

by low end walls on each abutment. This plan requires only 15
inches between the base of the rail and the top of the culvert
channel. It also gives a continuous ballasted roadbed.

ARCH CULVERTS.

191. Influence of design on flow. The variations in the
design of arch culverts have a very marked influence on the
cost and efficiency. To combine the least cost with the great-
est efficiency, due weight should be given to the following
elements: (*a*) the amount of masonry, (*b*) the simplicity of
the constructive work, (*c*) the design of the wing walls, (*d*)
the design of the junction of the wing walls with the barrel
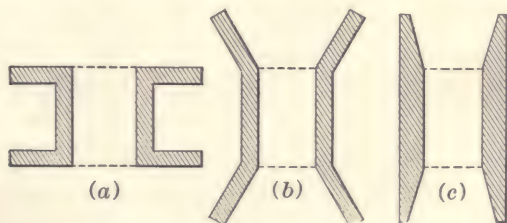
FIG. 102.—TYPES OF CULVERTS.

and faces of the arch, and (*e*) the safety and permanency of the
construction. These elements are more or less antagonistic to
each other, and the defects of most designs are due to a lack of
proper proportion in the design of these opposing interests. The
simplest construction (satisfying elements *b* and *e*) is the straight

barrel arch between two parallel vertical head walls, as sketched
in Fig. 102, *a*. From a hydraulic standpoint the design is poor,
as the water eddies around the corners, causing a great resistance
which decreases the flow. Fig. 102, *b*, shows a much better de-
sign in many respects, but much depends on the details of the
design as indicated in elements (*b*) and (*d*). As a general thing
a good hydraulic design requires complicated and expensive
masonry construction, i.e., elements (*b*) and (*d*) are opposed.
Design 102, *c*, is sometimes inapplicable because the water is
liable to work in behind the masonry during floods and perhaps
cause scour. This design uses less masonry than (*a*) or (*b*).

192. Example of arch culvert design. In Plate XV is shown
the design for an 8-foot arch culvert according to the standard
of the Norfolk and Western R.R. Note that the plan uses
the flaring wing walls (Fig. 102, *b*) on the up-stream side
(thus protecting the abutments from scour) and straight wing
walls (similar to Fig. 102, *c*) on the down-stream end. This
economizes masonry and also simplifies the constructive work.
Note also the simplicity of the junction of the wing walls with
the barrel of the arch, there being no re-entrant angles below
the springing line of the arch. The design here shown is but
one of a set of designs for arches varying in span from 6′ to 30′.

<div align="center">MINOR OPENINGS.</div>

193. Cattle-guards. (a) **Pit guards.** Cattle-guards will be
considered under the head of minor openings, since the old-
fashioned plan of pit guards, which are even now defended and
preferred by some railroad men, requires a break in the con-
tinuity of the roadbed. A pit about three feet deep, five feet
long, and as wide as the width of the roadbed, is walled up with
stone (sometimes with wood), and the rails are supported on heavy
timbers laid longitudinally with the rails. The break in the
continuity of the roadbed produces a disturbance in the elastic
wave running through the rails, the effect of which is noticeable
at high velocities. The greatest objection, however, lies in the

PLATE XV.

STANDARD ARCH CULVERT
8 FEET SPAN
NORFOLK & WESTERN R.R.
(1891)

(To face page 216.)
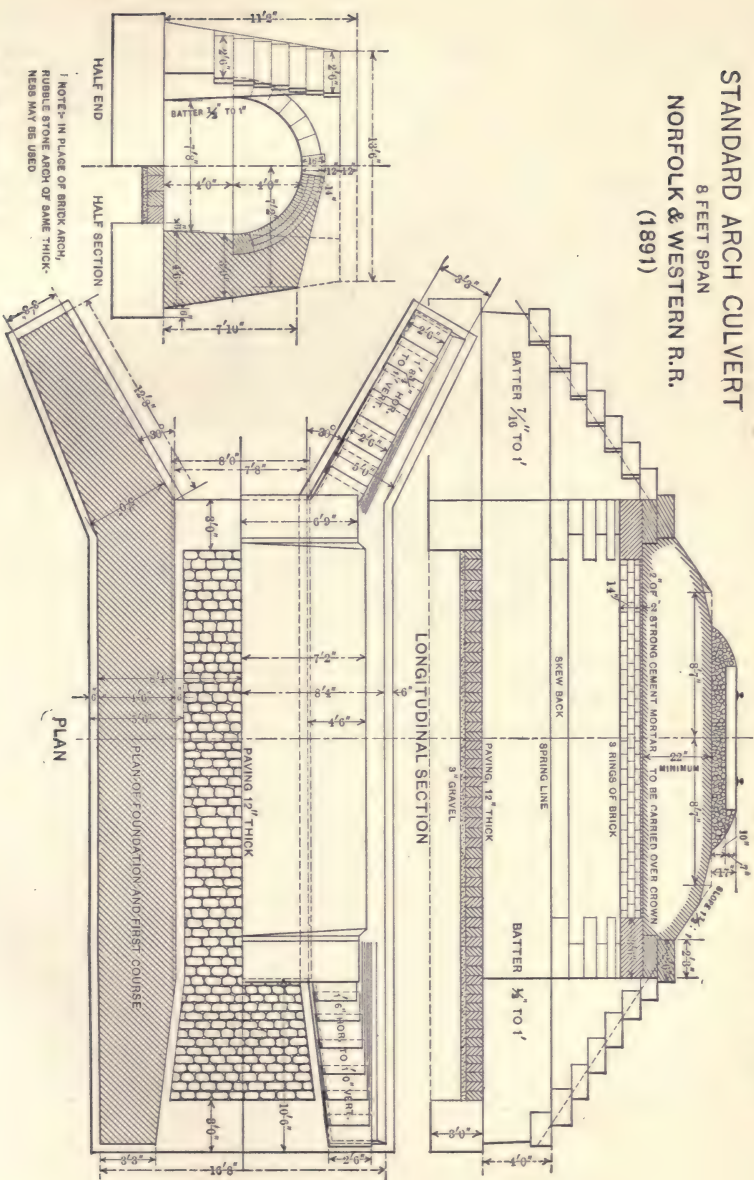
dangerous consequences of a derailment or a failure of the timbers owing to unobserved decay or destruction by fire—caused perhaps by sparks and cinders from passing locomotives. The very insignificance of the structure often leads to careless in-
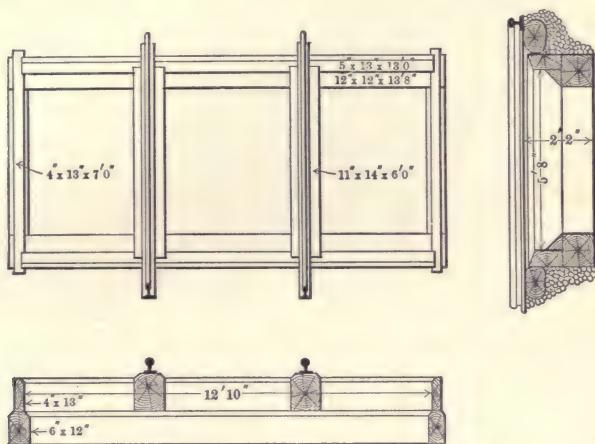
FIG. 103.—PIT CATTLE-GUARDS.   P. R.R.

spection. But if a single pair of wheels gets off the rails and drops into the pit, a costly wreck is inevitable. The (once) standard design for such a structure on the Pennsylvania R.R. is shown in Fig. 103.

(b) **Surface cattle-guards.** These are fastened on top of the ties; the continuity of the roadbed is absolutely unbroken and thus is avoided much of the danger of a bad wreck owing to a possible derailment. The device consists essentially of overlaying the ties (both inside and outside the rails) with a surface on which cattle will not walk. The multitudinous designs for such a surface are variously effective in this respect. An objection, which is often urged indiscriminately against all such designs, is the liability that a brake-chain which may happen to be dragging *may* catch in the rough bars which are used. The bars are sometimes "home-made," of wood, as shown in Fig. 104. Iron or steel bars are made as shown in Fig. 105. The general construction is the same as for the wooden bars. The

metal bars have far greater durability, and it is claimed that they are more effective in discouraging cattle from attempting to cross.
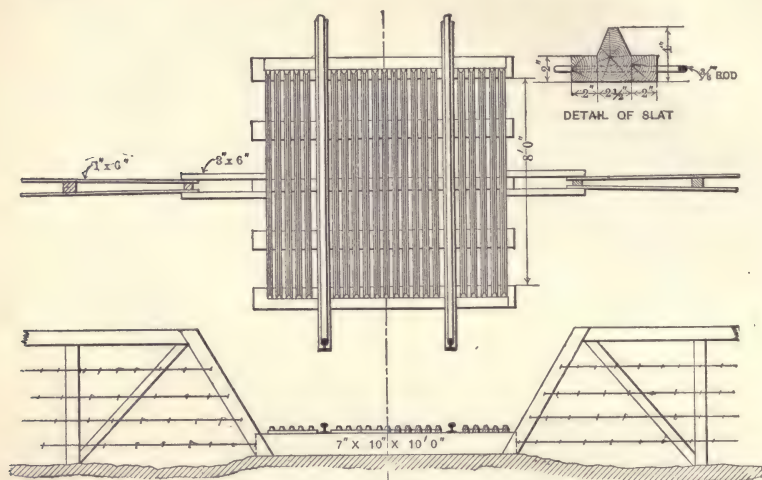
FIG. 104.—CATTLE-GUARD WITH WOODEN SLATS.
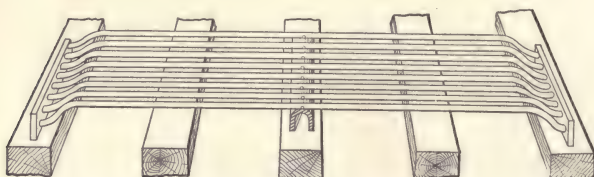
FIG. 105.—MERRILL-STEVENS STEEL CATTLE-GUARD.

194. Cattle-passes. Frequently when a railroad crosses a farm on an embankment, cutting the farm into two parts, the railroad company is obliged to agree to make a passageway through the embankment sufficient for the passage of cattle and perhaps even farm-wagons. If the embankment is high enough so that a stone arch is practicable, the initial cost is the only great objection to such a construction; but if an open wooden structure is necessary, all the objections against the old-fashioned cattle-guards apply with equal force here. The avoidance of a grade crossing which would otherwise be necessary is one of the

PLATE XVI.

¾ BOLT EVERY THIRD TIE.

6" X 8" GUARD RAIL

1'-6" 1'-0" 3'-9" 3'-9" 3'-0" 3'-9" 1'-0" 6" 1'-6"

15" I-BEAM (DOUBLE,) 90 LBS. PER FOOT.
17 FT. LONG, 14 FEET CLEAR SPAN.

11'-0"

8" X 10" TIES, 14'-0" LONG, 15" C. TO C.

10 ⌷—20# 6'-0" LONG.

¾" BOLTS, 8½" UNDER HEAD.

ANCHOR BOLT, 1" DIA.
10" LONG

PL. ⅜" X 12" X 18"

2'-0"

8" 15"

1¾" @ 3.8" 5" X 3½ X ½"
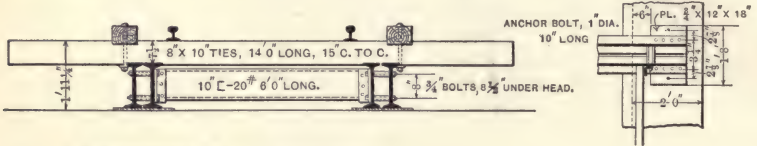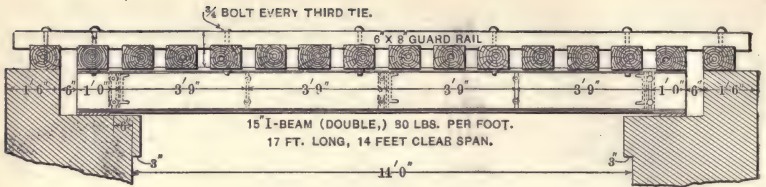
STANDARD I-BRIDGES—14-FT. SPAN.

NORFOLK AND WESTERN R.R.

(1891.)

TYPE "A₁"

45 FEET AND UNDER.

TYPE "B₂"
FROM 40 TO 70 FEET
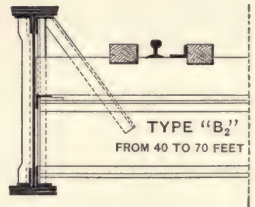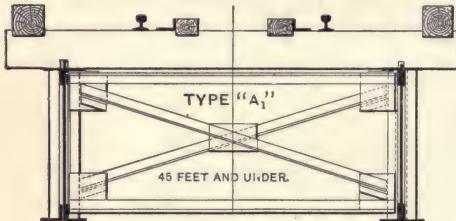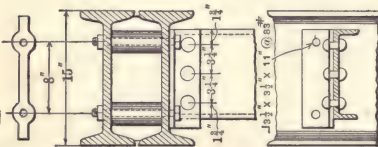
TYPES OF PLATE GIRDER BRIDGES.

C. M. & ST.P. RY.

(DEC. 1895.)

TYPE "E" GIRDER
25 FEET AND UNDER.

TYPE "C₄"
FROM 65 TO 100 FEET
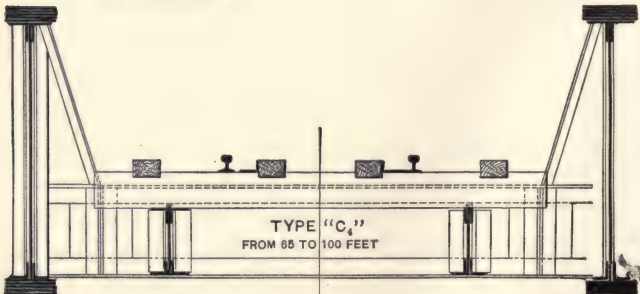
great compensations for the expense of the construction and maintenance of these structures. The construction is sometimes made by placing two pile trestle bents about 6 to 8 feet apart, supporting the rails by stringers in the usual way, the special feature of this construction being that the embankments are filled in behind the trestle bents, and the thrust of the embankments is mutually taken up through the stringers, which are notched at the ends or otherwise constructed so that they may take up such a thrust. The designs for old-rail culverts and arch culverts are also utilized for cattle-passes when suitable and convenient, as well as the designs illustrated in the following section.

195. Standard stringer and I-beam bridges. The advantages of standard designs apply even to the covering of short spans with wooden stringers or with **I** beams—especially since the methods do not require much vertical space between the rails and the upper side of the clear opening, a feature which is often of prime importance. These designs are chiefly used for culverts or cattle-passes and for crossing *over* highways—providing such a narrow opening would be tolerated. The plans all imply stone abutments, or at least abutments of sufficient stability to withstand all thrust of the embankments. Some of the designs are illustrated in Plate XVI. The preparation of these standard designs should be attacked by the same general methods as already illustrated in § 156. When computing the required transverse strength, due allowance should be made for lateral bracing, which should be amply provided for. Note particularly the methods of bracing illustrated in Plate XVI. The designs calling for iron (or steel) stringers may be classed as permanent constructions, which are cheap, safe, easily inspected and maintained and therefore a desirable method of construction.

CHAPTER VII.

BALLAST.

196. Purpose and requirements. "The object of the ballast is to transfer the applied load over a large surface; to hold the timber work in place horizontally; to carry off the rain-water from the superstructure and to prevent freezing up in winter; to afford means of keeping the ties truly up to the grade line; and to give elasticity to the roadbed." This extremely condensed statement is a description of an ideally perfect ballast. The value of any given kind of ballast is proportional to the extent to which it fulfills these requirements. The ideally perfect ballast is not necessarily the most economical ballast for all roads. Light traffic generally justifies something cheaper, but a very common error is to use a very cheap ballast when a small additional expenditure would procure a much better ballast which would be much more economical in the long run.

197. Materials. The materials most commonly employed are gravel and broken stone. Burnt clay, cinders, shells, and small coal are occasionally used as ballast when they are especially cheap and convenient or when better kinds are especially expensive. Although it is hardly correct to speak of the natural soil as ballast, yet many miles of cheap railways are "ballasted" with the natural soil, which is then called "mud ballast."

Mud ballast. When the natural soil is gravelly so that rain will drain through it quickly, it will make a fair roadbed for light traffic, but for heavy traffic, and for the greater part of the length of most roads, the natural soil is a very poor material for ballast; for, no matter how suitable the soil might be along

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limited sections of the road, it would practically never happen that the soil would be uniformly good throughout the whole length of the road. Considering that a heavy rain will in one day spoil the results of weeks of patient "surfacing" with mud ballast, it is seldom economical to use "mud" if there is a gravel-bed or other source of ballast anywhere on the line of the road.

Cinders. The advantages consist in the excellent facilities for drainage, ease of handling, and cheapness—after the road is in operation. One disadvantage is excessive dust in dry weather. Cinders are considered preferable to gravel in yards.

Slag. When slag is readily obtainable it furnishes an excellent ballast, free from dust and perfect in drainage qualities. Some kinds of slag are objectionable on account of their deleterious chemical effect on the ties and spikes—especially on metallic ties.

Shells, small coal, etc. These comparatively inferior kinds of ballast are used for light traffic when they are especially cheap and convenient. They are extremely dusty in dry weather, break up into very fine dust, and are but little better than mud.

Gravel. This is the most common form of ballast which may be called good ballast. In 1885, the Roadmasters Association of America voted in favor of gravel ballast as against rock ballast. Although not so stated, this action was perhaps due to a conviction of its real economy for the *average* railroad of this country, which may be called a "light traffic" road. Gravel should preferably be screened over a screen having a $\frac{1}{2}''$ mesh, so as to screen out all dirt and the finest stones. Generally a railroad will be able to find at some point along its line a "gravel-pit" affording a suitable supply. This may be dug out with a steam-shovel, screened if necessary, and sent out over the line by the train-load at a comparatively small cost.

Rock or broken stone. Rock ballast is generally specified to be such as will pass through a $1\frac{1}{2}''$ (or $2''$) ring. Although preferably broken by hand, machine-broken stone is much cheaper. It is most easily handled with forks. This also has the effect of

screening out the dirt and fine chips which would interfere with effectual drainage. Rock ballast is more expensive in first cost, and also more troublesome to handle, than any other kind, but under heavy traffic will keep in surface better and will require less work for maintenance after the ties have become thoroughly bedded. For roads with very light traffic, running few trains, at comparatively low velocities, the advantages of rock ballast over other kinds are not so pronounced. For such roads rock ballast is an expensive luxury. The amount of traffic which will justify the use of rock ballast will depend on the cost of obtaining ballast of the various kinds.

198. Cross-sections. A depth of 12″ under the tie is generally required on the best roads, but for light traffic this is sometimes reduced to 6″ and even less. The width is generally 1 to 2 feet less than the width of the roadbed proper—excluding ditches. If the ballast has an average width of 10 feet (12 feet at bottom and 8 feet at top) and an average depth of 15 inches (including that placed between the ties), it will require 2444 cubic yards per mile of track. The P. R.R. estimates 2500 cubic yards of gravel and 2800 cubic yards of stone ballast per mile of single track. On account of the requirements of drainage the best form of cross-section depends on the kind of ballast used.

Mud ballast. Since the great objection to mud ballast lies in its liability to become soft by soaking up the rain that falls, it becomes necessary that it should be drained as quickly and readily as its nature will permit. Fig. 106 shows a typical
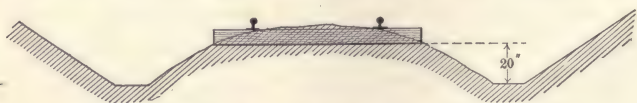
FIG. 106.—" MUD " BALLAST.

cross-section for mud ballast. It should be crowned 2″ above the top of the tie at the center, thence sloped so as to leave a slight clearance under the rail between the ties, thence sloping down to the bottom of the tie at each end and continuing to

slope down to the ditch (in cut), which should be 18″ or 20″ below the bottom of the tie.

Gravel, cinders, slag, etc. The subgrade is crowned 6″ or 8″ in the center, as shown in Fig. 107. The ballast is crowned
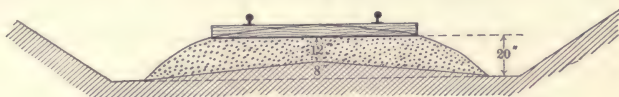
FIG. 107.—GRAVEL BALLAST.

to the top of the tie in the center, but is sloped down to the bottom of the tie at each end. This is necessary (and more especially so with mud ballast) to prevent a possible accumulation and settlement of water at the ends of the tie, which would readily soak into the end fibers and produce decay.

Broken stone. Stone ballast is shouldered out beyond the ends of the ties so as to afford greater lateral binding. The space between the ties is filled up level with the tops. The

FIG. 108.—BROKEN STONE BALLAST.

perfect drainage of stone ballast permits this to be done without any danger of causing decay of the ties by the accumulation and retention of water.

199. Methods of laying ballast. The cheapest method of laying ballast on new roads is to lay ties and rails directly on the prepared subgrade and run a construction train over the track to distribute the ballast. Then the track is lifted up until sufficient ballast is worked under the ties and the track is properly surfaced. This method, although cheap, is apt to injure the rails by causing bends and kinks, due to the passage of loaded construction trains when the ties are very unevenly and roughly supported, and the method is therefore condemned and prohibited in some specifications. The best method is to draw

in carts (or on a contractor's temporary track) the ballast that is required under the level of the *bottom* of the ties. Spread this ballast carefully to the required surface. Then lay the ties and rails, which will then have a very fair surface and uniform support. A construction train can then be run on the rails and distribute sufficient additional ballast to pack around and between the ties and make the required cross-section.

The necessity for constructing some lines at an absolute minimum of cost and of opening them for traffic as soon as possible has often led to the policy of starting traffic when there is little or no ballast—perhaps nothing more than a mere tamping of the natural soil under the ties. When this is done ballast may subsequently be drawn where required by the train-load on flat cars and unloaded at a minimum of cost by means of a "plough." The plough has the same width as the cars and is guided either by a ridge along the center of each car or by short posts set up at the sides of the cars. It is drawn from one end of the train to the other by means of a cable. The cable is sometimes operated by means of a small hoisting-engine carried on a car at one end of the train. Sometimes the locomotive is detached temporarily from the train and is run ahead with the cable attached to it.

200. Cost. The cost of ballast *in the track* is quite a variable item for different roads, since it depends (*a*) on the first cost of the material as it comes to the road, (*b*) on the distance from the source of supply to the place where it is used, and (*c*) on the method of handling. The first cost of cinder or slag is frequently insignificant. A gravel-pit may cost nothing except the price of a little additional land beyond the usual limits of the right of way. Broken stone will usually cost $1 or more per cubic yard. If suitable stone is obtainable on the company's land, the cost of blasting and breaking should be somewhat less than this. The cost of loading the ballast on to trains will be small (per cubic yard) if it is handled with steam-shovels—as in the case of gravel taken from a gravel-pit. Hand-shovelling will cost more. The cost of hauling will depend on the distance

hauled, and also, to a considerable extent, on the limitations on
the operation of the train due to the necessity of keeping out of
the way of regular trains.   There is often a needless waste in
this way.   The "mud train" is considered a pariah and entitled
to no rights whatever, regardless of the large daily cost of such
a train and of the necessary gang of men.   The cost of broken
stone ballast *in the track* is estimated at $1.25 per cubic yard.
The cost of gravel ballast is estimated at 60 c. per cubic yard
in the track.   The cost of placing and tamping gravel ballast is
estimated at 20 c. to 24 c. per cubic yard, for cinders 12 c. to
15 c. per cubic yard.   The cost of loading gravel on cars, using
a steam-shovel, is estimated at 6 c. to 10 c. per cubic yard.*

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* Report Roadmasters Association, 1885.

CHAPTER VIII.

TIES,

AND OTHER FORMS OF RAIL SUPPORT.

201. Various methods of supporting rails. It is necessary that the rails shall be sufficiently supported and braced, so that the gauge shall be kept constant and that the rails shall not be subjected to excessive transverse stress. It is also preferable that the rail support shall be neither rigid (as if on solid rock) nor too yielding, but shall have a *uniform* elasticity throughout. These requirements are more or less fulfilled by the following methods.

(a) Longitudinals. Supporting the rails throughout their entire length. This method is very seldom used in this country except occasionally on bridges and in terminals when the longitudinals are supported on cross-ties. In § 224 will be described a system of rails, used to some extent in Europe, having such broad bases that they are self-supporting on the ballast and are only connected by tie-rods to maintain the gauge.

(b) Cast-iron "bowls" or "pots." These are castings resembling large inverted bowls or pots, having suitable chairs on top for holding and supporting the rails, and tied together with tie-rods. They will be described more fully later (§ 223).

(b) Cross-ties of metal or wood. These will be discussed in the following sections.

202. Economics of ties. The true cost of ties depends on the relative total cost of maintenance for long periods of time. The first cost of the ties delivered to the road is but one item in the

economics of the question. Cheap ties require frequent renewals, which cost for the *labor* of each renewal practically the same whether the tie is of oak or hemlock. Cheap ties make a poor roadbed which will require more track labor to keep even in tolerable condition. The roadbed will require to be disturbed so frequently on account of renewals that the ties never get an opportunity to get settled and to form a smooth roadbed for any length of time. Irregularity in width, thickness, or length of ties is especially detrimental in causing the ballast to act and wear unevenly. The life of ties has thus a more or less direct influence on the life of the rails, on the wear of rolling stock, and on the speed of trains. These last items are not so readily reducible to dollars and cents, but when it can be shown that the total cost, for a long period of time, of several renewals of cheap ties, with all the extra track labor involved, is as great as or greater than that of a few renewals of durable ties, then there is no question as to the real economy. In the following discussions of the merits of untreated ties (either cheap or costly), chemically treated ties, or metal ties, the true question is therefore of the ultimate cost of maintaining any particular kind of ties for an indefinite period, the cost including the first cost of the ties, the labor of placing them and maintaining them to surface, and the somewhat uncertain (but not therefore non-existent) effect of frequent renewals on repairs of rolling stock, on possible speed, etc.

WOODEN TIES.

203. Choice of wood. This naturally depends, for any particular section of country, on the supply of wood which is most readily available. The woods most commonly used, especially in this country, are oak and pine, oak being the most durable and generally the most expensive. Redwood is used very extensively in California and proves to be extremely durable, so far as decay is concerned, but it is very soft and is much injured by " rail-cutting." This defect is being partly remedied by the

use of tie-plates, as will be explained later. Cedar, chestnut, hemlock, and tamarack are frequently used in this country. In tropical countries very durable ties are frequently obtained from the hard woods peculiar to those countries. According to a recent bulletin of the U. S. Department of Agriculture the proportions of the various kinds used in the United States are about as follows:

Oak................	60%	Chestnut............	5%	Cypress.............	2%
Pine..............	20	Hemlock and Tama-		Various............	1
Cedar............	6	rack..............	3		
		Redwood..........	3	Total.........	100%

204. Durability. The durability of ties depends on the climate; the drainage of the ballast; the volume, weight, and speed of the traffic; the curvature, if any; the use of tie-plates; the time of year of cutting the timber; the age of the timber and the degree of its seasoning before placing in the track; the nature of the soil in which the timber was grown; and, chiefly, on the species of wood employed. The variability in these items will account for the discrepancies in the reports on the life of various woods used for ties.

White oak is credited with a life of 5 to 12 years, depending principally on the traffic. Is is both hard and durable, the hardness enabling it to withstand the cutting tendency of the rail-flanges, and the durability enabling it to resist decay. *Pine* and *redwood* resist decay very well, but are so soft that they are badly cut by the rail-flanges and do not hold the spikes very well, necessitating frequent respiking. Since the spikes must be driven within certain very limited areas on the face of each tie, it does not require many spike-holes to "spike-kill" the tie. On sharp curves, especially with heavy traffic, the wheel-flange pressure produces a side pressure on the rail tending to overturn it, which tendency is resisted by the spike, aided sometimes by rail-braces. Whenever the pressure becomes too great the spike will yield somewhat and will be slightly withdrawn. The resistance is then somewhat less and the spike is soon so loose that it must be redriven in a new hole. If this occurs very

often, the tie may need to be replaced long before any decay has
set in.   When the traffic is very light, the wood very durable,
and the climate favorable ties have been known to last 25 years.

205. Dimensions. The usual dimensions for the best roads
(standard gauge) are 8′ to 8′ 6″ long, 6″ to 7″ thick, and 8″ to
10″ wide on top and bottom (if they are hewed) or 8″ to 9″
wide if they are sawed.   For cheap roads and light traffic the
length is shortened sometimes to 7′ and the cross-section also re-
duced.   On the other hand a very few roads use ties 9′ long.

Two objections are urged against sawed ties : first, that the
grain is torn by the saw, leaving a woolly surface which induces
decay ; and secondly, that, since timber is not perfectly straight-
grained, some of the fibers are cut obliquely, exposing their ends,
which are thus liable to decay.   The use of a " planer-saw " ob-
viates the first difficulty.   Chemical treatment of ties obviates
both of these difficulties.   Sawed ties are more convenient to
handle, are a necessity on bridges and trestles, and it is even
claimed, although against commonly received opinion, that
actual trial has demonstrated that they are more durable than
hewed ties.

206. Spacing. The spacing is usually 14 to 16 ties to a 30-
foot rail.   This number is sometimes reduced to 12 and even
10, and on the other hand occasionally increased to 18 or 20 by
employing narrower ties.   There is no economy in reducing the
number of ties very much, since for any required stiffness of
track it is more economical to increase the number of supports than
to increase the weight of the rail.   The decreasing cost of rails
and the increasing cost of ties have materially changed the rela-
tion between number of ties and weight of rail to produce a
given stiffness at minimum cost, but many roads have found it
economical to employ a large number of ties rather than increase
the weight of the rail.   On the other hand there is a practical
limit to the number that may be employed, on account of the
necessary space between the ties that is required for proper
tamping.   This width is ordinarily about twice the width of the
tie.   At this rate, with light ties 6″ wide and with 12″ clear

space, there would be 20 ties per 30-foot rail, or 3520 per mile. The smaller ties can generally be bought much cheaper (proportionately) than the larger sizes, and hence the economy.

Track instructions to foremen generally require that the spacing of ties shall *not* be uniform along the length of any rail. Since the joint is generally the weakest part of the rail structure, the joint requires more support than the center of the rail. Therefore the ties are placed with but 8″ or 10″ clear space between them at the joints, this applying to 3 or 4 ties at each joint; the remaining ties, required for each rail length, are equally spaced along the remaining distance.

207. Specifications. The specifications for ties are apt to include the items of size, kind of wood, and method of construction, besides other minor directions about time of cutting, seasoning, delivery, quality of timber, etc.

(a) Size. The particular size or sizes required will be somewhat as indicated in § 205.

(b) Kind of wood. When the kind or kinds of wood are specified, the most suitable kinds that are available in that section of country are usually required.

(c) Method of construction. It is generally specified that the ties shall be hewed on two sides; that the two faces thus made shall be parallel planes and that the bark shall be removed. It is sometimes required that the ends shall be sawed off square; that the timber shall be cut in the winter (when the sap is down); and that the ties shall be seasoned for six months. These last specifications are not required or lived up to as much as their importance deserves. It is sometimes required that the ties shall be delivered on the right of way, neatly piled in rows, the alternate rows at right angles, piled if possible on ground not lower than the rails and at least seven feet away from them, the lower row of ties resting on two ties which are themselves supported so as to be clear of the ground.

(d) Quality of timber. The usual specifications for sound timber are required, except that they are not so rigid as for a better class of timber work. The ties must be sound, reason-

ably straight-grained, and not very crooked—one test being that a line joining the center of one end with the center of the middle shall not pass outside of the other end. Splits or shakes, especially if severe, should cause rejection.

Specifications sometimes require that the ties shall be cut from single trees, making what is known as " pole ties " and definitely condemning those which are cut or split from larger trunks, giving two " slab ties " or four " quarter ties " for each cross-section, as is illustrated in Fig. 109. Even if pole ties are better, their exclusive use means the rapid destruction of forests of young trees.

POLE TIE.   -   SLAB TIE.   QUARTER TIE

FIG. 109.—METHODS OF CUTTING TIES.

208. Regulations for laying and renewing ties. The regulations issued by railroad companies to their track foremen will generally include the following, in addition to directions regarding dimensions, spacing, and specifications given in §§ 204–207. When hewn ties of somewhat variable size are used, as is frequently the case, the largest and best are to be selected for use as joint ties. If the upper surface of a tie is found to be warped (contrary to the usual specifications) so that one or both rails do not get a full bearing across the whole width of the tie, it must be adzed to a true surface along its whole length and not merely notched for a rail-seat. When respiking is necessary and spikes have been pulled out, the holes should be immediately plugged with " wooden spikes," which are supplied to the foremen for that express purpose, so as to fill up the holes and prevent the decay which would otherwise take place when the hole becomes filled with rain-water. Ties should always be laid at right angles to the rails and never obliquely. Minute regulations to prevent premature rejection and renewal of ties are frequently made. It is generally required that the requisitions for renewals shall be made by the actual count of the individual ties to be renewed instead of by any wholesale estimates. It is unwise to have ties of widely variable size, hardness, or durability adjacent to each

other in the track, for the uniform elasticity, so necessary for smooth riding, will be unobtainable under those circumstances.

209. Cost of ties. When railroads can obtain ties cut by farmers from woodlands in the immediate neighborhood, the price will frequently be as low as 20 c. for the smaller sizes, running up to 50 c. for the larger sizes and better qualities, especially when the timber is not very plentiful. Sometimes if a railroad cannot procure suitable ties from its immediate neighborhood, it will find that adjacent railroads control all adjacent sources of supply for their own use and that ties can only be procured from a considerable distance, with a considerable added cost for transportation. First-class oak ties cost about 75 to 80 c. and frequently much more. Hemlock ties can generally be obtained for 35 c. or less.

PRESERVATIVE PROCESSES FOR WOODEN TIES.

210. General principle. Wood has a fibrous cellular structure, the cells being filled with sap or air. The woody fiber is but little subject to decay unless the sap undergoes fermentation. Preservative processes generally aim at removing as much of the water and sap as possible and filling up the pores of the wood with an antiseptic compound. The most common methods (except one) all agree in this general process and only differ in the method employed to get rid of the sap and in the antiseptic chemical with which the fibers are filled. One valuable feature of these processes lies in the fact that the softer cheaper woods (such as hemlock and pine) are more readily treated than are the harder woods and yet will produce practically as good a tie as a treated hard-wood tie and a very much better tie than an untreated hard-wood tie. The various processes will be briefly described, taking up first the process which is fundamentally different from the others, viz., vulcanizing.

211. Vulcanizing. The process consists in heating the timber to a temperature of 300° to 500° F. in a cylinder, the air being under a pressure of 100 to 175 lbs. per square inch. By this process the albumen in the sap is coagulated, the water evap-

orated, and the pores are partially closed by the coagulation of the albumen. It is claimed that the heat sterilizes the wood and produces chemical changes in the wood which give it an antiseptic character. It has been very extensively used on the elevated lines of New York City, and it is claimed to give perfect satisfaction. The treatment has cost that road 25 c. per tie.

212. Creosoting. This process consists in impregnating the wood with *wood-creosote* or with *dead oil of coal-tar.* *Wood-creosote* is one of the products of the destructive distillation of wood—usually long-leaf pine. *Dead oil of coal-tar* is a product of the distillation of coal-tar at a temperature between 480° and 760° F. It would require about 35 to 50 pounds of creosote to completely fill the pores of a cubic foot of wood. But it would be impossible to force such an amount into the wood, nor is it necessary or desirable. About 10 pounds per cubic foot, or about 35 pounds per tie, is all that is necessary. For piling placed in salt water about 18 to 20 pounds per cubic foot is used, and the timber is then perfectly protected against the ravages of the *teredo navalis.* To do the work, long cylinders, which may be opened at the ends, are necessary. Usually the timbers are run in and out on iron carriages running on rails fastened to braces on the inside of the cylinder. When the load has been run in, the ends of the cylinder are fastened on. The water and air in the pores of the wood are first drawn out by subjecting the wood alternately to steam-pressure and to the action of a vacuum-pump. This is continued for several hours. Then, after one of the vacuum periods, the cylinder is filled with creosote oil at a temperature of about 170° F. The pumps are kept at work until the pressure is about 80 to 100 pounds per square inch, and is maintained at this pressure from one to two hours according to the size of the timber. The oil is then withdrawn, the cylinders opened, the train pulled out and another load made up in 40 to 60 minutes. The average time required for treating a load is about 18 or 20 hours, the absorption about 10 or 11 pounds of oil per cubic foot, and the cost (1894) from $12.50 to $14.50 per thousand feet B. M.

213. Burnettizing (chloride-of-zinc process). This process is very similar to the creosoting process except that the chemical is chloride of zinc, and that the chemical is not heated before use. The preliminary treatment of the wood to alternate vacuum and pressure is not continued for quite so long a period as in the creosoting process. Care must be taken, in using this process, that the ties are of as uniform quality as possible, for seasoned ties will absorb much more zinc chloride than unseasoned (in the same time), and the product will lack uniformity unless the seasoning is uniform. The A., T. & S. Fé R.R. has works of its own at which ties are treated by this process at a cost of about 25 c. per tie. The Southern Pacific R.R. also has works for burnettizing ties at a cost of 9.5 to 12 c. per tie. The zinc-chloride solution used in these works contains only 1.7% of zinc chloride instead of over 3% as used in the Santa Fé works, which perhaps accounts partially for the great difference in cost per tie. One great objection to burnettized ties is the fact that the chemical is somewhat easily washed out, when the wood again becomes subject to decay. Another objection, which is more forcible with respect to timber subject to great stresses, as in trestles, than to ties, is the fact that when the solution of zinc chloride is made strong (over 3%) the timber is made very brittle and its strength is reduced. The reduction in strength has been shown by tests to amount to $\frac{1}{4}$ to $\frac{1}{10}$ of the ultimate strength, and that the elastic limit has been reduced by about $\frac{1}{7}$.

214. Kyanizing (bichloride-of-mercury or corrosive-sublimate process). This is a process of "steeping." It requires a much longer time than the previously described processes, but does not require such an expensive plant. Wooden tanks of sufficient size for the timber are all that is necessary. The corrosive sublimate is first made into a concentrated solution of one part of chemical to six parts of *hot* water. When used in the tanks this solution is weakened to 1 part in 100 or 150. The wood will absorb about 5 to 6.5 pounds of the bichloride per 100 cubic feet, or about one pound for each 4 to 6 ties. The timber is allowed to soak in the tanks for several days, the general rule

being about one day for each inch of least thickness and one day over—which means seven days for six-inch ties, or thirteen (to fifteen) days for 12″ timber (least dimension). The process is somewhat objectionable on account of the chemical being such a virulent poison, workmen sometimes being sickened by the fumes arising from the tanks. On the Baden railway (Germany) kyanized ties last 20 to 30 years. On this railway the wood is always air-dried for two weeks after impregnation and before being used, which is thought to have an important effect on its durability. The solubility of the chemical and the liability of the chemical washing out and leaving the wood unprotected is an element of weakness in the method.

215. Wellhouse (or zinc-tannin) process. The last two methods described (as well as some others employing similar chemicals) are open to the objection that since the wood is impregnated with an aqueous solution. it is liable to be washed out very rapidly if the wood is placed under water, and will even disappear, although more slowly, under the action of moisture and rain. Several processes have been proposed or patented to prevent this. Many of them belong to one class, of which the Wellhouse process is a sample. By these processes the timber is successively subjected to the action of two chemicals, each individually soluble in water, and hence readily impregnating the timber, but the chemicals when brought in contact form insoluble compounds which cannot be washed out of the wood-cells. By the Wellhouse process, the wood is first impregnated with a solution of chloride of zinc and glue, and is then subjected to a bath of tannin under pressure. The glue and tannin combine to form an insoluble leathery compound in the cells, which will prevent the zinc chloride from being washed out. It is being used by the A., T. & S. Fé R.R., their works being located at Las Vegas, New Mexico, and also by the Union Pacific R.R. at their works at Laramie, Wyo. In 1897 Mr. J. M. Meade, a resident engineer on the A., T. & S. Fé, exhibited to the Roadmasters Association of America a piece of a tie treated by this process which had been taken from the tracks after

nearly 13 years' service. The tie was selected at random, was taken out for the sole purpose of having a specimen, and was still in sound condition and capable of serving many years longer. The cost of the treatment was then quoted as 13 c. per tie. It was claimed that the treatment trebled the life of the tie besides adding to its spike-holding power.

216. Cost of treating. The cost of treating ties by the various methods has been estimated as follows *—assuming that the plant was of sufficient capacity to do the work economically : creosoting, 25 c. per tie; vulcanizing, 25 c. per tie; burnettizing (chloride of zinc), 8.25 c. per tie ; kyanizing (steeping in corrosive sublimate), 14.6 c. per tie; Wellhouse process (chloride of zinc and tannin), 11.25 c. per tie. These estimates are only for the net cost at the works and do not include the cost of hauling the ties to and from the works, which may mean 5 to 10 c. per tie. Some of these processes have been installed on cars which are transported over the road and operated where most convenient.

217. Economics of treated ties. The fact that treated ties are not universally adopted is due to the argument that the added life of the tie is not worth the extra cost. If ties can be bought for 25 c., and cost 25 c. for treatment, and the treatment only doubles their life, there is apparently but little gained except the work of placing the extra tie in the track, which is more or less offset by the interest on 25 c. for the life of the untreated tie, and the larger initial outlay makes a stronger impression on the mind than the computed ultimate economy. But when ties cost 75 c. and treatment costs only 25 c., or perhaps less, then the economy is more apparent and unquestionable. But this analysis may be made more closely. As shown in § 202, the disturbance of the roadbed on account of frequent renewals of untreated ties is a disadvantage which would justify an appreciable expenditure to avoid, although it is

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*Bull. No. 9, U. S. Dept. of Agric., Div. of Forestry. App. No. 1, by Henry Flad.

very difficult to closely estimate its true value. The annual cost of a system of ties may be considered as the sum of (*a*) the interest on the first cost, (*b*) the annual sinking fund that would buy a new tie at the end of its life, and (*c*) the average annual cost of maintenance for the life of the tie, which includes the cost of laying and the considerable amount of subsequent tamping that must be done until the tie is fairly settled in the roadbed, beside the regular trackwork on the tie, which is practically constant. This last item is difficult to compute, but it is easy to see that, since the cost of laying the tie and the subsequent tamping to obtain proper settlement is the same for all ties (of similar form), the *average* annual charge on the longer-lived tie would be much less. In the following comparison item (*c*) is disregarded, simply remembering that the advantage is with the longer-lived tie.

	Untreated tie.	Treated tie.
Original cost	40 cents	65 cents
Life (assumed at).....................	7 years	14 years
Item (*a*)—interest on first cost @ 4%.....	1.6 cents	2.6 cents
" (*b*)—sinking fund @ 4%	5.1 "	3.6 "
" (*c*)—(considered here as offsetted) ...	——	——
Average annual cost (except item (*c*))	6.7 cents	6.2 cents

On this basis treated ties will cost 0.5 cent *less* per annum *besides* the advantage of item (*c*) and the still more indefinite advantages resulting from smoother running of trains, less wear and tear on rolling stock, etc., due to less disturbance of the roadbed.

In Europe, where wood is expensive, untreated ties are seldom used, as the treatment is always considered to be worth more than it costs. The rapid destruction of the forests of timber in this country is having the effect of increasing the price, so that it will not be long before treated ties (or metal ties) will be economical for a large majority of the railroads of the country.

METAL TIES.

218. Extent of use. In 1894 * there were nearly 35000 miles of " metal track " in various parts of the world. Of this total, there were 3645 miles of " longitudinals " (see § 224), found exclusively in Europe, nearly all of it being in Germany. There were over 12000 miles of " bowls and plates " (see § 223), found almost entirely in British India and in the Argentine Republic. The remainder, over 18000 miles, was laid with metal cross-ties of various designs. There were over 8000 miles of metal cross-ties in Germany alone, about 1500 miles in the rest of Europe, over 6000 miles in British India, nearly 1000 miles in the rest of Asia, and about 1500 miles more in various other parts of the world. Several railroads in this country have tried various designs of these ties, but their use has never passed the experimental stage. These 35000 miles represent about 9% of the total railroad mileage of the world—nearly 400000 miles. They represent about 17.6% of the total railroad mileage, exclusive of the United States and Canada, where they are not used at all, except experimentally. In the four years from 1890 to 1894 the use of metal track increased from less than 25000 miles to nearly 35000 miles. This increase was practically equal to the total increase in railroad mileage during that time, exclusive of the increase in the United States and Canada. This indicates a large growth in the percentage of metal track to total mileage, and therefore an increased appreciation of the advantages to be derived from their use.

219. Durability. The durability of metal track is still far from being a settled question, due largely to the fact that the best form for such track is not yet determined, and that a large part of the apparent failures in metal track have been evidently due to defective design. Those in favor of them estimate the life as from 30 to 50 years. The opponents place it as not more than 20 years, or perhaps as long as the best of wooden ties.

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* Bulletin No. 9, U. S. Dept. of Agriculture, Div. of Forestry.

Unlike the wooden tie, however, which deteriorates as much
with time as with usage, the life of a metal tie is more largely a
function of the traffic.   The life of a well-designed metal tie has
been estimated at 150000 to 200000 trains; for 20 trains per
day, or say 6000 per year, this would mean from 25 to 33 years.
20 trains per day on a *single* track is a much larger number than
will be found on the majority of railroads.   Metal ties are found
to be subject to rust, especially when in damp localities, such as
tunnels; but on the other hand it is in such confined localities,
where renewals are troublesome, that it is especially desirable to
employ the best and longest-lived ties.   Paint, tar, etc., have
been tried as a protection against rust, but the efficacy of such
protection is as yet uncertain, the conditions preventing any re-
newal of the protection—such as may be done by repainting a
bridge, for example.   Failures in metal cross-ties have been
largely due to cracks which begin at a corner of one of the *square*
holes which are generally *punched* through the tie, the holes
being made for the bolts by which the rails are fastened to the
tie.   The holes are generally *punched* because it is cheaper.
Reaming the holes after punching is thought to be a safeguard
against this frequent cause of failure.   Another method is to
round the corners of the square punch with a radius of about
$\frac{1}{8}''$.   If a crack has already started, the spread of the crack may
be prevented by drilling a small hole at the end of it.

220. Form and dimensions of metal cross-ties.   Since stability
in the ballast is an essential quality for a tie, this must be accom-
plished either by turning down the end of the tie or by having
some form of lug extending downward from one or more points
of the tie.   The ties are sometimes depressed in the center (see
Plate XVII, N. Y. C. & H. R. R.R. tie) to allow for a thick
covering of ballast on top in order to increase its stability in the
ballast.   This form requires that the ties should be sufficiently
well tamped to prevent a tendency to bend out straight, thus
widening the gauge.   Many designs of ties are objection-
able because they cannot be placed in the track without
disturbing adjacent ties.   The failure of many metal cross-

ties, otherwise of good design, may be ascribed to too light
weight. Those weighing much less than 100 pounds have
proved too light. From 100 to 130 pounds weight is being used
satisfactorily on German railroads. The general outside dimen-
sions are about the same as for wooden ties, except as to thick-
ness. The metal is generally from $\frac{1}{4}''$ to $\frac{3}{8}''$ thick. They are,
of course, only made of wrought iron or steel, cast iron being
used only for " bowls " or " plates " (see § 213). The details
of construction of some of the most commonly used ties may be
seen by a study of Plate XVII.

221. Fastenings. The devices for fastening the rails to the
ties should be such that the gauge may be widened if desired on
curves, also that the gauge can be made true regardless of slight
inaccuracies in the manufacture of the ties, and also that shims
may be placed under the rail if necessary during cold weather
when the tie is frozen into the ballast and cannot be easily
disturbed. Some methods of fastening require that the base of
the rail be placed against a lug which is riveted to the tie or
which forms a part of it. This has the advantage of reducing
the number of pieces, but is apt to have one or more of the
disadvantages named above. Metal keys or wooden wedges are
sometimes used, but the majority of designs employ some form
of bolted clamp. The form adopted for the experimental ties
used by the N. Y. C. & H. R. R.R. (see Plate XVII) is especially
ingenious in the method used to vary the gauge or allow for
inaccuracies of manufacture. Plate XVII shows some of the
methods of fastening adopted on the principal types of ties.

222. Cost. The cost of metal cross-ties in Germany averages
about 1.6 c. per pound or about $1.60 for a 100-lb. tie. The
ties manufactured for the N. Y. C. & H. R. R.R. in 1892
weighed about 100 lbs. and cost $2.50 per tie, but if they had
been made in larger quantities and with the present price of
steel the cost would possibly have been much lower. The
item of freight from the place of manufacture to the place where
used is no inconsiderable item of cost with some roads. Metal
cross-ties have been used by some street railroads in this country.

PLATE XVII.

"SYSTEM VAUTHERIN"

"SYSTEM COUILLET"
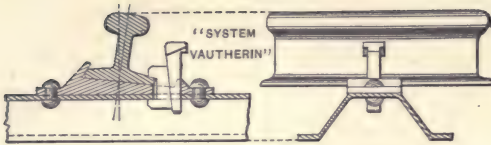
"SYSTEM GRIFFIN" (BOWL)

TIE ROD

N. Y. C. & H. R. R. R. (1892)

FOR LB. RAIL
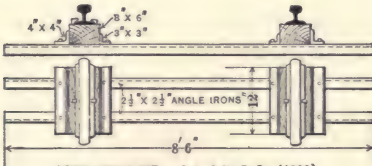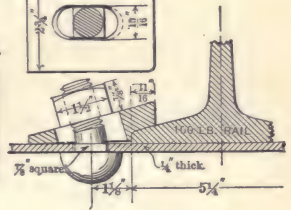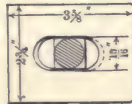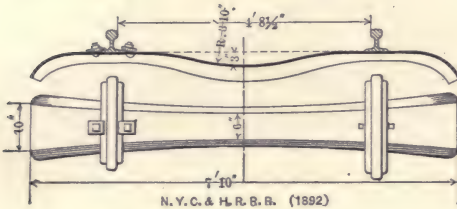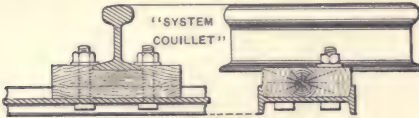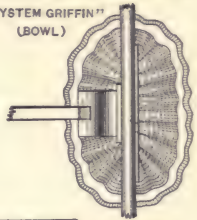
COOK-HICKS TIE. D. L. & W. R. R. (1890)
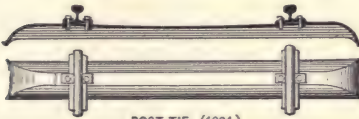
POST TIE. (1884)
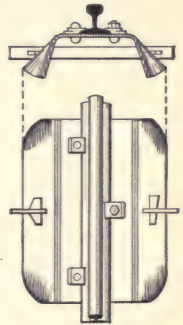
LIVESEY BOWL. (1864)

"SYSTEM KÖSTLIN U. BATTIG"—LONGITUDINAL.
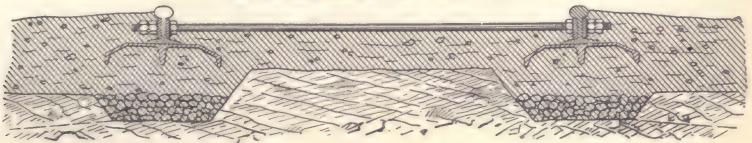
METAL TIES.

Those used on the Terre Haute Street Railway weigh 60 pounds and cost about 66 c. for the tie, or 74 c. per tie with the fastenings.

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223. Bowls or plates. As mentioned before, over 12000 miles of railway, chiefly in British India and in the Argentine Republic, are laid with this form of track. It consists essentially of large cast-iron inverted " bowls " laid at intervals under each rail and opposite each other, the opposite bowls being tied together with tie-rods. A suitable chair is riveted or bolted on to the top of each bowl so as to properly hold the rail. Being made of cast iron, they are not so subject to corrosion as steel or wrought iron. They have the advantage that when old and worn out their scrap value is from 60 to 80% of their initial cost, while the scrap value of a steel or wrought-iron tie is practically nothing. Failure generally occurs from breakage, the failures from this cause in India being about 0.4 per cent per annum. They weigh about 250 lbs. apiece and are therefore quite expensive in first cost and transportation charges. There are miles of them in India which have already lasted 25 years and are still in a serviceable condition. Some illustrations of this form of tie are shown in Plate XVII.

224. Longitudinals.* This form, the use of which is confined almost exclusively to Germany, is being gradually replaced on many lines by metal cross-ties. The system generally consists of a compound rail of several parts, the upper bearing rail being very light and supported throughout its length by other rails, which are suitably tied together with tie-rods so as to maintain the proper gauge, and which have a sufficiently broad

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* Although the discussion of longitudinals might be considered to belong more properly to the subject of RAILS, yet the essential idea of all designs must necessarily be the *support* of a rail-head on which the rolling stock may run, and therefore this form, unused in this country, will be briefly described here.

base to be properly supported in the ballast. One great objection to this method of construction is the difficulty of obtaining proper drainage especially on grades, the drainage having a

tendency to follow along the lines of the rails. The construction is much more complicated on sharp curves and at frogs and switches. Another fundamentally different form of longi-
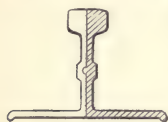
Fig. 110.

tudinal is the Haarman compound "self-bearing" rail, having a base 12″ wide and a height of 8″, the alternate sections breaking joints so as to form a practically continuous rail.

Some of the other forms of longitudinals are illustrated in Plate XVII.

For a very complete discussion of the subject of metal ties, see the "Report on the Substitution of Metal for Wood in Railroad Ties" by E. E. Russell Tratman, it being Bulletin No. 4, Forestry Division of the U. S. Dept. of Agriculture.

CHAPTER IX.

RAILS.

225. Early forms. The first rails ever laid were wooden stringers which were used on very short tram-roads around coal-mines. As the necessity for a more durable rail increased, owing chiefly to the invention of the locomotive as a motive power, there were invented successively the cast-iron "fish-belly" rail and various forms of wrought-iron strap rails which finally developed into the T rail used in this country and the double-headed rail, supported by chairs, used so extensively in England. The cast-iron rails were cast in lengths of about 3 feet and were supported in iron chairs which were sometimes set upon stone piers. A great deal of the first railroad track of this country was laid with longitudinal stringers of wood placed upon cross-ties, the inner edge of the stringers being

QUINCY R. R. 1826. BALT. & OHIO R.R. 1843. "BULL-HEAD." "FISH-BELLY"—CAST IRON.

VIGNOLES. 1836. CAST IRON.

CAMDEN & AMBOY. 1832. STEPHENSON. 1838. "PEAR." REYNOLDS—1767.
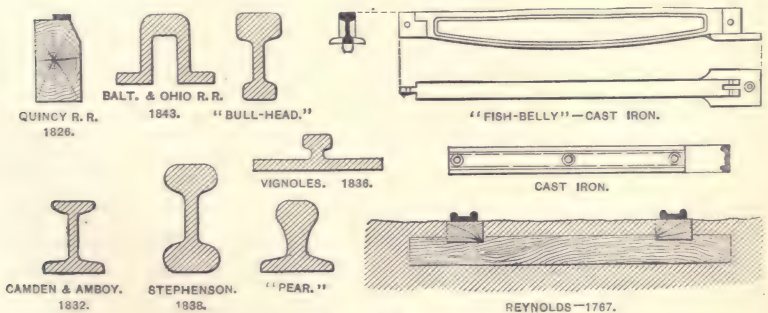
FIG. 111.—EARLY FORMS OF RAILS.

protected by wrought-iron straps. The "bridge" rails were first rolled in this country in 1844. The "pear" section was

an approach to the present form, but was very defective on
account of the difficulty of designing a good form of joint.   The
" Stevens " section was designed in 1830 by Col. Robert L.
Stevens, Chief Engineer of the Camden and Amboy Railroad;
although quite defective in its proportions, according to the
present knowledge of the requirements, it is essentially the pres-
ent form.   In 1836, Charles Vignoles invented essentially the
same form in England; this form is therefore known throughout
England and Europe as the Vignoles rail.

226. Present standard forms.   The larger part of modern
railroad track is laid with rails which are either " T " rails or
the double-headed or " bull-headed " rails which are carried in
chairs.   The double-headed rail was designed with a symmetri-
cal form with the idea that after one head had been worn out
by traffic the rail could be reversed, and that its life would be
practically doubled.   Experience has shown that the wear of the
rail in the chairs is very great; so much so that when one head
has been worn out by traffic the whole rail is generally useless.
If the rail is turned over, the worn places, caused by the chairs,
make a rough track and the rail appears to be more brittle and
subject to fracture, possibly due to the crystallization that may
have occurred during the previous usage and to the reversal of
stresses in the fibers.   Whatever the explanation, experience has
demonstrated the *fact*.   The " bull-headed " rail has the lower

head only large enough to properly hold
the wooden keys with which the rail is
secured to the chairs (see Fig. 112) and
furnish the necessary strength.   The use

Fig. 112.—Bull-headed of these rails requires the use of two cast-
RAIL AND CHAIR. iron chairs for each tie.   It is claimed that
such track is better for heavy and fast traffic, but it is more
expensive to build and maintain.   It is the standard form of
track in England and some parts of Europe.

Until a few years ago there was a very great multiplicity
in the designs of " T " rails as used in this country, nearly
every prominent railroad having its own special design, which

perhaps differed from that of some other road by only a very
minute and insignificant detail, but which nevertheless would
require a complete new set of rolls for rolling. This certainly
must have had a very appreciable effect on the cost of rails. In
1893, the American Society of Civil Engineers, after a very
exhaustive investigation of the subject, extending over several
years, having obtained the opinions of the best experts of the
country, adopted a series of sections which have been very ex-
tensively adopted by the railroads of this country. Instead of
having the rail sections for various weights to be geometrically
similar figures, certain dimensions are made constant, regardless
of the weight. It was decided that the metal should be dis-
tributed through the section in the proportions of—head 42%,
web 21%, and flange 37%. The top of the head should have a
radius of 12″; the top corner radius of head should be $\frac{5}{16}$″; the
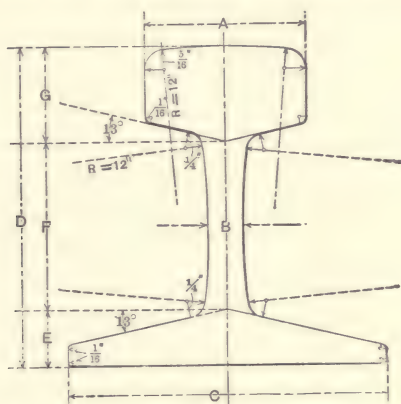
FIG. 113.—AM. SOC. C. E. STANDARD RAIL SECTION.

lower corner radius of head should be $\frac{1}{16}$″; the corners of the
flanges, $\frac{1}{16}$″ radius; side radius of web, 12″; top and bottom
radii of web corners, $\frac{1}{4}$″; and angles with the horizontal of the
under side of the head and the top of the flange, 13°. The
sides of the head are vertical.

The height of the rail (D) and the width of the base (C) are
always made equal to each other.

	Weight per Yard.												
	40	45	50	55	60	65	70	75	80	85	90	95	100
A	$1\frac{7}{8}''$	$2''$	$2\frac{1}{8}''$	$2\frac{1}{4}''$	$2\frac{3}{8}''$	$2\frac{13}{32}''$	$2\frac{7}{16}''$	$2\frac{15}{32}''$	$2\frac{1}{2}''$	$2\frac{9}{16}''$	$2\frac{5}{8}''$	$2\frac{11}{16}''$	$2\frac{3}{4}''$
B	$\frac{25}{64}$	$\frac{27}{64}$	$\frac{7}{16}$	$\frac{15}{32}$	$\frac{31}{64}$	$\frac{1}{2}$	$\frac{33}{64}$	$\frac{17}{32}$	$\frac{35}{64}$	$\frac{9}{16}$	$\frac{9}{16}$	$\frac{9}{16}$	$\frac{9}{16}$
$C\ \&\ D$	$3\frac{1}{2}$	$3\frac{11}{16}$	$3\frac{7}{8}$	$4\frac{1}{16}$	$4\frac{1}{4}$	$4\frac{7}{16}$	$4\frac{5}{8}$	$4\frac{13}{16}$	5	$5\frac{3}{16}$	$5\frac{3}{8}$	$5\frac{9}{16}$	$5\frac{3}{4}$
E	$\frac{9}{16}$	$\frac{21}{32}$	$\frac{11}{16}$	$\frac{23}{32}$	$\frac{49}{64}$	$\frac{25}{32}$	$\frac{13}{16}$	$\frac{27}{32}$	$\frac{7}{8}$	$\frac{57}{64}$	$\frac{59}{64}$	$\frac{15}{16}$	$\frac{31}{32}$
F	$1\frac{55}{64}$	$1\frac{31}{32}$	$2\frac{1}{16}$	$2\frac{11}{64}$	$2\frac{17}{64}$	$2\frac{3}{8}$	$2\frac{15}{32}$	$2\frac{35}{64}$	$2\frac{5}{8}$	$2\frac{3}{4}$	$2\frac{55}{64}$	$2\frac{63}{64}$	$3\frac{5}{64}$
G	$1\frac{1}{64}$	$1\frac{1}{16}$	$1\frac{1}{8}$	$1\frac{11}{64}$	$1\frac{7}{32}$	$1\frac{9}{32}$	$1\frac{11}{32}$	$1\frac{27}{64}$	$1\frac{1}{2}$	$1\frac{37}{64}$	$1\frac{41}{64}$	$1\frac{11}{16}$	$1\frac{45}{64}$

The chief features of disagreement among railroad men relate to the radius of the upper corner of the head and the slope of the side of the head. The radius ($\frac{5}{16}''$) adopted for the upper corner (constant for all weights) is a little more than is advocated by those in favor of "sharp corners" who often use a radius of $\frac{1}{4}''$. On the other hand it is much less than is advocated by those who consider that it should be nearly equal to (or even greater than) the larger
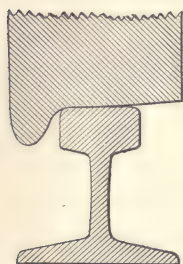
Fig. 114. — Rela-
tion of Rail to
Wheel-tread.

radius universally adopted for the corner of the wheel-flange. The discussion turns on the relative rapidity of rail wear and the wear of the wheel-flanges as affected by the relation of the form of the wheel-tread to that of the rail. It is argued that sharp rail corners wear the wheel-flanges so as to produce sharp flanges, which are liable to cause derailment at switches and also to require that the tires of engine-drivers must be more frequently turned down to their true form. On the other hand it is generally believed that rail wear is much less rapid while the area of contact between the rail and wheel-flange is small, and that when the rail has worn down, as it invariably does, to nearly the same form as the wheel-flange, the rail wears away very quickly.

227. Weight for various kinds of traffic. The heaviest rails in regular use weigh 100 lbs. per yard, and even these are only used on some of the heaviest traffic sections of such roads as the N. Y. Central, the Pennsylvania, the N. Y., N. H. & H., and

a few others. Probably the larger part of the mileage of the country is laid with 60- to 75-lb. rails—considering the fact that "the larger part of the mileage" consists of comparatively light-traffic roads and may exclude all the heavy trunk lines. Very light-traffic roads are sometimes laid with 56-lb. rails. Roads with fairly heavy traffic generally use 75- to 85-lb. rails, especially when grades are heavy and there is much and sharp curvature. The tendency on all roads is toward an increase in the weight, rendered necessary on account of the increase in the weight and capacity of rolling stock, and due also to the fact that the price of rails has been so reduced that it is both better and cheaper to obtain a more solid and durable track by increasing the weight of the rail rather than by attempting to support a weak rail by an excessive number of ties or by excessive track labor in tamping. It should be remembered that in buying rails the mere weight is, in one sense, of no importance. The important thing to consider is the STRENGTH and the STIFFNESS. If we assume that all weights of rails have *similar* cross-sections (which is nearly although not exactly true), then, since for beams of similar cross-sections the *strength* varies as the *cube* of the homologous dimensions and the *stiffness* as the *fourth power*, while the area (and therefore the weight per unit of length) only varies as the *square*, it follows that the stiffness varies as the square of the weight, and the strength as the $\frac{3}{2}$ power of the weight. Since for ordinary variations of weight the price per ton is the same, adding (say) 10% to the weight (and cost) adds 21% to the stiffness and over 15% to the strength. As another illustration, using an 80-lb. rail instead of a 75-lb. rail adds only $6\frac{2}{3}$% to the cost, but adds about 14% to the stiffness and nearly 11% to the strength. This shows why heavier rails are more economical and are being adopted even when they are not absolutely needed on account of heavier rolling stock. The stiffness, strength, and consequent durability are increased in a much greater ratio than the cost.

228. Effect of stiffness on traction. A very important but generally unconsidered feature of a stiff rail is its effect on trac-

tive force. An extreme illustration of this principle is seen
when a vehicle is drawn over a soft sandy road. The constant
compression of the sand in front of the wheel has virtually the
same effect on traction as drawing the wheel up a grade whose
steepness depends on the radius of the wheel and the depth of
the rut. On the other hand, if a wheel, made of perfectly
elastic material, is rolled over a surface which, while supported
with absolute rigidity, is also perfectly elastic, there would be a
forward component, caused by the expanding of the compressed
metal just behind the center of contact, which would just bal-
ance the backward component. If the rail was supported
throughout its length by an absolutely rigid support, the high
elasticity of the wheel-tires and rails would reduce this form of
resistance to an insignificant quantity, but the ballast and even
the ties are comparatively inelastic. When a weak rail yields,
the ballast is more or less compressed or displaced, and even
though the elasticity of the rail brings it back to nearly its
former place, the work done in compressing an inelastic material
is wholly lost. The effect of this on the fuel account is certainly
very considerable and yet is frequently entirely overlooked. It
is practically impossible to compute the saving in tractive power,
and therefore in cost of fuel, resulting from a given increase in
the weight and stiffness of the rail, since the yielding of the rail
is so dependent on the spacing of the ties, the tamping, etc. But
it is not difficult to perceive in a general way that such an econ-
omy is possible and that it should not be neglected in considering
the value of stiffness in rails.

229. Length of rails. The standard length of rails with most
railroads is 30 feet. In recent years many roads have been try-
ing 45-foot and even 60-foot rails. The argument in favor of
longer rails is chiefly that of the reduction in track-joints, which
are costly to construct and to maintain and are a fruitful source
of accidents. Mr. Morrison of the Lehigh Valley R.R.* de-
clares that, as a result of extensive experience with 45-foot rails

on that road, he finds that they are much less expensive to handle, and that, being so long, they can be laid around sharp curves without being curved in a machine, as is necessary with the shorter rails. The great objection to longer rails lies in the difficulty in allowing for the expansion, which will require, in the coldest weather, an opening at the joint of nearly $\frac{3}{4}''$ for a 60-foot rail. The Pennsylvania R.R. and the Norfolk and Western R.R. each have a considerable mileage laid with 60-foot rails.

230. Expansion of rails. Steel expands at the rate of .0000065 of its length per degree Fahrenheit. The extreme range of temperature to which any rail will be subjected will be about 160°, or say from $- 20°$ F. to $+ 140°$ F. With the above coefficient and a rail length of 60 feet the expansion would be 0.0624 foot, or about $\frac{3}{4}$ inch. But it is doubtful whether there would ever be such a range of motion even if there were such a range of temperature. Mr. A. Torrey, chief engineer of the Mich. Cent. R.R., experimented with a section over 500 feet long, which, although not a single rail, was made " continuous " by rigid splicing, and he found that there was no appreciable additional contraction of the rail at any temperature below $+ 20$ F. The reason is not clear, but the *fact* is undeniable.

The heavy girder rails, used by the street railroads of the country, are bonded together with perfectly tight rigid joints which do not permit expansion. If the rails are laid at a temperature of 60° F. and the temperature sinks to 0°, the rails have a *tendency* to contract .00039 of their length. If this tendency is resisted by the friction of the pavement in which the rails are buried, it only results in a tension amounting to .00039 of the modulus of elasticity, or say 10920 pounds per square inch, assuming 28 000 000 as the modulus of elasticity. This stress is not dangerous and may be permitted. If the temperature rises to 120° F., a tendency to expansion and buckling will take place, which will be resisted as before by the pavement, and a compression of 10920 pounds per square inch will be induced, which will likewise be harmless. The range of tempera-

ture of rails which are buried in pavement is much less than when they are entirely above the ground and will probably never reach the above extremes.   Rails supported on ties which are only held in place by ballast must be allowed to expand and contract almost freely, as the ballast cannot be depended on to resist the distortion induced by any considerable range of temperature, especially on curves.

231. Rules for allowing for temperature. Track regulations generally require that the track foremen shall use iron (*not* wooden) shims for placing between the ends of the rails while splicing them.   The thickness of these shims should vary with the temperature.   Some roads use such approximate rules as the following : " The proper thickness for coldest weather is $\frac{5}{16}$ of an inch; during spring and fall use $\frac{1}{8}$ of an inch, and in the very hottest weather $\frac{1}{16}$ of an inch should be allowed."   This is on the basis of a 30-foot rail.   When a more accurate adjustment than this is desired, it may be done by assuming some very high temperature (120° to 150° F.) as a maximum, when the joints should be *tight;* then compute in tabular form the spacing for each temperature, varying by 20°, allowing 0″.0468 (almost exactly $\frac{3}{64}$″) for each 20° change.   Such a tabular form would be about as follows (rail length 30 feet):

Temperature.	150°	130°	110°	90°	70°	50°	30°	10°	− 10°	− 30°
Rail opening...	0	$\frac{3}{64}$″	$\frac{3}{32}$″	$\frac{9}{64}$″	$\frac{3}{16}$″	$\frac{15}{64}$″	$\frac{9}{32}$″	$\frac{21}{64}$″	$\frac{3}{8}$″	$\frac{27}{64}$″

One practical difficulty in the way of great refinement in this work is the determination of the real temperature of the rail when it is laid.   A rail lying in the hot sun has a very much higher temperature than the air.   The temperature of the rail cannot be obtained even by exposing a thermometer directly to the sun, although such a result might be the best that is easily obtainable.   On a cloudy or rainy day the rail has practically the same temperature as the air; therefore on such days there need be no such trouble.

232. Chemical composition. About 98 to 99.5% of the composition of steel rails is iron, but the value of the rail, as a rail, is almost wholly dependent upon the large number of other chemical elements which are, or may be, present in very small amounts. The manager of a steel-rail mill once declared that their aim was to produce rails having in them—

Carbon.	0.32 to 0.40%
Silicon.	0.04 to 0.06%
Phosphorus.	0.09 to 0.105%
Manganese.	1.00 to 1.50%

The analysis of 32 specimens of rails on the Chic., Mil. & St. Paul R.R. showed variations as follows:

Carbon.	0.211 to 0.52%
Silicon.	0.013 to 0.256%
Phosphorus.	0.055 to 0.181%
Manganese.	0.35 to 1.63%

These quantities have the same general relative proportions as the rail-mill standard given above, the differences lying mainly in the broadening of the limits. Increasing the percentage of carbon by even a few hundredths of one per cent makes the rail harder, but likewise more brittle. If a track is well ballasted and not subject to heaving by frost, a harder and more brittle rail may be used without excessive danger of breakage, and such a rail will wear much longer than a softer tougher rail, although the softer tougher rail may be the better rail for a road having a less perfect roadbed.

A small but objectionable percentage of sulphur is sometimes found in rails, and very delicate analysis will often show the presence, in very minute quantities, of several other chemical elements. The use of a very small quantity of nickel or aluminum has often been suggested as a means of producing a more durable rail. The added cost and the uncertainty of

the amount of advantage to be gained has hitherto prevented the practical use or manufacture of such rails.

233. Testing. Chemical and mechanical testing are both necessary for a thorough determination of the value of a rail. The chemical testing has for its main object the determination of those minute quantities of chemical elements which have such a marked influence on the rail for good or bad. The mechanical testing consists of the usual tests for elastic limit, ultimate strength, and elongation at rupture, determined from pieces cut out of the rail, besides a "drop test." The drop test consists in dropping a weight of 2000 lbs. from a height of 16 to 20 feet on to the center of a rail which is supported on abutments placed three or four feet apart. The number of blows required to produce rupture or to produce a permanent set of specified magnitude gives a measure of the strength and toughness of the rail.

234. Rail wear on tangents. When the wheel loads on a rail are abnormally heavy, and particularly when the rail has but little carbon and is unusually soft, the concentrated pressure on

FIG. 115.

the rail is frequently greater than the elastic limit, and the metal "flows" so that the head, although greatly abraded, will spread somewhat outside of its original lines, as shown in Fig. 115. The rail wear that occurs on tangents is almost exclusively on top. Statistics show that the rate of rail wear on tangents decreases as the rails are more worn. Tests of a large number of rails on tangents have shown a rail wear averaging nearly one pound per yard per 10 000 000 tons of traffic. There is about 33 pounds of metal in one yard of the head of an 80-lb. rail. As an extreme value this may be worn down one-half, thus giving a tonnage of 165 000 000 tons for the life of the rail. Other estimates bring the tonnage down to 125 000 000 tons. Since the locomotive is considered to be responsible for one half (and possibly more) of the damage done to the rail, it is found that the rate of wear on roads with shorter trains is more rapid in proportion to the tonnage, and it

is therefore thought that the life of a rail should be expressed in terms of the number of trains. This has been estimated at 300 000 to 500 000 trains.

235. Rail wear on curves. On curves the maximum rail wear occurs on the inner side of the head of the outer rail, giving a worn form somewhat as shown in Fig. 116. The dotted line shows the nature and progress of the rail wear on the inner rail of a curve. Since the pressure on the outer rail is somewhat lateral rather than vertical, the " flow " does not take place to the same extent, if at all, on the outside, and whatever flow would take place on the inside is

Fig. 116.

immediately worn off by the wheel-flange.. Unlike the wear on tangents, the wear on curves is at a greater rate as the rail becomes more worn.
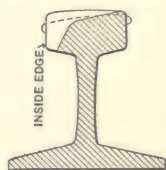
The inside rail on curves wears chiefly on top, the same as on a tangent, except that the wear is much greater owing to the longitudinal slipping of the wheels on the rail, and the lateral slipping that must occur when a rigid four-wheeled truck is guided around a curve. The outside rail is subjected to a greater or less proportion of the longitudinal slipping, likewise to the lateral slipping, and, worst of all, to the grinding action of the flange of the wheel, which grinds off the side of the head.

The results of some very elaborate tests, made by Mr. A. M. Wellington, on the Atlantic and Great Western R.R., on the wear of rails, seem to show that the rail wear on curves may be expressed by the formula: " Total wear of rails on a d degree curve in pounds per yard per 10 000 000 tons duty $= 1 + 0.03d^2$." " It is not pretended that this formula is strictly correct even in theory, but several theoretical considerations indicate that it may be nearly so." According to this formula the average rail wear on a 6° curve will be about twice the rail wear on a tangent. While this is approximately true, the various causes modifying the rate of rail wear (length of trains, age and quality of rails, etc.) will result in numerous and

large variations from the above formula, which should only be taken as indicating an approximate law.

236. Cost of rails. In 1873 the cost of steel rails was about $120 per ton, and the cost of iron rails about $70 per ton. Although the steel rails were at once recognized as superior to iron rails on account of more uniform wear, they were an expensive luxury. The manufacture of steel rails by the Bessemer process created a revolution in prices, and they have steadily dropped in price until, during the last few years, steel rails have been manufactured and sold for $22 per ton. At such prices there is no longer any demand for iron rails, since the cost of manufacturing them is substantially the same as that of steel rails, while their durability is unquestionably inferior to that of steel rails.

CHAPTER X.

RAIL-FASTENINGS.

RAIL-JOINTS.

237. Theoretical requirements for a perfect joint. A perfect rail-joint is one that has the *same strength* and *stiffness*—no more and no less—as the rails which it joins, and which will not interfere with the regular and uniform spacing of ties. It should also be reasonably cheap both in first cost and in cost of maintenance. Since the action of heavy loads on an elastic rail is to cause a wave of translation in front of each wheel, any change in the stiffness or elasticity of the rail structure will cause more or less of a shock, which must be taken up and resisted by the joint. The greater the change in stiffness the greater the shock, and the greater the destructive action of the shock. The perfect rail-joint must keep both rail ends truly in line both laterally and vertically, so that the flange or tread of the wheel need not jump or change its direction of motion suddenly in passing from one rail to the other. A consideration of all the above requirements will show that only a perfect welding of rail-ends would produce a joint of uniform strength and stiffness which would give a uniform elastic wave ahead of each wheel. As welding is impracticable for ordinary railroad work (see § 230), some other contrivance is necessary which will approach this ideal as closely as may be.

238. Efficiency of the ordinary angle-bar. Throughout the middle portion of a rail the rail acts as a continuous girder. If we consider for simplicity that the ties are unyielding, the deflection of such a continuous girder between the ties will be but

one-fourth of the deflection that would be found if the rail were cut half-way between the ties and an equal concentrated load were divided equally between the two unconnected ends. The maximum stress for the continuous girder would be but one-half of that in the cantilevers. Joining these ends with rail-joints will give the ordinary "suspended" joint. In order to maintain uniform strength and stiffness the angle-bars must supply the deficiency. These theoretical relations are modified to an unknown extent by the unknown and variable yielding of the ties. From some experiments made by the Association of Engineers of Maintenance of Way of the P. R.R.* the following deductions were made:

1. The capacity of a "suspended" joint is greater than that of a "supported" joint—whether supported on one or three ties. (See § 240.)

2. That (with the particular patterns tested) the angle-bars alone can carry only 53 to 56% of a concentrated load placed on a joint.

3. That the capacity of the whole joint (angle-bars and rail) is only 52.4% of the strength of the unbroken rail.

4. That the ineffectiveness of the angle-bar is due chiefly to a deficiency in compressive resistance.

Although it has been universally recognized that the angle-bar is not a perfect form of joint, its simplicity, cheapness, and reliability have caused its almost universal adoption. Within a very few years other forms (to be described later) have been adopted on trial sections and have been more and more extended, until their present use is very large. The present time (1900) is evidently a transition period, and it is quite probable that within a very few years the now common angle-plate will be as unknown in standard practice as the old-fashioned "fish-plate" is at the present time.

239. Effect of rail gap at joints. It has been found that the jar at a joint is due almost entirely to the *deflection* of the joint

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* Roadmasters Association of America—Reports for 1897.

and scarcely at all to the small gap required for expansion.
This gap causes a drop equal to the versed sine of the arc hav-
ing a chord equal to the gap and a radius equal to the radius of
the wheel.   Taking the extreme case (for a 30-foot rail) of a $\frac{3}{8}''$
gap and a 33″ freight-car wheel, the drop is about $\frac{1}{1000}''$.   In
order to test how much the jarring at a joint is due to a gap be-
tween the rails, the experiment was tried of cutting shallow
notches in the top of an otherwise solid rail and running a loco-
motive and an inspection car over them.   The resulting jarring
was practically imperceptible and not comparable to the jar pro-
duced at joints.   Notwithstanding this fact, many plans have
been tried for avoiding this gap.   The most of these plans con-
sist essentially of some form of compound rail, the sections
breaking joints.   (Of course the design of the compound rail
has also several other objects in view.)   In Fig. 117 are shown a

FIG. 117.—COMPOUND RAIL SECTIONS.

few of the very many designs which have been proposed.   These
designs have invariably been abandoned after trial.   Another
plan, which has been extensively tried on the Lehigh Valley
R. R., is the use of mitered joints.   The advantages gained by
their use are as yet doubtful, while the added expense is unques-
tionable.   The " Roadmasters Association of America " in 1895
adopted a resolution recommending mitered joints for double
track, but their use does not seem to be growing.

240. "Supported," "suspended," and "bridge" joints.   In a
supported joint the ends of the rails are on a tie.   If the angle-
plates are short, the joint is entirely supported on one tie; if
very long, it may be possible to place three ties under one angle-
bar and thus the joint is virtually supported on three ties rather
than one.   In a suspended joint the ends of the rails are midway
between two ties and the joint is supported by the two.   There

have always been advocates of both methods, but suspended joints are more generally used than supported joints. The opponents of three-tie joints claim that either the middle tie will be too strongly tamped, thus making it a supported joint, or that, if the middle tie is weakest, the joint becomes a very long (and therefore weak) suspended joint between the outer joint-ties, or that possibly one of the outer joint-ties gives way, thus breaking the angle-plate at the joint. Another objection which is urged is that unless the bars are very long (say 44 inches, as used on the Mich. Cent. R.R.) the ties are too close for proper tamping. The best answer to these objections is the successful use of these joints on several heavy-traffic roads.

"Bridge"-joints are similar to suspended joints in that the joint is supported on two ties, but there is the important difference that the bridge-joint supports the rail from *underneath* and there is no transverse stress in the rail, whereas the supported joint requires the combined transverse strength of both angle-bars and rail. A serious objection to bridge-joints lies in the fact of their considerable thickness between the rail base and the tie. When joints are placed "staggered" rather than "opposite" (as is now the invariable standard practice), the ties supporting a bridge-joint must either be notched down, thus weakening the tie and promoting decay at the cut, or else the tie must be laid on a slope and the joint and the opposite rail do not get a fair bearing.

241. Failures of rail-joints. It has been observed on double-track roads that the maximum rail wear occurs a few inches beyond the rail gap at the joint in the direction of the traffic. On single-track roads the maximum rail wear is found a few inches *each* side of the joint rather than at the extreme ends of the rail, thus showing that the rail end deflects down under the wheel until (with fast trains especially) the wheel actually jumps the space and strikes the rail a few inches beyond the joint, the impact producing excessive wear. This action, which is called the "drop," is apt to cause the first tie beyond the joint to become depressed, and unless this tie is carefully watched and main-

tained at its proper level, the stresses in the angle-bar may actually become reversed and the bar may break at the top. The angle-bars of a suspended joint are normally in compression at the top. The mere reversal of the stresses would cause the bars
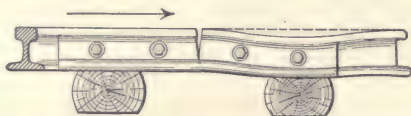
FIG. 118.—EFFECT OF " WHEEL DROP " (EXAGGERATED).

to give way with a less stress than if the stress were always the same in kind. A supported joint, and especially a three-tie joint (see § 240), is apt to be broken in the same manner.

242. Standard angle-bars.—An angle-bar must be so made as to closely fit the rails. The great multiplicity in the designs of rails (referred to in Chapter IX) results in nearly as great variety in the detailed dimensions of the angle-bars. The sections here illustrated must be considered only as types of the variable forms necessary for each different shape of rail. The absolutely essential features required for a fit are (1) the angles
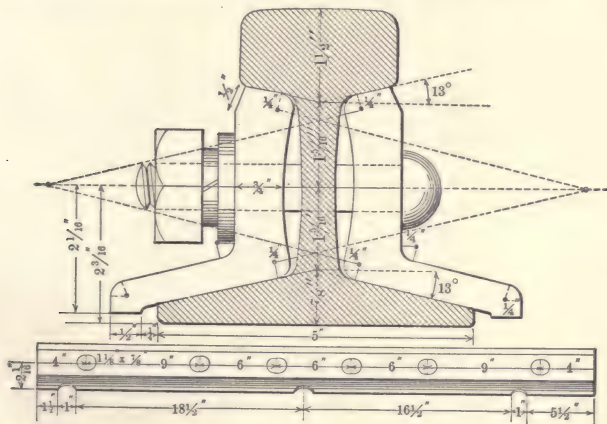
FIG. 119.—STANDARD ANGLE-BAR—80-LB. RAIL.   M. C. R.R.

of the upper and lower surfaces of the bar where they fit against the rail, and (2) the height of the bar. The bolt-holes in the

bar and rail must also correspond. The holes in the angle-plates are elongated or made oval, so that the track-bolts, which are made of corresponding shape immediately under the head, will not be turned by jarring or vibration. The holes in the rails are made of larger diameter (by about $\frac{1}{4}''$) than the bolts, so as to allow the rail to expand with temperature.

243. Later designs of rail-joints. In Plate XVIII are shown various designs which are competing for adoption. The most prominent of these (judging from the discussion in the convention of the Roadmasters Association of America in 1897) are the " Continuous " and the " Weber." Each of them has been very extensively adopted, and where used are universally preferred to angle-plates. Nearly all the later designs embody more or less directly the principle of the bridge-joint, i.e., support the rail from underneath. An experience of several years will be required to demonstrate which form of joint best satisfies the somewhat opposed requirements of minimum cost (both initial and for maintenance) and minimum wear of rails and rolling stock.

<center>TIE-PLATES.</center>

244. Advantages. (*a*) As already indicated in § 204, the life of a soft-wood tie is very much reduced by " rail-cutting " and " spike-killing," such ties frequently requiring renewal long before any serious decay has set in. It has been practically demonstrated that the " rail-cutting " is not due to the mere pressure of the rail on the tie, even with a maximum load on the rail, but is due to the impact resulting from vibration and to the longitudinal working of the rail. It has been proved that this rail-cutting is practically prevented by the use of tie-plates. (*b*) On curves there is a tendency to overturn the outer rail due to the lateral pressure on the side of the head. This produces a concentrated pressure of the outer edge of the base on the tie which produces rail-cutting and also draws the inner spikes. Formerly the only method of guarding

PLATE XVIII.
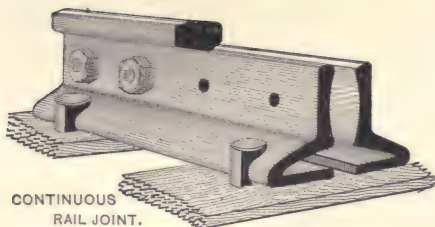
CONTINUOUS RAIL JOINT.
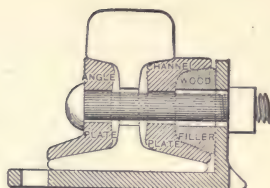
WEBER RAIL JOINT.
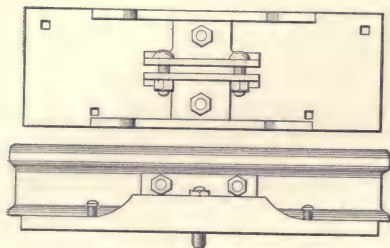
FISHER BRIDGE JOINT.
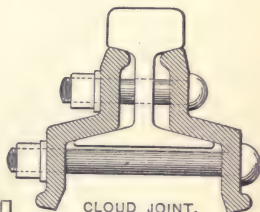
CLOUD JOINT.
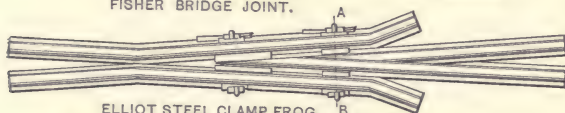
ELLIOT STEEL CLAMP FROG.
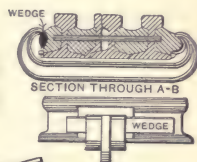
SECTION THROUGH A-B

WEIR BOLTED STIFF FROG.

ELLIOT PLATE RIVETED FROG.

SECTION THROUGH C-D.   SECTION THROUGH A-B.
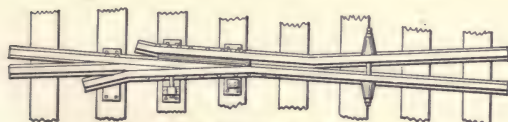
WEIR SPRING-RAIL FROG.

SECTION THROUGH PLATE AT POINT.   SECTION THROUGH SPRING-HOUSING.

RAIL JOINTS AND FROGS.

(*To face page* 260.)

against this was by the use of "rail-braces," one pattern of which is shown in Fig. 120. But it has been found that tie-
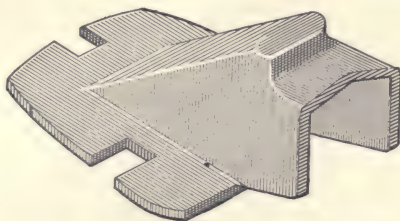
FIG. 120.

plates serve the purpose even better, and rail-braces have been abandoned where tie-plates are used. (c) Driving spikes through holes in the plate enables the spikes on *each* side of the rail to mutually support each other, no matter in which (lateral) direction the rail may tend to move, and this probably accounts in large measure for the added stability obtained by the use of tie-plates. (d) The wear in spikes, called "necking," caused by the vertical vibration of the rail against them, is very greatly reduced. (e) The cost is very small compared with the value of the added life of the tie, the large reduction in the work of track maintenance, and the smoother running on the better track which is obtained. It has been estimated that by the use of tie-plates the life of hard-wood ties is increased from one to three years, and the life of soft-wood ties is increased from three to six years. From the very nature of the case, the value of tie-plates is greater when they are used to protect soft ties.

245. Elements of the design. The earliest forms of tie-plates were flat on the bottom, but it was soon found that they would work loose, allow sand and dirt to get between the rail and the plate and also between the plate and the tie, which would cause excessive wear. Such plates are also apt to produce an objectionable rattle. Another fault of the earlier designs was the use of plates so thin that they would buckle. The latest designs have flanges or "teeth" formed on the lower surface which penetrate the tie about $\frac{3}{4}''$ to $1\frac{3}{8}''$. Opinion is still divided on the question of whether these teeth should run with the grain

or across the grain. If the flanges run with the grain, they generally extend the whole length of the tie-plate—as in the Wolhaupter design. If the grain is to be cut crosswise, several teeth about 1″ wide will be used—as in the Goldie design.
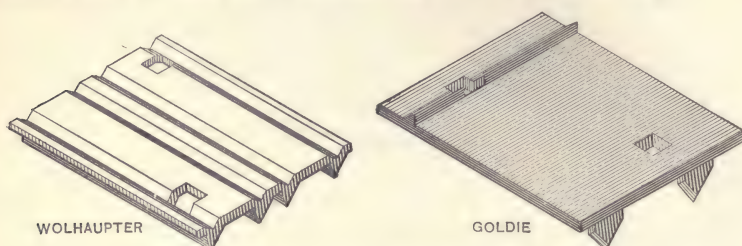
FIG. 121.—TIE-PLATES.

It is a very important feature that the spike-holes should be so punched that the spikes will fit closely to the base of the rail. Otherwise a lateral motion of the rail will be permitted which will defeat one of the main objects of the use of the plate.

Another unsettled detail is the use of "shoulders" on the upper surface. On the one hand it is claimed that the use of shoulders relieves the spikes of side pressure from the rail and prevents "necking." On the other hand it is claimed that if the plain plate is once properly set with new spikes (at least with spikes not already necked) the spikes will not neck appreciably, and that, as the shouldered plates cost more, the additional expenditure is unnecessary.

The above designs should be studied with reference to the manner in which they fulfill the requirements which have been already stated. As in the case of rail-joints, the best forms of tie-plates are of comparatively recent design, and experience with them is still insufficient to determine beyond all question which designs are the best.

246. Methods of setting. A very important detail in the process of setting the tie-plates on the ties is that the flanges or teeth should penetrate the tie as far as desired when the plates are first put in position. It requires considerable force to press the teeth into a tie. In a few cases trackmen have depended on the easy process of waiting for passing trains to force the teeth

down.   Until the teeth are down the spikes cannot be driven
home, and this apparently cheap and easy process results in loose
spikes and rails.   If the trackmen neglect even temporarily to
tighten these spikes, it will become impossible to make them
tight ultimately.   The plates are generally pounded into place
with a 10- to 16-pound sledge-hammer.   A very good method
was adopted once during the construction of a bridge when a
pile-driver was at hand.   The bridge-ties were placed under the
pile-hammer.   The plates, accurately set to gauge, were then
forced in by a blow from the 3000-lb. hammer falling 2 or 3
feet.

<div align="center">SPIKES.</div>

247. Requirements.   The rails must be held to the ties by a
fastening which will not only give sufficient resistance, but which
will retain its capacity for resistance.   It must also be cheap
and easily applied.   The ordinary track-spike fulfills the last
requirements, but has comparatively small resisting power, com-
pared with screws or bolts.   Worse than all, the tendency to
vertical vibration in the rail produces a series of upward pulls on
the spike that soon loosens it.   When motion has once begun
the capacity for resistance is greatly reduced, and but little more

vibration is required to pull the spike out
so much that redriving is necessary.
Driving the spike to place again in the
same hole is of small value except as a
very temporary expedient, as its holding
power is then very small.   Redriving the
spikes in new holes very soon "spike-kills"
the tie.   Many plans have been devised to
increase the holding power of spikes, such
as making them jagged, twisting the spike,
swelling the spike at about the center of its
length, etc.   But it has been easily demon-
strated that the fibers of the wood are gen-

FIG. 122.

erally so crushed and torn by driving such spikes that their
holding power is less than that of the plain spike.

The ordinary spike (see Fig. 122) is made with a square cross-section which is uniform through the middle of its length, the lower $1\frac{3}{4}''$ tapering down to a chisel edge, the upper part swelling out to the head. The Goldie spike (see Fig. 123) aims to improve this form by reducing to a minimum the destruction of the fibers. To this end, the sides are made smooth, the edges are clean-cut, and the point, instead of being chisel-shaped, is ground down to a pyramidal form. Such fiber-cutting as occurs is thus accomplished without much crushing, and the fibers are thus pressed away from the spike and slightly downward. Any tendency to draw the spike will therefore cause

FIG. 123.   the fibers to press still harder on the spike and thus increase the resistance.

248. Driving. The holding power of a spike depends largely on how it is driven. If the blows are eccentric and irregular in direction, the hole will be somewhat enlarged and the holding power largely decreased. The spikes on each side of the rail in any one tie should not be directly opposite, but should be staggered. Placing them directly opposite will tend to split the tie, or at least decrease the holding power of the spikes. The direction of staggering should be reversed in

the two pairs of spikes in any one tie   FIG. 124.   SPIKE-DRIVING.

(see Fig. 124). This will tend to prevent any twisting of the tie in the ballast, which would otherwise loosen the rail from the tie.
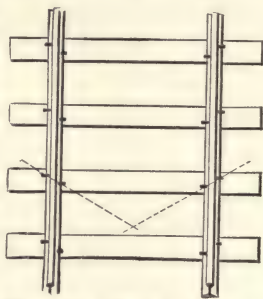
249. Screws and bolts. The use of these abroad is very extensive, but their use in this country has not passed the experimental stage. The screws are "wood"-screws (see Fig. 125), having large square heads, which are screwed down with a track-wrench. Holes, having the same diameter as the *base* of the screw-threads, should first be bored into the tie, at exactly the right position and at the proper angle with the vertical.

A light wooden frame is sometimes used to guide the auger at the proper angle.   Sometimes the large head of the screw bears directly against the base of the rail, as with the ordinary spike.   Other designs employ a plate, made to fit the rail on one side, bearing on the tie on the other side, and through which the screw passes.   These screws cost much more than spikes and require more work to put in place, but their holding power is much greater and the work of track maintenance is very · much less.   Screw-bolts, passing entirely through the tie, having the head at the bottom of the tie and the nut on

Fɪɢ. 125.

the upper side, are also used abroad.   These are quite difficult to replace, requiring that the ballast be dug out beneath the tie, but on the other hand the occasions for replacing such a bolt are comparatively rare, as their durability is very great.   The
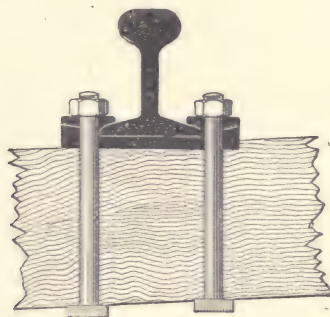
Fɪɢ. 126.

use of screws or bolts increases the life of the tie by the avoidance of " spike-killing."   It is capable of demonstration that the reduced cost of maintenance and the resulting improvement in track would much more than repay the added cost of screws and bolts, but it seems impossible to induce railroad directors to authorize a large and immediate additional expenditure to make an annual saving whose value, although unquestionably considerable, cannot be exactly computed.

250. "Wooden spikes." Among the regulations for track-laying given in § 208, mention was made of wooden "spikes," or plugs, which are used to fill up the holes when spikes are withdrawn. The value of the policy of filling up these holes is unquestionable, since the expense is insignificant compared with the loss due to the quick and certain decay of the tie if these holes are allowed to fill with water and remain so. But the method of making these plugs is variable. On some roads they are "hand-made" by the trackmen out of otherwise useless scraps of lumber, the work being done at odd moments. This policy, while apparently cheap, is not necessarily so, for the hand-made plugs are ir-regular in size and therefore more or less inefficient. It is also quite probable that if a track gang is required to make their own plugs, they may spend time on these very cheap articles which could be more profitably employed otherwise. Since the holes made by the spikes are larger at the top than they are near the bottom, the plugs should *not* be of uniform cross-section but should be slightly wedge-shaped. The "Goldie tie-plug" (see Fig. 127) has been de-signed to fill these requirements. Being machine-

FIG. 127.

made, they are uniform in size; they are of a shape which will best fit the hole; they can be furnished of any desired wood, and at a cost which makes it a wasteful economy to at-tempt to cut them by hand.

TRACK-BOLTS AND NUT-LOCKS.

251. Essential requirements. The track-bolts must have sufficient strength and must be screwed up tight enough to hold the angle-plates against the rail with sufficient force to develop the full transverse strength of the angle-bars. On the other hand the bolts should not be screwed so tight that slipping may not take place when the rail expands or contracts with temperature. It would be impossible to screw the bolts tight enough to prevent

slipping during the contraction due to a considerable fall of
temperature on a straight track, but when the track is curved,
or when expansion takes place, it is conceivable that the resist-
ance of the ties in the ballast to lateral motion may be less than
the resistance at the joint.   A test to determine this resistance
was made by Mr. A. Torrey, chief engineer of the Mich. Cent.
R.R., using 80-lb. rails and ordinary angle-bars, the bolts being
screwed up as usual.   It required a force of about 31000 to
35000 lbs. to start the joint, which would be equivalent to the
stress induced by a change of temperature of about 22°.   But
if the central angle of any given curve is small, a comparatively
small lateral component will be sufficient to resist a compression
of even 35000 lbs. in the rails.   Therefore there will ordinarily
be no trouble about having the joints screwed too tight.   The
vibration caused by the passage of a train reduces the resistance
to slipping.   This vibration also facilitates an objectionable
feature, viz., loosening of the nuts of the track-bolts.   The bolt
is readily prevented from turning by giving it a form which is
not circular immediately under the head and making corre-
sponding holes in the angle-plate.   Square holes would answer
the purpose, except that the square corners in the holes in the
angle-plates would increase the danger of fracture of the plates.
Therefore the holes (and also the bolts, under the head) are
made of an oval form, or perhaps a square form with rounded
corners, avoiding angles in the outline.

    The nut-locks should be simple and cheap, should have a life
at least as long as the bolt, should be effective, and should not
lose their effectiveness with age.   Many of the designs that
have been tried have been failures in one or more of these
particulars, as will be described in detail below.

    252. Design of track-bolts.   In Fig. 128 is shown a common
design of track-bolt.   In its general form this represents
the bolt used on nearly all roads, being used not only
with the common angle-plates, but also with many of the im-
proved designs of rail-joints.   The variations are chiefly a
general increase in size to correspond with the increased

weight of rails, besides variations in detail dimensions which are frequently unimportant.   The diameter is usually $\frac{3}{4}''$ to $\frac{7}{8}''$; $1''$ bolts are sometimes used for the heaviest sections of rails.   As to length, the bolts should not extend more than $\frac{1}{2}''$ outside of the nut when it is screwed up.   If it extends farther than this, it is liable to be broken off by a possible derailment at that point.   The lengths used vary from $3\frac{1}{4}''$, which may be used with 60 lbs. rails, to $5''$, which is required with 100-lb. rails.   The length required depends somewhat on the type of nut-lock used.
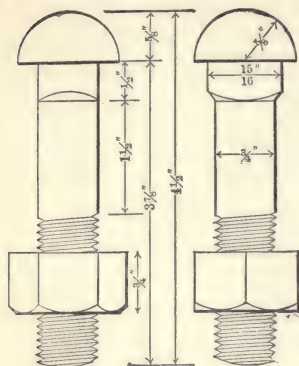
FIG. 128.—TRACK-BOLT.

253. Design of nut-locks. The designs for nut-locks may be divided into three classes: (*a*) those depending entirely on an elastic washer which absorbs the vibration which might otherwise induce turning; (*b*) those which jam the threads of the bolt and nut so that, when screwed up, the frictional resistance is too great to be overcome by vibration; (*c*) the " positive " nut-locks—those which mechanically hold the nut from turning. Some of the designs combine these principles to some extent. The " vulcanized fiber " nut-lock is an example of the first class.   It consists essentially of a rubber washer which is protected by an iron ring.   When first placed this lock is effective, but the rubber soon hardens and loses its elasticity and it is then ineffective and worthless.   Another illustration of class (*a*) is the use of wooden blocks, generally of $1''$ to $2''$ oak, which extend the entire length of the angle-bar, a single piece forming the washer for the four or six bolts of a joint.   This form is cheap, but the wood soon shrinks, loses its elasticity, or decays so that it soon becomes worthless, and it requires constant adjustment to keep it in even tolerable condition.   The " Verona " nut-lock is another illustration of class (*a*) which also combines some of the positive elements of class (*c*).   It is made of

tempered steel and, as shown in Fig. 129, is warped and has sharp edges or points. The warped form furnishes the element of elastic pressure when the nut is screwed up. The steel being harder than the iron of the angle-bar or of the nut, it bites into them, owing to the great pressure that must exist

VERONA

VULCANIZED FIBRE

IMPROVED VERONA

NATIONAL

EXCELSIOR.

JONES
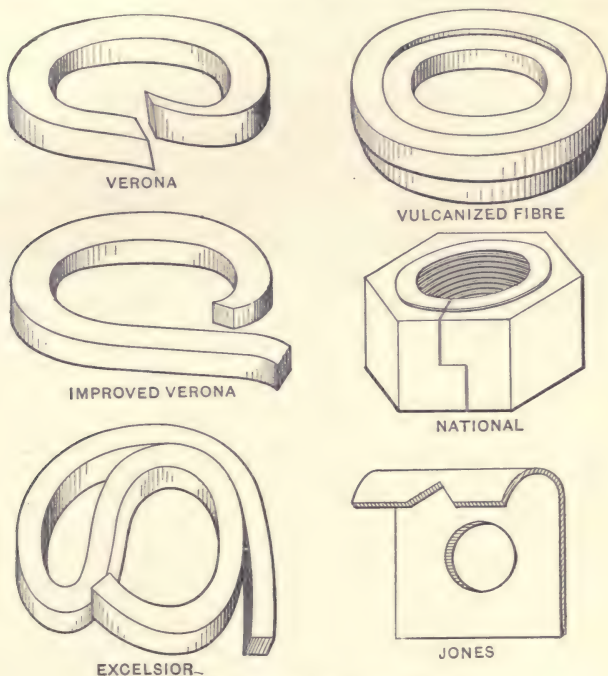
FIG. 129.—TYPES OF NUT-LOCKS.

when the washer is squeezed nearly flat, and thus prevents any *backward* movement, although forward movement (or tightening the bolt) is not interfered with. The " National " nut-lock is a type of the second class (*b*), in which, like the " Harvey " nut-lock, the nut and lock are combined in one piece. With six-bolt angle-bars and 30-foot rails, this means a saving of 2112 pieces on each mile of single track. The " National " nuts are open on one side. The hole is drilled and the thread is cut slightly smaller than the bolt, so that when the nut is screwed

up it is forced slightly open and therefore presses on the threads
of the bolt with such force that vibration cannot jar it loose.
Unlike the " National " nut, the " Harvey " nut is solid, but
the form of the thread is progressively varied so that the thread
pinches the thread of the bolt and the frictional resistance to
turning is too great to be affected by vibration.

The " Jones " nut-lock, belonging to class (*c*), is a type of
a nut-lock that does not depend on elasticity or jamming of
screw-threads. It is made of a thin flexible plate, the square
part of which is so large that it will not turn after being placed
on the bolt. After the nut is screwed up, the thin plate is bent
over so that the re-entrant angle of the plate engages the corner
of the nut and thus mechanically prevents any turning. The
metal is supposed to be sufficiently tough to endure without
fracture as many bendings of the plate as will ever be desired.
Nut-locks of class (*c*) are not in common use.

SWITCHES AND CROSSINGS.

SWITCH CONSTRUCTION.

254. Essential elements of a switch. Flanges of some sort are a necessity to prevent car-wheels from running off from the rails on which they may be moving. But the flanges, although a necessity, are also a source of complication in that they require some special mechanism which will, when desired, guide the wheels out from the controlling influence of the main-line rails. This must either be done by raising the wheels high enough so that the flanges may pass *over* the rails, or by breaking the continuity of the rails in such a way that channels or " flange spaces " are formed *through* the rails. An ordinary stub switch breaks the continuity of the main-line rails in three places, two of them at the switch-block and one at the frog. The Wharton switch avoids two of these breaks by so placing inclined planes that the wheels, rolling on their flanges, will surmount these inclines until they are a little higher than the rails. Then the wheels on the side toward which the switch runs are guided over and across the main rail on that side. This rise being accomplished in a short distance, it becomes impracticable to operate these switches except at slow speeds, as any sudden change in the path of the center of gravity of a car causes very destructive jars both to the switch and to the rolling stock. The other general method makes a break in one main rail (or both) at the switch-block. In both methods the wheels are led to one side by means of the "lead rails," and finally one line of wheels passes *through* the main rail on that side by means of a " frog." There are some designs by which even this break in the main rail is avoided, the wheels being led *over* the main rail by means

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of a short *movable* rail which is on occasion placed across the
main rail, but such designs have not come into general use.

255. Frogs. Frogs are provided with two channel-ways or
" flange spaces " through which the flanges of the wheels move.
Each channel cuts out a parallelogram from the tread area.
Since the wheel-tread is always wider than the rail, the wing
rails will support the wheel not only across the space cut out by
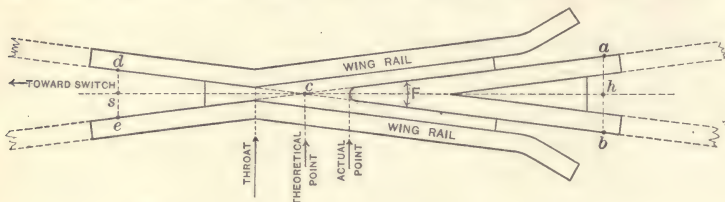
FIG. 130.—DIAGRAMMATIC DESIGN OF FROG.

the channel, but also until the tread has passed the point of the
frog and can obtain a broad area of contact on the tongue of the
frog.   This is the theoretical idea, but it is very imperfectly
realized.   The wing rails are sometimes subjected to excessive
wear owing to " hollow treads " on the wheels—owing also to
the frog being so flexible that the point " ducks " when the
wheel approaches it.   On the other hand the sharp point of the
frog will sometimes cause destructive wear on the tread of the
wheel.   Therefore the tongue of the frog is not carried out to
the sharp theoretical point, but is purposely somewhat blunted.
But the break which these channels make in the continuity of
the tread area becomes extremely objectionable at high speeds,
being mutually destructive to the rolling stock and to the frog.
The jarring has been materially reduced by the device of
" spring frogs "—to be described later.   Frogs were originally
made of cast iron—then of cast iron with wearing parts of cast
steel, which were fitted into suitable notches in the cast iron.
This form proved extremely heavy and devoid of that elasticity
of track which is necessary for the safety of rolling stock
and track at high speeds.   The present universal practice is to
build the frog up of pieces of rails which are cut or bent as re-
quired.   These pieces of rails (at least four) are sometimes

assembled by riveting them to a flat plate, but this method is now but little used, except for very light work. The usual practice is now chiefly divided between " bolted " and " keyed " frogs. In each case the space between the rails, except a sufficient flange-way, is filled with a cast-iron filler and the whole assemblage of parts is suitably bolted or clamped together, as is illustrated in Plate XVIII. The operation of a spring-rail frog is evident from the figure. Since a siding is usually operated at slow speed, while the main track may be operated at fast speed, a spring-rail frog will be so set that the tread is continuous for the main track and broken for the siding. This also means that the spring rail will only be moved by trains moving at a (presumably) slow speed on to the siding. For the fast trains on the main line such a frog is substantially a " fixed " frog and has a tread which is practically continuous.

256. To find the frog number. The frog number (n) equals the ratio of the distance of any point on the tongue of the frog from the theoretical point of the frog divided by the width of the tongue at that point, i.e. $= hc \div ab$ (Fig. 130). This value may be directly measured by applying any convenient unit of measure (even a knife, a short pencil, etc.) to some point of the tongue where the width just equals the unit of measure, and then noting how many times the unit of measure is contained in the distance from that place to the theoretical point. But since c, the theoretical point, is not so readily determinable with exactitude, it being the imaginary intersection of the gauge lines, it may be more accurate to measure de, ab, and hs; then n, the frog number, $= hs \div (ab + de)$. If the frog angle be called F, then

$$n = hc \div ab = hs \div (ab + de) = \tfrac{1}{2} \cot \tfrac{1}{2} F;$$

i,e.. $cot \tfrac{1}{2} F = 2n.$

257. Stub switches. The use of these, although once nearly universal, has been practically abandoned as turnouts from *main track* except for the poorest and cheapest roads. In some States, their use on main track is prohibited by law. They

have the sole merit of cheapness with adaptability to the circumstances of very light traffic operated at slow speed when a considerable element of danger may be tolerated for the sake of economy. The rails from *A* to *B* (see Fig. 131*) are not fastened
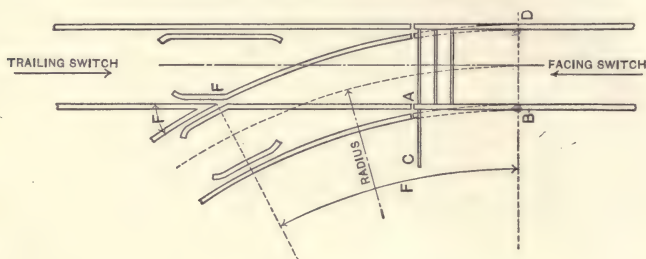
FIG. 131.—STUB SWITCH.

to the ties; they are fastened to each other by tie-rods which keep them at the proper gauge; at and back of *B* they are securely spiked to the ties, and at *A* they are kept in place by the connecting bar (*C*) fastened to the switch-stand. One great objection to the switch is that, in its usual form, when operated as a trailing switch, a derailment is inevitable if the switch is misplaced. The very least damage resulting from such a derailment must include the bending or breaking of the tie-rods of the switch-rail. Several devices have been invented to obviate this objection, some of which succeed very well mechanically, although their added cost precludes any economy in the total cost of the switch. Another objection to the switch is the looseness of construction which makes the switches objectionable at high speeds. The gap of the rails at the head-block is always considerable, and is sometimes as much as two inches. A

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* The student should at once appreciate that in Fig. 131, as well as in nearly all the remaining figures in this chapter, it becomes necessary to use excessively large frog angles, short radii, and a very wide gauge in order to illustrate the desired principles with figures which are sufficiently small for the page. In fact, the proportions used in the figures are such that serious mechanical difficulties would be encountered if they were used. These difficulties are here ignored because they can be neglected in the proportions used in practice.

driving-wheel with a load of 12000 to 20000 pounds, jumping this gap with any considerable velocity, will do immense damage to the farther rail end, besides producing such a stress in the construction that a breakage is rendered quite likely, and such a breakage might have very serious consequences.

258. Point switches. The essential principle of a point switch is illustrated in Fig. 132. As is shown, one main rail and also one of the switch-rails is unbroken and immovable.
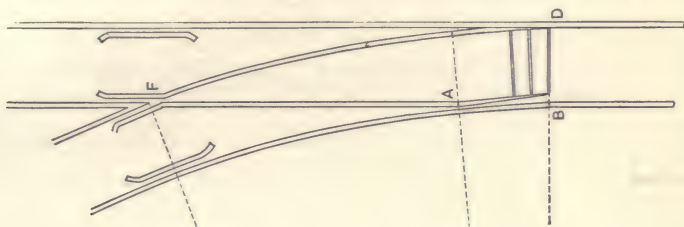
FIG. 132.—POINT SWITCH.

The other main rail (from A to F) and the corresponding portion of the other lead rail are substantially the same as in a stub switch. A portion of the main rail (AB) and an equal length of the opposite lead rail (usually 15 to 24 feet long) are fastened together by tie-rods. The end at A is jointed as usual and the other end is pointed, both sides being trimmed down so that the feather edge at B includes the web of the rail. In order to retain in it as much strength as possible, the point-rail is raised so that it rests on the base of the stock-rail, one side of the base of the point-rail being entirely cut away. As may be seen in Fig. 133, although the influence of the point of the rail in moving the wheel-flange away from the stock-rail is really zero at that point, yet the rail has all the strength of the web and about one-half that of the base—a very fair angle-iron.
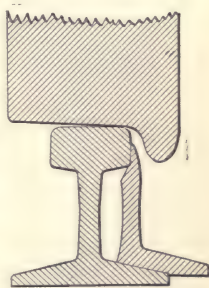
FIG. 133.

The planing runs back in *straight* lines, until at about six or seven feet back from the point the full width of the head is

obtained.   The full width of the base will only be obtained at about 13 feet from the point.   An 80-lb. rail is 5 inches
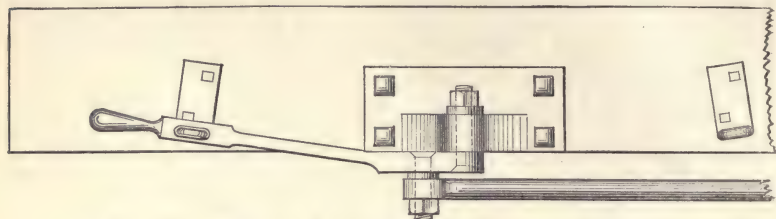
FIG. 134.—GROUND LEVER FOR THROWING A SWITCH.

wide at the base.   Allowing $\frac{3}{4}''$ more for a spike between the rails, this gives $5\frac{3}{4}''$ as the minimum width between rail centers at the joint.   The minimum angle of the switch-point (using a 15-foot point rail) is therefore the angle whose tangent is $\frac{5.75}{15 \times 12} = .03914$, which is the tangent of $1° 50'$.   Switch-rails are sometimes used with a length of 24 feet, which reduces the angle of the switch-point to $1° 09'$.
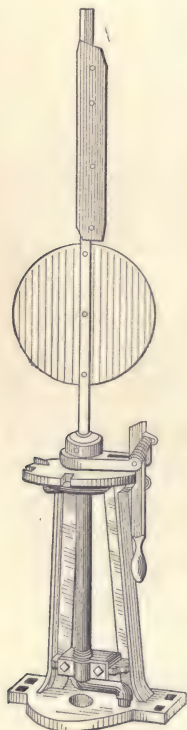
FIG. 135.

259. Switch-stands.   The simplest and cheapest form is the "ground lever," which has no target.   The radius of the circle described by the connecting-rod pin is precisely one-half the throw.   From the nature of the motion the device is practically self-locking in either position, padlocks being only used to prevent malicious tampering.   The numerous designs of upright stands are always combined with targets, one design of which is illustrated in Fig. 135.   When the road is equipped with interlocking signals, the switch-throw mechanism forms a part of the design.

260. Tie-rods.   These are fastened to the webs of the rails by means of lugs which are bolted on, there

being usually a hinge-joint between the rod and the lug.   Four
such tie-rods are generally necessary.   The first rod is some-
times made without hinges, which gives additional stiffness to
the comparatively weak rail-points.   The old fashioned tie-rod,
having jaws fitting the base of the rail, was almost universally
used in the days of stub switches.   One great inconvenience
in their use lies in the fact that they must be slipped on, one by
one, over the *free* ends of the switch-rails.   Sometimes the
lugs are fastened to the rail-webs by rivets instead of bolts.
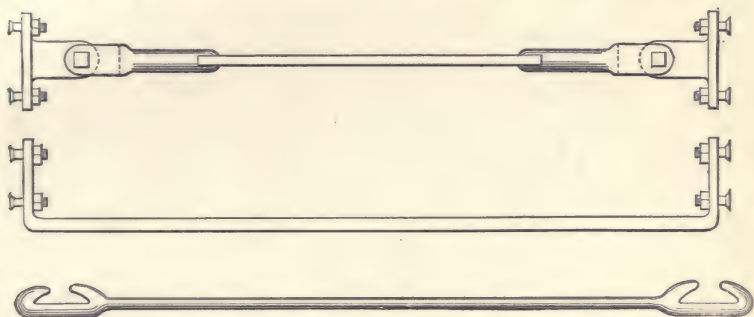
FIG. 136.—FORMS OF TIE-RODS.

261. Guard-rails.   As shown in Figs. 131 and 132, guard-
rails are used on both the main and switch tracks opposite the
frog-point.   Their function is not only to prevent the possibil-
ity of the wheel-flanges passing on the wrong side of the frog-
point, but also to save the side of the frog-tongue from exces-
sive wear.   The necessity for their use may be realized by
noting the very apparent wear usually found on the side of the
head of the guard-rail.   The flange-way space between the
heads of the guard-rail and wheel-rail therefore becomes a
definite quantity and should equal about two inches.   Since this
is less than the space between the heads of ordinary (say
80-pound) rails when placed base to base, to say nothing of the
$\frac{3}{4}''$ necessary for spikes, it becomes necessary to cut the flange
of the guard-rail.   The length of the rail is made from 10 to
15 feet, the ends being bent as shown in Fig. 132, so as to

prevent the possibility of the end of the rail being struck by a wheel-flange.

In all of the following demonstrations regarding switches, turnouts, and crossovers, the lines are assumed to represent the *gauge-lines*—i.e., the lines of the *inside* of the head of the rails.

262. Design with circular lead-rails. The simplest method
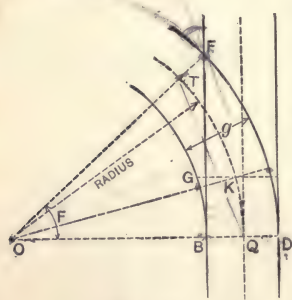
Fig. 137.

is to consider that the lead-rails curve out from the main track-rails by arcs of circles which are tangent to the main rails and which extend to the frog-point F. The simple curve from D to F is of such radius that $(r + \frac{1}{2}g)$ vers $F = g$, in which $F =$ the frog angle; $g =$ gauge, $L =$ the "lead" (BF), and $r =$ the radius of the *center* of the switch-rails.

$$\therefore \quad r + \tfrac{1}{2}g = \frac{g}{\text{vers } F} \quad \cdots \quad (74)$$

Also　　$BF \div BD = \cot \tfrac{1}{2}F; \quad BD = g; \quad BF = L.$

$$\therefore \quad L = g \cot \tfrac{1}{2}F \quad \cdots \quad \cdots \quad (75)$$

Also　　　　　　$L = (r + \tfrac{1}{2}g) \sin F; \quad \cdots \quad (76)$

$$QT = 2r \sin \tfrac{1}{2}F. \quad \cdots \quad \cdots \quad (77)$$

These formulæ involve the angle F. As shown in Table III, the angles (F) are always odd quantities, and their trigonometric functions are somewhat troublesome to obtain closely with ordinary tables. The formulæ may be simplified by substituting the frog-number n, from the relation that $n = \frac{1}{2} \cot \frac{1}{2}F$. Since

$$r - \tfrac{1}{2}g = L \cot F \text{ and } r + \tfrac{1}{2}g = L \text{ cosec } F,$$

then $\qquad\qquad r = \tfrac{1}{2}L\,(\cot F + \operatorname{cosec} F)$

$\qquad\qquad\qquad = \tfrac{1}{2}g \cot \tfrac{1}{2}F\,(\cot F + \operatorname{cosec} F)$

$\qquad\qquad\qquad = \tfrac{1}{2}g \cot^2 \tfrac{1}{2}F,$ since $(\cot \alpha + \operatorname{cosec} \alpha) = \cot \tfrac{1}{2}\alpha$

$\qquad\qquad\qquad = 2gn^2.\quad.\quad.\quad.\quad.\quad.\quad.\quad.\quad.\quad.\quad.\quad(78)$

Also $\qquad\qquad L = 2gn,\quad.\quad.\quad.\quad.\quad.\quad.\quad.\quad.\quad.\quad.\quad(79)$

from which $\quad r = n \times L.\quad.\quad.\quad.\quad.\quad.\quad.\quad.\quad.\quad.\quad(80)$

These extremely simple relations may obviate altogether the necessity for tables, since they involve only the frog-number and the gauge. On account of the great simplicity of these rules, they are frequently used as they are, regardless of the fact that the curve is never a uniform simple curve from switch-block to frog. In the first place there is a considerable length of the gauge-line within the frog, which is straight unless it is purposely curved to the proper curve while being manufactured, which is seldom if ever done—except for the very large-angled frogs used for street-railway work, etc. It is also doubtful whether the switch-rails (BA, Fig. 131) are bent to the computed curve when the rails are set for the switch. The switch-rails of point switches are *straight*, thus introducing a stretch of straight track which is about one-fifth of the total length of the lead-rails. The effect of these modifications on the length and radius of the lead-rails will be developed and discussed in the next four sections.

The throw (t) of a stub switch depends on the weight of the rail, or rather on the width of its base. The throw must be at least $\tfrac{3}{4}''$ more than that width. The head-block should therefore be placed at such a distance from the heel of the switch (B) that the versed sine of the arc equals the throw. These points *must* be opposite on the two rails, but the points on the two rails where these relations are exactly true will not be opposite. Therefore, instead of considering either of the two radii ($r + \tfrac{1}{2}g$) and ($r - \tfrac{1}{2}g$), the mean radius r is used. Then (see Fig. 137)

$$\text{vers } KOQ = t \div r,$$

and the length of the switch-rails is

$$QK = r \sin KOQ. \quad . \quad . \quad . \quad . \quad (81)$$

These relations develop another disadvantage in the use of a stub switch. The required value of BG, using a No. 10 frog and 80-pound rail, is 30.1 feet—slightly more than a full rail length. It would be unsafe to leave so much of the track unspiked from the ties. Whether this is obviated by spiking down a portion of the switch-rails (virtually shortening the lead) or by moving the switch-block nearer the heel of the switch (shortening the switch-rails), but still maintaining the required throw, the theoretical accuracy of the curve is hopelessly lost.

263. Effect of straight frog-rails. A portion of the ends of the rails of a frog are free and *may* be bent to conform to the switch-rail curve, but there is a considerable portion which is fitted to the cast-iron filler, and this portion is always straight. Call the length of this straight portion back from the frog-point f ($= FH$, Fig. 138). Then we have

FH = f
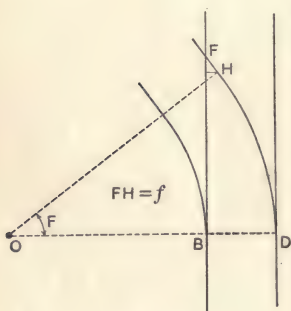
FIG. 138.

$$r + \tfrac{1}{2}g = (g - f \sin F) \div \text{vers } F$$

$$= \frac{g}{\text{vers } F} - f \cot \tfrac{1}{2}F$$

$$= \frac{g}{\text{vers } F} - 2fn. \quad . \quad . \quad (82)$$

$$BF = L = (g - f \sin F) \cot \tfrac{1}{2}F + f \cos F$$

$$= 2gn - f \sin F \cot \tfrac{1}{2}F + f \cos F$$

$$= 2gn - f(1 + \cos F) + f \cos F$$

$$= 2gn - f. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (83)$$

Since $r - \tfrac{1}{2}g = (L - f \sec F) \cot F$, and

$$r + \tfrac{1}{2}g = (L - f \cos F) \text{ cosec } F,$$

$$r = \tfrac{1}{2}L \left(\cot F + \operatorname{cosec} F\right) - \tfrac{1}{2}f \sec F \cot F - \tfrac{1}{2}f \cos F \operatorname{cosec} F$$

$$= Ln - \tfrac{1}{2}f \left(\frac{1 + \cos F}{\sin F}\right).$$

$$r = Ln - \tfrac{1}{2}f \cot \tfrac{1}{2}F$$
$$= Ln - fn. \quad \text{Then from (83)}$$
$$r = 2gn^2 - 2fn. \quad . \quad . \quad . \quad . \quad . \quad (84)$$

264. Effect of straight point-rails. The " point switches," now so generally used, have *straight* switch-rails. This requires an *angle* in the alignment rather than turning off by a tangential curve. The angle is, however, very small (between 1° and 2°), and the disadvantages of this angle are small compared with the very great advantages of the device.
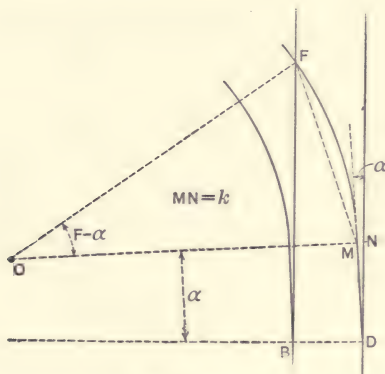
FIG. 139.

$$FM = \frac{g - k}{\sin \tfrac{1}{2}(F + \alpha)};$$

$$r + \tfrac{1}{2}g = \frac{FM}{2 \sin \tfrac{1}{2}(F - \alpha)}$$

$$= \frac{g - k}{2 \sin \tfrac{1}{2}(F + \alpha) \sin \tfrac{1}{2}(F - \alpha)}$$

$$= \frac{g - k}{\cos \alpha - \cos F}. \quad . \quad . \quad . \quad . \quad (85)$$

$$BF = L = FM \cos \tfrac{1}{2}(F + \alpha) + DN$$
$$= (g - k) \cot \tfrac{1}{2}(F + \alpha) + DN. \quad . \quad (86)$$

265. Combined effect of straight frog-rails and straight point-rails. It becomes necessary in this case to find a curve which shall be tangent to both the point-rail and the frog-rail. The curve therefore begins at M, its tangent making an angle of α (usually $1° 50'$) with the main rail, and runs to H. The central
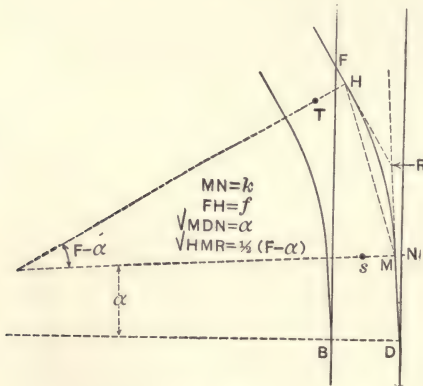
Fig. 140.

angle of the curve is therefore $(F - \alpha)$. The angle of the chord HM with the main rails is therefore

$$\tfrac{1}{2}(F - \alpha) + \alpha = \tfrac{1}{2}(F + \alpha);$$

$$HM = \frac{g - f \sin F - k}{\sin \tfrac{1}{2}(F + \alpha)};$$

$$r + \tfrac{1}{2}g = \frac{HM}{2 \sin \tfrac{1}{2}(F - \alpha)}$$

$$= \frac{g - f \sin F - k}{2 \sin \tfrac{1}{2}(F + \alpha) \sin \tfrac{1}{2}(F - \alpha)}$$

$$= \frac{g - f \sin F - k}{\cos \alpha - \cos F}; \quad . \quad . \quad . \quad . \quad (87)$$

$$ST = 2r \sin \tfrac{1}{2}(F - \alpha). \quad . \quad . \quad . \quad . \quad (88)$$

$$BF = L = HM \cos \tfrac{1}{2}(F + \alpha) + f \cos F + DN$$
$$= (g - f \sin F - k) \cot \tfrac{1}{2}(F + \alpha) + f \cos F + DN. \quad (89)$$

It may be more simple, if $(r + \tfrac{1}{2}g)$ has already been computed, to write

$$L = 2(r + \tfrac{1}{2}g) \sin \tfrac{1}{2}(F - \alpha) \cos \tfrac{1}{2}(F + \alpha) + f \cos F + DN$$
$$= (r + \tfrac{1}{2}g)(\sin F - \sin \alpha) + f \cos F + DN. \quad . \quad . \quad (90)$$

266. Comparison of the above methods. Computing values for r and L by the various methods, on the uniform basis of a No. 9 frog, standard gauge $4'\ 8\tfrac{1}{2}''$, $f = 3'.37$, $k = 5\tfrac{3}{4}'' = 0'.479$, $DN = 15'\ 0''$, and $\alpha = 1° 50'$, we may tabulate the comparative results:

	§ 262. Simple circle Curved frog r. Curved switch-r.	§ 263. Straight frog-r. Curved switch-r.	§ 264. Curved frog-r. Straight switch-r.	§ 265. Straight frog-r. Straight switch-r.
r	762.75	702.00	747.48	681.16
Deg. of curve	$7° 31'$	$8° 10'$	$7° 40'$	$8° 25'$
L	84.75	81.37	74.00	72.13

This shows that the effect of using straight frog-rails and straight switch-rails is to sharpen the curve and shorten the lead in each case separately, and that the combined effect is still greater. The effect of the straight switch-rails is especially marked in reducing the length of lead, and therefore Eq. 78 to 80, although having the advantage of extreme simplicity, cannot be used for point-switches without material error. The effect of the straight frog-rail is less, and since it can be materially reduced by bending the free end of the frog-rails, the influence of this feature is frequently ignored, the frog-rails are assumed to be curved and Eq. 85 and 86 are used. (See § 276 for a further discussion of this point.)

267. Dimensions for a turnout from the OUTER side of a curved track. In this demonstration the switch-rails will be considered as uniformly circular from the switch-points to the frog-point.

FIG. 141.

In the triangle FCD (Fig. 141) we have

$$(FC+CD):(FC-CD)::\tan \tfrac{1}{2}(FDC+DFC):\tan \tfrac{1}{2}(FDC-DFC);$$

but $\qquad \tfrac{1}{2}(FDC + DFC) = 90° - \tfrac{1}{2}\theta$

and $\qquad \tfrac{1}{2}(FDC - DFC) = \tfrac{1}{2}F.$

Also $\qquad FC + CD = 2R$ and $FC - CD = g;$

$$\therefore \; 2R : g :: \cot \tfrac{1}{2}\theta : \tan \tfrac{1}{2}F$$
$$:: \cot \tfrac{1}{2}F : \tan \tfrac{1}{2}\theta;$$

$$\therefore \; \tan \tfrac{1}{2}\theta = \frac{gn}{R}. \quad . \quad . \quad . \quad . \quad . \quad (91)$$

Also $\quad OF : FC :: \sin \theta : \sin \phi; \qquad$ but $\quad \phi = (F - \theta);$

then $\qquad r + \tfrac{1}{2}g = (R + \tfrac{1}{2}g)\dfrac{\sin \theta}{\sin (F - \theta)}. \quad . \quad . \quad (92)$

$$BF = L = 2(R + \tfrac{1}{2}g) \sin \tfrac{1}{2}\theta. \quad . \quad . \quad (93)$$

If the curvature of the main track is very sharp or the frog angle unusually small, F may be less than θ; in which case the center O will be on the same side of the main track as C. Eq. 92 will become (by calling $r = -r$ and changing the signs)

$$(r - \tfrac{1}{2}g) = (R + \tfrac{1}{2}g)\frac{\sin \theta}{\sin (\theta - F)}. \quad . \quad . \quad (94)$$

If we call d the degree of curve corresponding to the radius r, and D the degree of curve corresponding to the radius R, also d' the degree of curve of a turnout from a straight track (the frog angle F being the same), it may be shown that $d = d' - D$ (very nearly).   To illustrate we will take three cases, a number 6 frog (very blunt), a number 9 frog (very commonly used), and a number 12 frog (unusually sharp).   Suppose $\dot{D} = 4° \ 0'$; also $D = 10° \ 0'$; $g = 4' \ 8\frac{1}{2}'' = 4'.708$.

Frog number.	$D = 4°$.					"L" for straight track.
	d		$d' - D$		Error.	L	
6	$12°$ $54'$ $20''$		$12°$ $57'$ $52''$		$0°$ $03'$ $32''$	56.57	56.50
9	3 30 27		3 31 04		0 0 37	84.85	84.75
12	0 13 33		0 13 36		0 0 03	112.72	113.00

Frog number.	$D = 10°$					"L" for straight track.
	d		$d' - D$		Error.	L	
6	$6°$ $53'$ $24''$		$6°$ $57'$ $52''$		$0°$ $04'$ $28''$	56.66	56.50
9	2 27 54		2 28 56		0 01 02	84.86	84.75
12	5 44 26		5 46 24		0 01 58	112.91	113.00

A brief study of the above tabular form will show that the error involved in the use of the approximate rule for ordinary curves ($4°$ or less) and for the usual frogs (about No. 9) is really insignificant, and that, even for sharper curves ($10°$ or more), or for very blunt frogs, the error would never cause damage, considering the lower probable speed.   In the most unfavorable case noted above the change in radius is about 1%.   On account of the closeness of the approximation the method is frequently used.   The remarkable agreement of the computed values of L with the corresponding values for a straight main track (the lead

rails circular throughout) shows that the error is insignificant in using the more easily computed values.

268. **Dimensions for a turnout from the INNER side of a curved**

Fig. 142.

track. (Lead rails circular throughout.) From Fig. 142 we have

$$DC + FC : DC - FC :: \tan \tfrac{1}{2}(DFC + FDC) : \tan \tfrac{1}{2}(DFC - FDC);$$

but $$\tfrac{1}{2}(DFC + FDC) = 90° - \tfrac{1}{2}\theta$$

and $$\tfrac{1}{2}(DFC - FDC) = \tfrac{1}{2}F;$$
$$\therefore\ 2R : g :: \cot \tfrac{1}{2}\theta : \tan \tfrac{1}{2}F$$
$$:: \cot \tfrac{1}{2}F : \tan \tfrac{1}{2}\theta;$$

$$\therefore\ \tan \tfrac{1}{2}\theta = \frac{gn}{R}. \quad . \quad . \quad . \quad . \quad . \quad . \quad (95)$$

$$OF : FC :: \sin \theta : \sin (F + \theta).$$

$$(r + \tfrac{1}{2}g) = (R - \tfrac{1}{2}g)\frac{\sin \theta}{\sin (F + \theta)}. \quad . \quad . \quad (96)$$

$$L = BF = 2(R - \tfrac{1}{2}g) \sin \tfrac{1}{2}\theta. \quad . \quad . \quad (97)$$

As in § 267, it may be readily shown that the degree of the turnout (d) is *nearly* the *sum* of the degree of the main track (D) and the degree (d') of a turnout from a straight track when the frog angle is the same. The discrepancy in this case is

somewhat greater than in the other, especially when the curvature of the main track is sharp. If the frog angle is also large, the curvature of the turnout is excessively sharp. If the frog angle is very small, the liability to derailment is great. Turnouts to the inside of a curved track should therefore be avoided, unless the curvature of the main track is small.

269. **Double turnout from a straight track**. In Fig. 143 the frogs F_l and F_r are generally made equal. Then, if there are
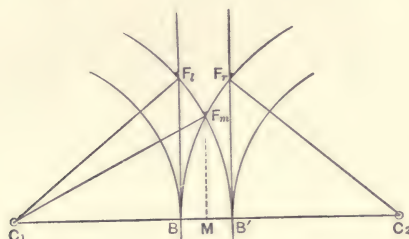
Fig. 143.

uniform curves from B' to F_l and from B to F_r, the required value of F_m is obtained from

$$\text{vers } \tfrac{1}{2}F_m = \frac{g}{2(r + \tfrac{1}{2}g)}, \qquad . \quad . \quad . \quad (98)$$

r being found from Eq. 78, in which n is the frog number of F_l or F_r.

$$MF_m = r \tan \tfrac{1}{2}F_m;$$

but since $n_m = \tfrac{1}{2} \cot \tfrac{1}{2}F_m$,

$$MF_m = \frac{r}{2n_m}. \quad . \quad . \quad . \quad . \quad . \quad (99)$$

Since $\text{vers } F_l = \dfrac{g}{(r + \tfrac{1}{2}g)}$,

$$\text{vers } \tfrac{1}{2}F_m = \tfrac{1}{2} \text{ vers } F_l, \quad . \quad . \quad . \quad . \quad (100)$$

Also, since $(C_iF_m)^2 = (MF_m)^2 + (C_iM)^2$, we have

$$(r + \tfrac{1}{2}g)^2 = \left(\frac{r}{2n_m}\right)^2 + r^2;$$

$$r^2 + rg + \tfrac{1}{4}g^2 = \frac{r^2}{4n_m{}^2} + r^2.$$

Simplifying and substituting $r = 2gn^2$, we have

$$2g^2n^2 + \tfrac{1}{4}g^2 = \frac{4g^2n^4}{4n_m{}^2};$$

$$n_m{}^2 = \frac{n^4}{2n^2 + \tfrac{1}{4}}.$$

Dropping the $\tfrac{1}{4}$, which is always insignificant in comparison with $2n^2$, we have

$$n_m = \frac{n}{\sqrt{2}} = n \times .707 \text{ (approx.).} \quad . \quad . \quad (101)$$

Frogs are usually made with angles corresponding to integral values of n, or sometimes in "half" sizes, e.g. 6, $6\tfrac{1}{2}$, 7, $7\tfrac{1}{2}$, etc. If No. $8\tfrac{1}{2}$ frogs are used for F_l and F_r, the exact frog number for F_m is 6.01. This is so nearly 6 that a No. 6 frog may be used without sensible inaccuracy. Numbers 7 and 10 are a less perfect combination. If sharp frogs must be used, $8\tfrac{1}{2}$ and 12 form a very good combination.

If it becomes necessary to use other frogs because the right combination is unobtainable, it may be done by compounding the curve at the middle frog. F_l and F_r should be greater than $\tfrac{1}{2}F_m$. If equal to $\tfrac{1}{2}F_m$, the rails would be straight from the middle frog to the outer frogs. In Fig. 144, $\theta_1 = F_l - \tfrac{1}{2}F_m$. Drawing the *chord* $\overline{F_lF_m}$,

$$\overline{KF_lF_m} = F_l - \tfrac{1}{2}\theta_1 = F_l - \tfrac{1}{2}F_l + \tfrac{1}{4}F_m = \tfrac{1}{2}(F_l + \tfrac{1}{2}F_m);$$

$$\overline{F_l F_m} = \frac{\overline{KF_m}}{\sin \overline{KF_l F_m}} = \frac{g}{2 \sin \frac{1}{2}(F_l + \frac{1}{2}F_m)}; \quad . \quad . \quad . \quad (102)$$

$$\overline{KF_l} = \overline{KF_m} \cot \overline{KF_l F_m} = \frac{1}{2}g \cot \frac{1}{2}(F_l + \frac{1}{2}F_m); \quad (103)$$

$$(r_1 + \tfrac{1}{2}g) = \frac{\overline{F_l F_m}}{2 \sin \frac{1}{2}\theta} = \frac{g}{4 \sin \frac{1}{2}(F_l + \frac{1}{2}F_m) \sin \frac{1}{2}(F_l - \frac{1}{2}F_m)}$$

$$= \frac{\frac{1}{2}g}{\cos \frac{1}{2}F_m - \cos F_l}. \quad . \quad . \quad . \quad (104)$$
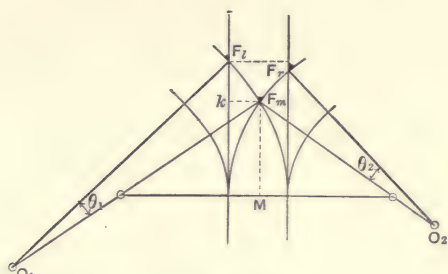
FIG. 144.

If three frogs, all different, *must* be used, the largest may be selected as F_m; the radius of the lead rails may be found by an inversion of Eq. 98; F_m may be located in the center of the tracks by Eq. 99; then each of the smaller frogs may be located by separate applications of Eq. 102 or 103, the radius being determined by Eq. 104.

270. Two turnouts on the same side. In Fig. 145, let O_1 bisect $O_2 D$. Then $(r_1 + \frac{1}{2}g) = \frac{1}{2}(r_2 + \frac{1}{2}g)$; also, $O_1 O_2 = O_1 F_l$ and $F_r = F_l$.

$$\text{vers } F_m = \frac{g}{r_1 + \frac{1}{2}g} = \frac{2g}{r_2 + \frac{1}{2}g}; \quad . \quad . \quad . \quad (105)$$

$$BF_m = (r_1 + \tfrac{1}{2}g) \sin F_m. \quad . \quad . \quad . \quad (106)$$

It may readily be shown that the relative values of F_r, F_l, and F_m are almost identical with those given in § 269; as may

be apparent when it is considered that the middle switch may be regarded simply as a curved main track, and that, as

Fig. 145.

developed in § 267, the dimensions of turnouts are nearly the same whether the main track is straight or slightly curved.

271. Connecting curve from a straight track. The "connecting curve" is the track lying between the frog and the side track where it becomes parallel to the main track (*FS* in Fig. 146 or 147). Call *d* the distance between track centers. The angle $FO_1R = F$ (see Fig. 146). Call r' the radius of the connecting curve. Then

Fig. 146.

$$(r' - \tfrac{1}{2}g) = \frac{d - g}{\text{vers } F}; \quad . \quad . \quad . \quad . \quad (107)$$

$$FR = (r' - \tfrac{1}{2}g) \sin F. \quad . \quad . \quad (108)$$

If it is considered that the distance *FR* consumes too much track room, it may be shortened by the method indicated in Fig. 151.

272. Connecting curve from a curved track to the OUTSIDE. When the main track is curved, the required quantities are the radius *r* of the connecting curve from *F* to *S*, Fig. 147, and its length or central angle. In the triangle *CSF*

$$CS + CF : CS - CF :: \tan \tfrac{1}{2}(CFS + CSF) : \tan \tfrac{1}{2}(CFS - CSF);$$

but $\frac{1}{2}(CFS + CSF) = 90 - \frac{1}{2}\psi$; and, since the triangle O_1SF is isosceles, $\frac{1}{2}(CFS - CSF) = \frac{1}{2}F$;

$$\therefore \quad 2R + d : d - g :: \cot \tfrac{1}{2}\psi : \tan \tfrac{1}{2}F$$
$$:: \cot \tfrac{1}{2}F : \tan \tfrac{1}{2}\psi;$$

$$\therefore \quad \tan \tfrac{1}{2}\psi = \frac{2n(d - g)}{2R + d}. \quad \cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot \quad (109)$$
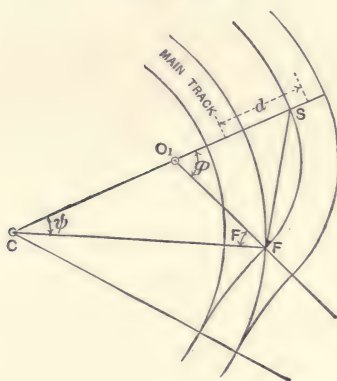
FIG. 147.

From the triangle CO_1F we may derive

$$r - \tfrac{1}{2}g : R + \tfrac{1}{2}g :: \sin \psi : \sin (F + \psi);$$

$$r - \tfrac{1}{2}g = (R + \tfrac{1}{2}g) \frac{\sin \psi}{\sin (F + \psi)} \quad \cdot \quad \cdot \quad \cdot \quad \cdot \quad (110)$$

Also $$FS = 2(r - \tfrac{1}{2}g) \sin \tfrac{1}{2}(F + \psi). \quad \cdot \quad \cdot \quad \cdot \quad (111)$$

273. Connecting curve from a curved track to the INSIDE. As above, it may readily be deduced from the triangle CFS (see Fig. 148) that

$$(2R - d) : (d - g) :: \cot \tfrac{1}{2}\psi : \tan \tfrac{1}{2}F,$$

and finally that

$$\tan \tfrac{1}{2}\psi = \frac{2n(d - g)}{2R - d}. \quad \cdot \quad \cdot \quad \cdot \quad \cdot \quad (112)$$

Similarly we may derive (as in Eq. 110)

$$(r - \tfrac{1}{2}g) = (R - \tfrac{1}{2}g) \frac{\sin \psi}{\sin (F - \psi)}. \quad . \quad . \quad (113)$$

Also $$FS = 2(r - \tfrac{1}{2}g) \sin \tfrac{1}{2}(F - \psi). \quad . \quad . \quad (114)$$
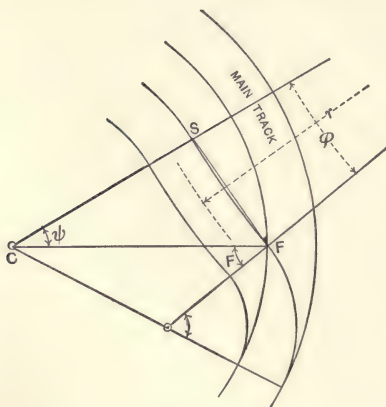
Fig. 148.

Two other cases are possible. (a) r may increase until it becomes infinite (see Fig. 149), then $F = \psi$. In such a case we may write, by substituting in Eq. 112,

$$2R - d = 4n^2(d - g). \quad . \quad . \quad (115)$$

This equation shows the value of R, which renders this case possible with the given values of n, d, and g. (b) ψ may be greater than F. As before (see Fig. 150)

$$2R - d : d - g :: \cot \tfrac{1}{2}\psi : \tan \tfrac{1}{2}F;$$
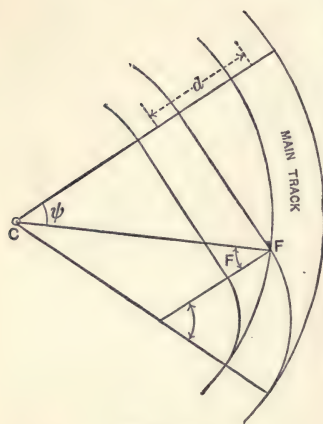
$$\tan \tfrac{1}{2}\psi = \frac{2n(d - g)}{2R - d},$$

Fig. 149.

the same as Eq. 112, but

$$r + \tfrac{1}{2}g = (R - \tfrac{1}{2}g)\frac{\sin \psi}{\sin (\psi - F)}. \quad . \quad . \quad (116)$$
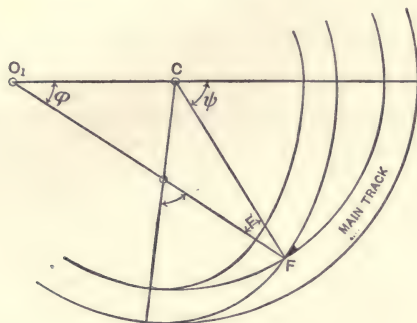
FIG. 150.

274. Crossover between two parallel straight tracks. (See Fig. 151.) The turnouts are as usual. The crossover track may be straight, as shown by the full lines, or it may be a reversed curve, as shown by the dotted lines. The reversed curve shortens the total length of track required, but is somewhat objectionable. The first method requires that both frogs must be equal. The second method permits unequal frogs, although equal frogs are preferable. The length of straight crossover track is $F_1 T$.
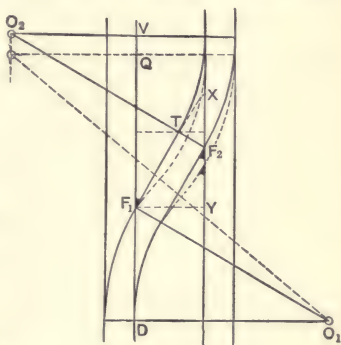
FIG. 151.

$$F_1 T \sin F_1 + g \cos F_1 = d - g;$$

$$F_1 T = \frac{d - g}{\sin F_1} - g \cot F_1. \quad . \quad . \quad . \quad . \quad (117)$$

The total distance along the track may be derived as follows:

$$DV = 2DF_1 + F_2Y = 2DF_1 + XY - XF_2;$$

$$XY = (d - g) \cot F_1; \qquad XF_2 = g \div \sin F_2;$$

$$\therefore \quad DV = 2DF_1 + (d - g) \cot F_1 - \frac{g}{\sin F_2}. \quad . \quad . \quad (118)$$

If a reversed curve with equal frogs is used, we have

$$\text{vers } \theta = \frac{d}{2r}; \quad . \quad . \quad . \quad . \quad . \quad (119)$$

also

$$DQ = 2r \sin \theta. \quad . \quad . \quad . \quad . \quad (120)$$
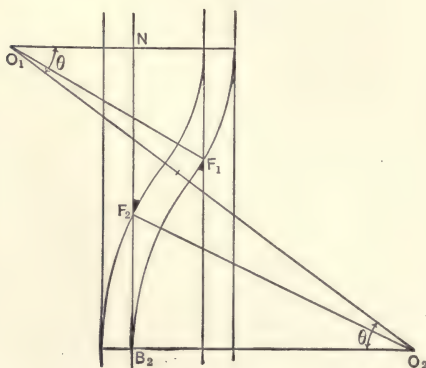
FIG. 152.

If the frogs are unequal, we will have (see Fig. 152)

$$r_2 \text{ vers } \theta + r_1 \text{ vers } \theta = d;$$

$$\therefore \quad \text{vers } \theta = \frac{d}{r_1 + r_2}; \quad . \quad . \quad . \quad (121)$$

also the distance along the track

$$B_2N = (r_1 + r_2) \sin \theta. \quad . \quad . \quad . \quad (122)$$

275. Crossover between two parallel curved tracks. **(a) Using a straight connecting curve.** This solution has limitations. If one frog (F_1) is chosen, F_2 becomes determined, being a function of F_1. If F_1 is less than some limit, depending on the width

FIG. 153.

(d) between the parallel tracks, this solution becomes impossible. In Fig. 153 assume F_1 as known. Then $F_1H = g \sec F_1$. In the triangle HOF_2 we have

$$\sin HF_2O : \sin F_2HO :: HO : F_2O;$$

$$\sin F_2HO = \cos F_1; \quad HF_2O = 90° + F_2;$$

$$\therefore \ \sin HF_2O = \cos F_2.$$

$$HO = R + \tfrac{1}{2}d - \tfrac{1}{2}g - g \sec F_1; \quad F_2O = R - \tfrac{1}{2}d + \tfrac{1}{2}g;$$

$$\therefore \ \cos F_2 = \cos F_1 \frac{R + \tfrac{1}{2}d - \tfrac{1}{2}g - g \sec F_1}{R - \tfrac{1}{2}d + \tfrac{1}{2}g}. \quad \cdot \quad \cdot \quad \cdot \quad (123)$$

Knowing F_2, θ_2 is determinable from Eq. 91. Fig. 153 shows the case where θ_2 is greater than F_2. Fig. 154 shows the case where it is less. The demonstration of Eq. 123 is applicable to
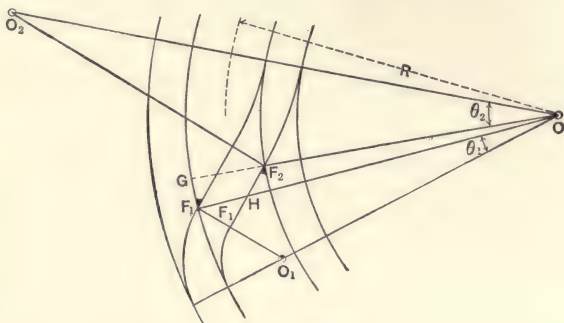
FIG. 154.

both figures. The relative position of the frogs F_1 and F_2 may be determined as follows, the solution being applicable to both Figs. 153 and 154:

$$HOF_2 = 180° - (90° - F_1) - (90° + F_2) = F_1 - F_2.$$

Then

$$GF_1 = 2(R + \tfrac{1}{2}d - \tfrac{1}{2}g) \sin \tfrac{1}{2}(F_1 - F_2). \quad . \quad . \quad . \quad (124)$$

Since F_2 comes out *any* angle, its value will not be in general that of an even frog number, and it will therefore need to be made to order.

(b) **Continuing the switch-rail curves until they meet as a reversed curve.** In this case F_1 and F_2 may be chosen at pleasure (within limitations), and they will of course be of regular sizes and equal or unequal as desired. F_1 and F_2 being known, θ_1 and θ_2 are computed by Eq. 95 and 91. In the triangle OO_1O_2 (see Fig. 155)

$$\text{vers } \psi = \frac{2(S - OO_2)(S - OO_1)}{OO_2 - OO_1},$$

in which $\qquad S = \tfrac{1}{2}(OO_1 + OO_2 + O_1O_2);$

but
$$OO_1 = R + \tfrac{1}{2}d - r_1,$$

$$OO_2 = R - \tfrac{1}{2}d + r_2,$$

$$O_1O_2 = r_1 + r_2;$$

$$\therefore \quad S = \tfrac{1}{2}(2R + 2r_2) = R + r_2;$$

$$S - OO_2 = R + r_2 - R + \tfrac{1}{2}d - r_2 = \tfrac{1}{2}d;$$

$$S - OO_1 = R + r_2 - R - \tfrac{1}{2}d + r_1 = r_1 + r_2 - \tfrac{1}{2}d;$$

Fig. 155.

$$\text{vers } \psi = \frac{d(r_1 + r_2 - \tfrac{1}{2}d)}{(R - \tfrac{1}{2}d + r_2)(R + \tfrac{1}{2}d - r_1)}; \quad \cdot \quad \cdot \quad (125)$$

$$\sin OO_2O_1 = \sin \psi \frac{OO_1}{O_1O_2} = \sin \psi \frac{R + \tfrac{1}{2}d - r_1}{r_1 + r_2}; \quad \cdot \quad \cdot \quad (126)$$

$$O_2O_1D = \psi + O_1O_2O; \quad \cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot \quad (127)$$

$$NF_2 = 2(R - \tfrac{1}{2}d + \tfrac{1}{2}g) \sin \tfrac{1}{2}(\psi - \theta_1 - \theta_2). \quad \cdot \quad (128)$$

Although the above method introduces a reversed curve, yet it uses up less track than the first method and permits the use of ordinary frogs rather than those having some special angle which must be made to order.

276. Practical rules for switch-laying. A consideration of the previous sections will show that the formulæ are comparatively simple when the lead rails are assumed as circular; that they become complicated, even for turnouts from a straight main track, when the effect of straight frog and point rails is allowed for, and that they become hopelessly complicated when allowing for this effect on turnouts from a curved main track. It is also shown (§ 267) that the length of the lead is practically

$$MN = k$$
$$FH = f$$
$$MDN = \alpha$$
$$HMR = \tfrac{1}{2} (F - \alpha)$$
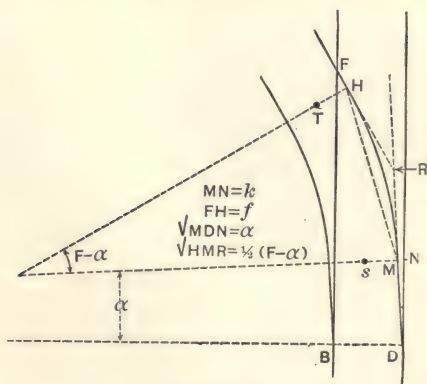
Fig. 140.

the same whether the main track is straight or is curved with such curves as are commonly used, and that the degree of curve of the lead rails from a curved main track may be found with close approximation by mere addition or subtraction. From this it may be assumed that, if the length of lead (L) and the radius of the lead rails (r) are computed from Eq. 87 and 90 for various frog angles, the same leads may be used for curved main track; also, that the degree of curve of the lead rails may be found by addition or subtraction, as indicated in § 267, and that the approximations involved will not be of practical detriment.

In accordance with this plan Table III has been computed from Eq. 87, 88, and 90. The *leads* there given may be used for all main tracks straight or curved. The table gives the degree of curve of the lead rails for *straight* main track; for a turnout to the *inside*, *add* the degree of curve of the main track; for a turnout to the *outside*, *subtract* it.

If the position of the switch-block is definitely determined, then the rails must be cut accordingly; but when some freedom is allowable (which never need exceed 15 feet and may require but a few inches), one rail-cutting may be avoided. Mark on the rails at B, F, and D; measure off the length of the switch-rails DN; offset $\frac{1}{2}g + k$ from N for the point S. The point H may be located (temporarily) by measuring along the rail a distance $FH (= f)$ and then swinging out a distance of $f \div n$ (n being the frog number). $HT = \frac{1}{2}g$ and is measured at right angles to FH. Points for track centers between S and T may be laid off by a transit or by the use of a string and tape. Substituting in Eq. 31 the value of R and of *chord* $(= ST)$, we may compute x $(= db)$. Locate the middle point d and the quarter points a'' and c''. Then $a''a$ and $c''c$ each equal three-fourths of db. Theoretically this gives a parabola rather than a circle, but the difference for all practical cases is too small for measurement.
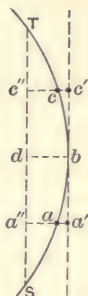
FIG. 156.

Example. Given a main track on a 4° curve; a turnout to the outside, using a number 9 frog; gauge $4'\ 8\frac{1}{2}''$; $f = 3'.37$; $k = 5\frac{3}{4}''$; $DN = 15'\ 0''$ and $\alpha = 1°\ 50'$. Then for a *straight* track r would equal 681.16 $[d = 8°\ 25']$. For this curved track d will be nearly $(8°\ 25' - 4°) = 4°\ 25'$, or r will be 1297.6. L for the *straight* track would be 72.20; but since the lead is slightly increased (see § 267) when the turnout is on the outside of a curve, L may here be called 72.5. $FH = f = 3'.37$; $f \div n = 3.37 \div 9 = 0'.375 = 4''.5$. H, T, and S may be located as described above. ST may be measured on the ground, or it may be computed from Eq. 88, giving the value

of 53.80 feet for straight track. Since it is slightly more for a turnout to the outside of a curve, it may be called 54.0. Then

$$x = db = \frac{(54.0)^2}{8 \times 1297.6} = 0.281 \text{ feet, and } aa'' \text{ and } cc'' = 0.21$$

foot.

<center>CROSSINGS.</center>

277. Two straight tracks. When two straight tracks cross each other, four frogs are necessary, the angles of two of them being supplementary to the angles of the other. Since such crossings are sometimes operated at high speeds, they should be

SECTION ON A-B   SECTION ON C-D
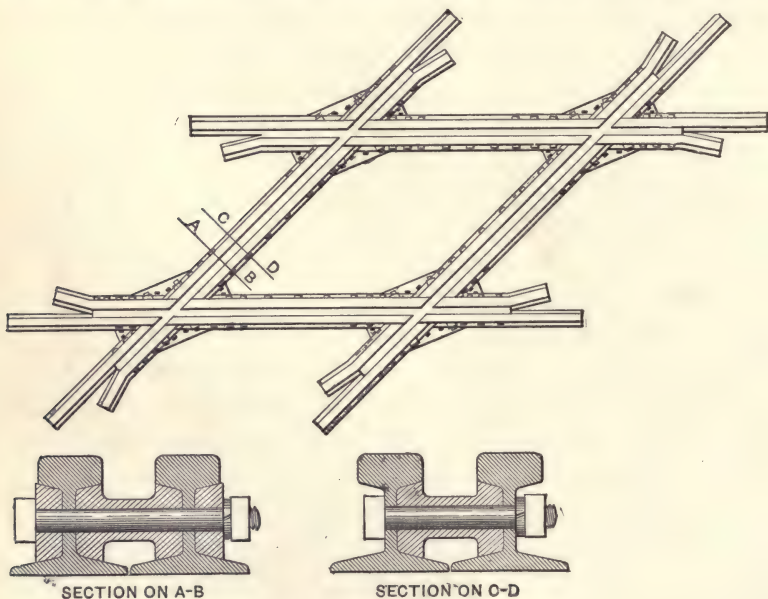
FIG. 157.—CROSSING.

very strongly constructed, and the angles should preferably be 90° or as near that as possible. The frogs will not in general be "stock" frogs of an even number, especially if the angles are large, but must be made to order with the required angles as measured. In Fig. 157 are shown the details of such a crossing. Note the fillers, bolts, and guard-rails.

278. One straight and one curved track. Structurally the crossing is about the same as above, but the frog angles are all unequal. In Fig. 158, R is known, and the angle M, made by the center lines of the tracks at their point of intersection, is also known.

$$M = NCM. \quad NC = R \cos M.$$

$$\left.\begin{array}{l} (R - \tfrac{1}{2}g) \cos F_1 = NC + \tfrac{1}{2}g\,; \\ \quad \therefore \cos F_1 = \dfrac{R \cos M + \tfrac{1}{2}g}{R - \tfrac{1}{2}g}. \\[4pt] \text{Similarly } \cos F_2 = \dfrac{R \cos M + \tfrac{1}{2}g}{R + \tfrac{1}{2}g}, \\[4pt] \quad \cos F_3 = \dfrac{R \cos M - \tfrac{1}{2}g}{R + \tfrac{1}{2}g}, \\[4pt] \quad \cos F_4 = \dfrac{R \cos M - \tfrac{1}{2}g}{R - \tfrac{1}{2}g}. \end{array}\right\} (129)$$
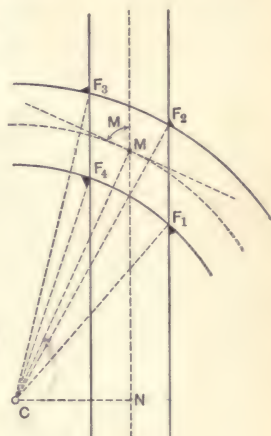
Fig. 158.

279. Two curved tracks. The four frogs are unequal, and the angle of each must be computed. The radii R_1 and R_2 are
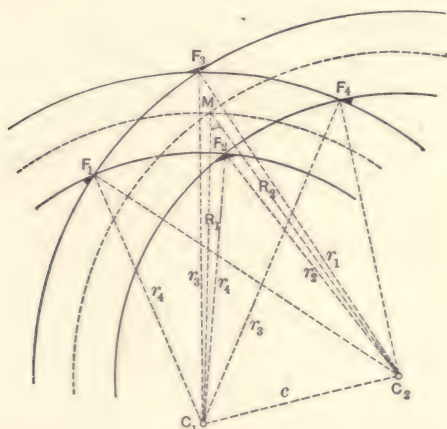
Fig. 159.

known; also the angle M. r_1, r_2, r_3, and r_4 are therefore known by adding or subtracting $\tfrac{1}{2}g$, but the lines are so indi-

cated for brevity. Call the angle $MC_1C_2 = C_1$, the angle $MC_2C_1 = C_2$, and the line $C_1C_2 = c$. Then

$$\tfrac{1}{2}(C_1 + C_2) = 90° - \tfrac{1}{2} M$$

and

$$\tan \tfrac{1}{2}(C_1 - C_2) = \cot \tfrac{1}{2} M \frac{R_2 - R_1}{R_2 + R_1}.$$

C_1 and C_2 then become known and

$$c = C_1C_2 = R_2 \frac{\sin M}{\sin C_1}.$$

In the triangle $F_1C_1C_2$, call $\tfrac{1}{2}(c + r_1 + r_4) = s_1$; then

$$\text{Similarly} \quad
\left.
\begin{aligned}
\text{vers } F_1 &= \frac{2(s_1 - r_1)(s_1 - r_4)}{r_1 r_4}. \\[4pt]
\text{vers } F_2 &= \frac{2(s_2 - r_2)(s_2 - r_4)}{r_2 r_4}, \\[4pt]
\text{vers } F_3 &= \frac{2(s_3 - r_1)(s_3 - r_3)}{r_1 r_3}, \\[4pt]
\text{vers } F_4 &= \frac{2(s_4 - r_2)(s_4 - r_3)}{r_2 r_3}.
\end{aligned}
\right\}
\qquad . \quad . \quad (130)$$

In the above equations

$$s_2 = \tfrac{1}{2}(c + r_2 + r_4),$$
$$s_3 = \tfrac{1}{2}(c + r_1 + r_3),$$
$$s_4 = \tfrac{1}{2}(c + r_2 + r_3).$$

APPENDIX.

THE ADJUSTMENTS OF INSTRUMENTS.

THE accuracy of instrumental work may be vitiated by any one of a large number of inaccuracies in the geometrical relations of the parts of the instruments. Some of these relations are so apt to be altered by ordinary usage of the instrument that the makers have provided adjusting-screws so that the inaccuracies may be readily corrected. There are other possible defects, which, however, will seldom be found to exist, provided the instrument was properly made and has never been subjected to treatment sufficiently rough to distort it. Such defects, when found, can only be corrected by a competent instrument maker or repairer.

A WARNING is necessary to those who would test the accuracy of instruments, and especially to those whose experience in such work is small. Lack of skill in handling an instrument will often indicate an apparent error of adjustment when the real error is very different or perhaps non-existent. It is always a safe plan when testing an adjustment to note the amount of the apparent error; then, beginning anew, make another independent determination of the amount of the error. When two or more *perfectly independent* determinations of such an error are made it will generally be found that they differ by an appreciable amount. The *differences* may be due in variable measure to careless inaccurate manipulation and to instrumental *defects* which are wholly independent of the particular test being made. Such careful determinations of the amounts of the errors are generally advisable in view of the next paragraph.

Do not disturb the adjusting-screws any more than necessary. Although metals are apparently rigid, they are really elastic and yielding. If some parts of a complicated mechanism, which is held together largely by friction, are subjected to greater internal stresses than other parts of the mechanism, the jarring resulting from handling will frequently cause a slight readjustment in the parts which will tend to more nearly equalize the internal stresses. Such action frequently occurs with the adjusting mechanism of instruments. One screw may be strained more than others. The friction of parts may prevent the opposing screw from *immediately* taking up an equal stress. Perhaps the adjustment appears perfect under these conditions. Jarring diminishes the friction between the parts, and the unequal stresses tend to equalize. A motion takes place which, although microscopically minute, is sufficient to indicate an error of adjustment. A readjustment, made by unskillful hands, may not make the final adjustment any more perfect. The frequent shifting of adjusting-screws wears them badly, and when the screws are worn it is still more difficult to keep them from moving enough to vitiate the adjustments. It is therefore preferable in many cases to refrain from disturbing the adjusting-screws, especially as the accuracy of the work done is not *necessarily* affected by errors of adjustment, as may be illustrated :

(*a*) Certain operations are *absolutely* unaffected by certain errors of adjustment.

(*b*) Certain operations are so slightly affected by certain *small* errors of adjustment that their effect may properly be neglected.

(*c*) Certain errors of adjustment may be readily allowed for and neutralized so that no error results from the use of the unadjusted instrument. Illustrations of all these cases will be given under their proper heads.

ADJUSTMENTS OF THE TRANSIT.

1. *To have the plate-bubbles in the center of the tubes when the axis is vertical.* Clamp the upper plate and, with the lower

clamp loose, swing the instrument so that the plate-bubbles are
parallel to the lines of opposite leveling-screws. Level up until
both bubbles are central. Swing the instrument 180°. If the
bubbles again settle at the center, the adjustment is perfect. If
either bubble does not settle in the center, move the leveling-
screws until the bubble is *half-way* back to the center. Then,
before touching the adjusting-screws, note carefully the position
of the bubbles and observe whether the bubbles always settle at
the *same* place in the tube, no matter to what position the in-
strument may be rotated. When the instrument is so leveled,
the axis is truly vertical and the discrepancies between this con-
stant position of the bubbles and the centers of the tubes measure
the errors of adjustment. By means of the adjusting-screws
bring each bubble to the center of the tube. If this is done so
skillfully that the true level of the instrument is not disturbed,
the bubbles should settle in the center for all positions of the
instrument. Under unskillful hands, two or more such trials
may be necessary.

When the plates are not horizontal, the measured angle is greater than
the true horizontal angle by the difference between the measured angle
and its projection on a horizontal plane. When this angle of inclination
is small, the difference is insignificant. Therefore when the plate-bubbles
are *very nearly* in adjustment, the error of measurement of horizontal
angles may be far within the lowest unit of measurement used. A *small*
error of adjustment of the plate-bubble *perpendicular* to the telescope will
affect the horizontal angles by only a small proportion of the error, which
will be perhaps imperceptible. Vertical angles will be affected by the
same insignificant amount. A *small* error of adjustment of the plate-
bubble *parallel* to the telescope will affect horizontal angles very slightly,
but will affect vertical angles by the full amount of the error.

All error due to unadjusted plate-bubbles may be avoided by noting in
what positions in the tubes the bubbles will remain fixed for all positions
of azimuth and then keeping the bubbles adjusted to these positions, for
the axis is then truly vertical. It will often save time to work in this way
temporarily rather than to stop to make the adjustments. This should
especially be done when accurate vertical angles are required.

When the bubbles are truly adjusted, they should remain stationary,
regardless of whether the telescope is revolved with the upper plate loose
and the lower plate clamped or whether the whole instrument is revolved,
the plates being clamped together. If there is any appreciable difference,

it shows that the two vertical axes or "centers" of the plates are not con-
centric. This may be due to cheap and faulty construction or to the exces-
sive wear that may be sometimes observed in an old instrument originally
well made. In either case it can only be corrected by a maker.

2. *To make the revolving axis of the telescope perpendicular
to the vertical axis of the instrument.* This is best tested by
using a long plumb-line, so placed that the telescope must be
pointed upward at an angle of about 45° to sight at the top of
the plumb-line and downward about the same amount, if pos-
sible, to sight at the lower end. The vertical axis of the transit
must be made truly vertical. Sight at the upper part of the
line, clamping the horizontal plates. Swing the telescope down
and see if the cross-wire again bisects the cord. If so, the
adjustment is *probably* perfect (a conceivable exception will be
noted later); if not, raise or lower one end of the axis by means
of the adjusting-screws, placed at the top of one of the stan-
dards, until the cross-wire will bisect the cord both at top and
bottom. The plumb-bob may be steadied, if necessary, by
hanging it in a pail of water. As many telescopes cannot be
focused on an object nearer than 6 or 8 feet from the telescope,
this method requires a long plumb-line swung from a high point,
which may be inconvenient.

Another method is to set up the instrument about 10 feet
from a high wall. After leveling, sight at some convenient
mark high up on the wall. Swing the telescope down and make
a mark (when working alone some convenient natural mark may
generally be found) low down on the wall. Plunge the telescope
and revolve the instrument about its vertical axis and again sight
at the upper mark. Swing down to the lower mark. If the
wire again bisects it, the adjustment is perfect. If not, fix a
point *half-way* between the two positions of the lower mark.
The plane of this point, the upper point, and the center of the
instrument is truly vertical. Adjust the axis to these upper and
lower points as when using the plumb-line.

3. *To make the line of collimation perpendicular to the
revolving axis of the telescope.* With the instrument level and

the telescope nearly horizontal point at some well-defined point at a distance of 200 feet or more. Plunge the telescope and establish a point in the opposite direction. Turn the whole instrument about the vertical axis until it again points at the first mark. Again plunge to "direct position" (i.e., with the level-tube *under* the telescope). If the vertical cross-wire again points at the second mark, the adjustment is perfect. If not, the error is *one-fourth* of the distance between the two positions of the second mark. Loosen the capstan-screw on one side of the telescope and tighten it on the other side until the vertical wire is set at the one-fourth mark. Turn the whole instrument by means of the tangent screw until the vertical wire is *midway* between the two positions of the second mark. Plunge the telescope. If the adjusting has been skillfully done, the cross-wire should come exactly to the first mark. As an "erecting eyepiece" reinverts an image already inverted, the ring carrying the cross-wires must be moved in the *same* direction as the *apparent* error in order to correct that error.

The necessity for the third adjustment lies principally in the practice of producing a line by plunging the telescope, but when this is required to be done with great accuracy it is always better to obtain the forward point by reversion (as described above for making the test) and take the *mean* of the two forward points. Horizontal and vertical angles are practically unaffected by *small* errors of this adjustment, unless, in the case of horizontal angles, the vertical angles to the points observed are very different.

Unnecessary motion of the adjusting-screws may sometimes be avoided by carefully establishing the forward point on line by repeated reversions of the instrument, and thus determining by repeated trials the exact amount of the error. *Differences* in the amount of error determined would be evidence of inaccuracy in manipulating the instrument, and would show that an adjustment based on the first trial would *probably* prove unsatisfactory.

The 2d and 3d adjustments are mutually dependent. If either adjustment is badly out, the other adjustment cannot be made except as follows :

(*a*) The second adjustment can be made regardless of the third when the lines to the high point and the low point make *equal* angles with the horizontal.

(b) The third adjustment can be made regardless of the second when the front and rear points are *on a level* with the instrument.

When both of these requirements are *nearly* fulfilled, and especially when the error of either adjustment is small, no trouble will be found in perfecting either adjustment on account of a small error in the other adjustment.

If the test for the second adjustment is made by means of the plumb-line and the vertical cross-wire intersects the line at all points as the telescope is raised or lowered, it not only demonstrates at once the accuracy of that adjustment, but also shows that the third adjustment is either perfect or has so small an error that it does not affect the second.

4. *To have the bubble of the telescope-level in the center of the tube when the line of collimation is horizontal.* The line of collimation should coincide with the optical axis of the telescope. If the object-glass and eyepiece have been properly centered, the previous adjustment will have brought the vertical cross-wire to the center of the field of view. The horizontal cross-wire should also be brought to the center of the field of view, and the bubble should be adjusted to it.

a. Peg method. Set up the transit at one end of a nearly level stretch of about 300 feet. Clamp the telescope with its bubble in the center. Drive a stake vertically under the eye-piece of the transit, and another about 300 feet away. Observe the height of the center of the eyepiece (the telescope being level) above the stake (calling it a); observe the reading of the rod when held on the other stake (calling it b); take the instrument to the other stake and set it up so that the eyepiece is vertically over the stake, observing the height, c; take a reading on the first stake, calling it d. If this adjustment is perfect, then

$$a - d = b - c,$$

or $$(a - d) - (b - c) = 0.$$

Call $$(a - d) - (b - c) = 2m.$$

When m is positive, the line points downward;
" m " negative, "   "   "   upward.

To adjust: if the line points *up*, sight the horizontal cross-wire (by moving the vertical tangent screw) at a point which is *m* lower, then adjust the bubble so that it is in the center.

By taking several independent values for *a*, *b*, *c*, and *d*, a mean value for *m* is obtained, which is more reliable and which may save much unnecessary working of the adjusting-screws.

b. Using an auxiliary level. When a carefully adjusted level is at hand, this adjustment may sometimes be more easily made by setting up the transit and level, so that their lines of collimation are as nearly as possible at the same height. If a point may be found which is half a mile or more away and which is on the horizontal cross-wire of the level, the horizontal cross-wire of the transit may be pointed directly at it, and the bubble adjusted accordingly. Any slight difference in the heights of the lines of collimation of the transit and level (say $\frac{1}{4}''$) may almost be disregarded at a distance of $\frac{1}{2}$ mile or more, or, if the difference of level would have an appreciable effect, even this may be practically eliminated by making an estimated allowance when sighting at the distant point. Or, if a distant point is not available, a level-rod with target may be used at a distance of (say) 300 feet, making allowance for the carefully determined difference of elevation of the two lines of collimation.

5. Zero of vertical circle. When the line of collimation is truly horizontal and the vertical axis is truly vertical, the reading of the vertical circle should be 0°. If the arc is adjustable, it should be brought to 0°. If it is not adjustable, the *index error* should be observed, so that it may be applied to all readings of vertical angles.

ADJUSTMENTS OF THE WYE LEVEL.

1. To make the line of collimation coincide with the center of the rings. Point the intersection of the cross-wires at some well-defined point which is at a considerable distance. The instrument need not be level, which allows much greater liberty in choosing a convenient point. The vertical axis should be

clamped, and the clips over the wyes should be loosened and raised. Rotate the telescope in the wyes. The intersection of the cross-wires should be continually on the point. If it is not, it requires adjustment. Rotate the telescope 180° and adjust *one-half* of the error by means of the capstan-headed screws that move the cross-wire ring. It should be remembered that, with an erecting telescope, on account of the inversion of the image, the ring should be moved in the direction of the *apparent* error. Adjust the other half of the error with the leveling-screws. Then rotate the telescope 90° from its usual position, sight accurately at the point, and then rotate 180° from that position and adjust any error as before. It may require several trials, but it is necessary to adjust the ring until the intersection of the cross-wires will remain on the point for any position of rotation.

If such a test is made on a very distant point and again on a point only 10 or 15 feet from the instrument, the adjustment may be found correct for one point and incorrect for the other. This indicates that the object-slide is improperly centered. Usually this defect can only be corrected by an instrument-maker. If the difference is very small it may be ignored, but the adjustment should then be made on a point which is at about the mean distance for usual practice—say 150 feet.

If the whole image appears to shift as the telescope is rotated, it indicates that the eyepiece is improperly adjusted. This defect is likewise usually corrected only by the maker. It does not interfere with instrumental accuracy, but it usually causes the intersection of the cross-wires to be eccentric with the field of view.

2. *To make the axis of the level tube parallel to the line of collimation.* Raise the clips as far as possible. Swing the level so that it is parallel to a pair of opposite leveling-screws and clamp it. Bring the bubble to the middle of the tube by means of the leveling-screws. Take the telescope out of the wyes and replace it end for end, using *extreme care* that the wyes are not jarred by the action. If the bubble does not come to the center, correct *one-half* of the error by the vertical adjusting-screws at one end of the bubble. Correct the other half by the leveling-screws. Test the work by again changing the telescope end for end in the wyes.

Care should be taken while making this adjustment to see that the level-tube is vertically under the telescope. With the bubble in the center of the tube, rotate the telescope in the wyes for a considerable angle each side of the vertical. If the first half of the adjustment has been made and the bubble moves, it shows that the axis of the wyes and the axis of the level-tube are not in the same vertical plane although both have been made horizontal. By moving one end of the level-tube *sidewise* by means of the horizontal screws at one end of the tube, the two axes may be brought into the same plane. As this adjustment is liable to disturb the other, both should be alternately tested until both requirements are complied with.

By these methods the axis of the bubble is made parallel to the axis of the wyes; and as this has been made parallel to the lines of collimation by means of the previous adjustment, the axis of the bubble is therefore parallel to the line of collimation.

3. *To make the line of collimation perpendicular to the vertical axis.* Level up so that the instrument is approximately level over both sets of leveling-screws. Then, after leveling carefully over one pair of screws, revolve the telescope 180°. If it is not level, adjust half of the error by means of the capstan-headed screw under one of the wyes, and the other half by the leveling-screws. Reverse again as a test.

When the first two adjustments have been accurately made, good leveling may always be done by bringing the bubble to the center by means of the leveling-screws, at every sight if necessary, even if the third adjustment is not made. Of course this third adjustment should be made as a matter of convenience, so that the line of collimation may be always level no matter in what direction it may be pointed, but it is not *necessary* to stop work to make this adjustment every time it is found to be defective.

ADJUSTMENTS OF THE DUMPY LEVEL.

1. *To make the axis of the level-tube perpendicular to the vertical axis.* Level up so that the instrument is approximately level over both sets of leveling-screws. Then, after leveling carefully over one pair of screws, revolve the telescope 180°. If

it is not level, adjust *one-half* of the error by means of the adjusting-screws at one end of the bubble, and the other half by means of the leveling-screws.   Reverse again as a test.

2. *To make the line of collimation perpendicular to the vertical axis.*   The method of adjustment is identical with that for the transit (No. 4, p. 308) except that the cross-wire must be adjusted to agree with the level-bubble rather than *vice versa*, as is the case with the corresponding adjustment of the transit; i.e., with the level-bubble in the center, raise or lower the horizontal cross-wire until it points at the mark known to be on a level with the center of the instrument.

If the instrument has been well made and has not been distorted by rough usage, the cross-wires will intersect at the center of the field of view when adjusted as described.   If they do not, it indicates an error which ordinarily can only be corrected by an instrument-maker.   The error may be due to any one of several causes, which are

(*a*) faulty centering of object-slide;

(*b*) faulty centering of eyepiece;

(*c*) distortion of instrument so that the geometric axis of the telescope is not perpendicular to the vertical axis.   If the error is only just perceptible, it will not probably cause any error in the work.

EXPLANATORY NOTE ON THE USE OF THE TABLES.

The logarithms here given are "five-place," but the last figure sometimes has a special mark over it (e.g., $\hat{6}$) which indicates that one-half a unit in the last place should be *added*. For example:

the value	includes all values between
.69586	.6958575000 $+$ and .6958624999 . . .
.6958$\hat{6}$	.6958625000 $+$ and .6958674999 . . .

The maximum error in any one value therefore does not exceed one-quarter of a fifth-place unit.

When adding or subtracting such logarithms allow a half-unit for such a sign. For example:

.69586	.69586	.6958$\hat{6}$
.10841	.1084$\hat{1}$	.1084$\hat{1}$
.1294$\hat{7}$	.1294$\hat{7}$	.1294$\hat{7}$
.9337$\hat{4}$	.93375	.9337$\hat{5}$

All other logarithmic operations are performed as usual and are supposed to be understood by the student.

TABLE I.—RADII OF CURVES.

Deg.	0°		1°		2°		3°		Deg.
Min.	Radius.	Log R	Radius.	Log R	Radius.	Log R	Radius.	Log R	Min.
0	∞	∞	5729.6	3.75813	2864.9	3.45711	1910.1	3.28105	0
1	343775	5.53627	5635.7	.75095	2841.3	.45351	1899.5	.27864	1
2	171887	5.23524	5544.8	.74389	2818.0	.44993	1889.1	.27625	2
3	114592	5.05915	5456.8	.73694	2795.1	.44639	1878.8	.27387	3
4	85944	4.93421	5371.6	.73010	2772.5	.44287	1868.6	.27151	4
5	68755	4.83736	5288.9	.72336	2750.4	.43939	1858.5	.26915	5
6	57296	4.75812	5208.8	3.71673	2728.5	3.43593	1848.5	3.26681	6
7	49111	.69117	5131.0	.71020	2707.0	.43249	1838.6	.26448	7
8	42972	.63318	5055.6	.70377	2685.9	.42909	1828.8	.26217	8
9	38197	.58203	4982.3	.69743	2665.1	.42571	1819.1	.25986	9
10	34377	.53627	4911.2	.69118	2644.6	.42235	1809.6	.25757	10
11	31252	4.49488	4842.0	3.68502	2624.4	3.41903	1800.1	3.25529	11
12	28648	.45709	4774.7	.67895	2604.5	.41572	1790.7	.25303	12
13	26444	.42233	4709.3	.67296	2584.9	.41245	1781.5	.25077	13
14	24555	.39014	4645.7	.66705	2565.6	.40919	1772.3	.24853	14
15	22918	.36018	4583.8	.66122	2546.6	.40597	1763.2	.24629	15
16	21486	4.33215	4523.4	3.65547	2527.9	3.40276	1754.2	3.24407	16
17	20222	.30582	4464.7	.64979	2509.5	.39958	1745.3	.24186	17
18	19099	.28100	4407.5	.64419	2491.3	.39642	1736.5	.23967	18
19	18093	.25752	4351.7	.63865	2473.4	.39329	1727.8	.23748	19
20	17189	.23524	4297.3	.63319	2455.7	.39017	1719.1	.23536	20
21	16370	4.21405	4244.1	3.62780	2438.3	3.38708	1710.6	3.23314	21
22	15626	.19385	4192.5	.62247	2421.1	.38401	1702.1	.23098	22
23	14947	.17454	4142.0	.61720	2404.2	.38097	1693.7	.22884	23
24	14324	.15606	4092.7	.61200	2387.5	.37794	1685.4	.22670	24
25	13751	.13833	4044.5	.60686	2371.0	.37494	1677.2	.22458	25
26	13222	4.12130	3997.5	3.60178	2354.8	3.37195	1669.1	3.22247	26
27	12732	.10491	3951.5	.59676	2338.8	.36899	1661.0	.22037	27
28	12278	.08911	3906.6	.59180	2323.0	.36604	1653.0	.21827	28
29	11854	.07387	3862.7	.58689	2307.4	.36312	1645.1	.21619	29
30	11459	.05915	3819.8	.58204	2292.0	.36021	1637.3	.21412	30
31	11090	4.04491	3777.9	3.57724	2276.8	3.35733	1629.5	3.21206	31
32	10743	.03112	3736.8	.57250	2261.9	.35446	1621.8	.21000	32
33	10417	.01776	3696.6	.56780	2247.1	.35162	1614.2	.20796	33
34	10111	4.00479	3657.3	.56316	2232.5	.34879	1606.7	.20593	34
35	9822.2	3.99221	3618.8	.55856	2218.1	.34598	1599.2	.20390	35
36	9549.3	3.97997	3581.1	3.55401	2203.9	3.34318	1591.8	3.20189	36
37	9291.3	.96807	3544.2	.54951	2189.8	.34041	1584.5	.19988	37
38	9046.7	.95649	3508.0	.54506	2176.0	.33765	1577.2	.19789	38
39	8814.8	.94521	3472.6	.54065	2162.3	.33491	1570.0	.19590	39
40	8594.4	.93421	3437.9	.53629	2148.8	.33219	1562.9	.19392	40
41	8384.8	3.92349	3403.8	3.53197	2135.4	3.32949	1555.8	3.19195	41
42	8185.2	.91302	3370.5	.52769	2122.3	.32680	1548.8	.18999	42
43	7994.8	.90281	3337.7	.52345	2109.2	.32412	1541.9	.18804	43
44	7813.1	.89282	3305.7	.51925	2096.4	.32147	1535.0	.18610	44
45	7639.5	.88306	3274.2	.51510	2083.7	.31883	1528.2	.18417	45
46	7473.4	3.87352	3243.3	3.51098	2071.1	3.31621	1521.4	3.18224	46
47	7314.4	.86418	3213.0	.50691	2058.7	.31360	1514.7	.18032	47
48	7162.0	.85503	3183.2	.50287	2046.5	.31101	1508.1	.17842	48
49	7015.9	.84608	3154.0	.49886	2034.4	.30843	1501.5	.17652	49
50	6875.6	.83731	3125.4	.49490	2022.4	.30587	1495.0	.17462	50
51	6740.7	3.82871	3097.2	3.49097	2010.6	3.30332	1488.5	3.17274	51
52	6611.1	.82027	3069.6	.48707	1998.9	.30079	1482.1	.17087	52
53	6486.4	.81200	3042.4	.48321	1987.3	.29827	1475.7	.16900	53
54	6366.3	.80388	3015.7	.47939	1975.9	.29577	1469.4	.16714	54
55	6250.5	.79591	2989.5	.47559	1964.6	.29328	1463.2	.16529	55
56	6138.9	3.78809	2963.7	3.47183	1953.5	3.29081	1457.0	3.16344	56
57	6031.2	.78040	2938.4	.46811	1942.4	.28835	1450.8	.16161	57
58	5927.2	.77285	2913.5	.46441	1931.5	.28590	1444.7	.15978	58
59	5826.8	.76542	2889.0	.46075	1920.7	.28347	1438.7	.15796	59
60	5729.6	.75813	2864.9	.45711	1910.1	.28105	1432.7	.15615	60

TABLE I.—RADII OF CURVES.

Deg. Min.	4° Radius.	4° Log R	5° Radius.	5° Log R	6° Radius.	6° Log R	7° Radius.	7° Log R	Deg. Min.
0	1432.7	3.15615	1146.3	3.05929	955.37	2.98017	819.02	2.91329	0
1	1426.7	.15434	1142.5	.05784	952.72	.97896	817.08	.91226	1
2	1420.8	.15255	1138.7	.05640	950.09	.97776	815.14	.91123	2
3	1415.0	.15076	1134.9	.05497	947.48	.97657	813.22	.91021	3
4	1409.2	.14897	1131.2	.05354	944.88	.97537	811.30	.90918	4
5	1403.5	.14720	1127.5	.05211	942.29	.97418	809.40	.90816	5
6	1397.8	3.14543	1123.8	3.05069	939.72	2.97300	807.50	2.90714	6
7	1392.1	.14367	1120.2	.04928	937.16	.97181	805.61	.90612	7
8	1386.5	.14191	1116.5	.04787	934.62	.97063	803.73	.90511	8
9	1380.9	.14017	1112.9	.04646	932.09	.96945	801.86	.90410	9
10	1375.4	.13843	1109.3	.04506	929.57	.96828	800.00	.90309	10
11	1369.9	3.13669	1105.8	3.04366	927.07	2.96711	798.14	2.90208	11
12	1364.5	.13497	1102.2	.04227	924.58	.96594	796.30	.90107	12
13	1359.1	:13325	1098.7	.04088	922.10	.96478	794.46	.90007	13
14	1353.8	.13154	1095.2	.03949	919.64	.96361	792.63	.89907	14
15	1348.4	.12983	1091.7	.03811	917.19	.96246	790.81	.89807	15
16	1343.2	3.12813	1088.3	3.03674	914.75	2.96130	789.00	2.89708	16
17	1338.0	.12644	1084.8	.03537	912.33	.96015	787.20	.89608	17
18	1332.8	.12475	1081.4	.03400	909.92	.95900	785.41	.89509	18
19	1327.6	.12307	1078.1	.03264	907.52	.95785	783.62	.89410	19
20	1322.5	.12140	1074.7	.03128	905.13	.95671	781.84	.89312	20
21	1317.5	3.11974	1071.3	3.02992	902.76	2.95557	780.07	2.89213	21
22	1312.4	.11808	1068.0	.02857	900.40	.95443	778.31	.89115	22
23	1307.4	.11642	1064.7	.02723	898.05	.95330	776.55	.89017	23
24	1302.5	.11477	1061.4	.02589	895.71	.95217	774.81	.88919	24
25	1297.6	.11313	1058.2	.02455	893.39	.95104	773.07	.88821	25
26	1292.7	3.11150	1054.9	3.02322	891.08	2.94991	771.34	2.88724	26
27	1287.9	.10987	1051.7	.02189	888.78	.94879	769.61	.88627	27
28	1283.1	.10825	1048.5	.02056	886.49	.94767	767.90	.88530	28
29	1278.3	.10663	1045.3	.01924	884.21	.94655	766.19	.88433	29
30	1273.6	.10502	1042.1	.01792	881.95	.94544	764.49	.88337	30
31	1268.9	3.10341	1039.0	3.01661	879.69	2.94433	762.80	2.88241	31
32	1264.2	.10182	1035.9	.01530	877.45	.94322	761.11	.88145	32
33	1259.6	.10022	1032.8	.01400	875.22	.94212	759.43	.88049	33
34	1255.0	.09864	1029.7	.01270	873.00	.94101	757.76	.87953	34
35	1250.4	.09705	1026.6	.01140	870.80	.93991	756.10	.87858	35
36	1245.9	3.09548	1023.5	3.01010	868.60	2.93882	754.44	2.87762	36
37	1241.4	.09391	1020.5	.00882	866.41	.93772	752.80	.87668	37
38	1236.9	.09234	1017.5	.00753	864.24	.93663	751.16	.87573	38
39	1232.5	.09079	1014.5	.00625	862.07	.93554	749.52	.87478	39
40	1228.1	.08923	1011.5	.00497	859.92	.93446	747.89	.87384	40
41	1223.7	3.08769	1008.6	3.00370	857.78	2.93337	746.27	2.87290	41
42	1219.4	.08614	1005.6	.00243	855.65	.93229	744.66	.87196	42
43	1215.1	.08461	1002.7	3.00116	853.53	.93122	743.06	.87102	43
44	1210.8	.08308	999.76	2.99989	851.42	.93014	741.46	.87008	44
45	1206.6	.08155	996.87	.99863	849.32	.92907	739.86	.86915	45
46	1202.4	3.08003	993.99	2.99738	847.23	2.92800	738.28	2.86822	46
47	1198.2	.07852	991.13	.99613	845.15	.92693	736.70	.86729	47
48	1194.0	.07701	988.28	.99488	843.08	.92587	735.13	.86636	48
49	1189.9	.07550	985.45	.99363	841.02	.92480	733.56	.86544	49
50	1185.8	.07400	982.64	.99239	838.97	.92374	732.01	.86451	50
51	1181.7	3.07251	979.84	2.99115	836.93	2.92269	730.45	2.86359	51
52	1177.7	.07102	977.06	.98992	834.90	.92163	728.91	.86267	52
53	1173.6	.06954	974.29	.98869	832.89	.92058	727.37	.86175	53
54	1169.7	.06806	971.54	.98746	830.88	.91953	725.84	.86084	54
55	1165.7	.06658	968.81	.98624	828.88	.91849	724.31	.85992	55
56	1161.8	3.06511	966.09	2.98501	826.89	2.91744	722.79	2.85901	56
57	1157.9	.06365	963.39	.98380	824.91	.91640	721.28	.85810	57
58	1154.0	.06219	960.70	.98258	822.93	.91536	719.77	.85719	58
59	1150.1	.06074	958.03	.98137	820.97	.91433	718.27	.85629	59
60	1146.3	.05929	955.37	.98017	819.02	.91329	716.78	.85538	60

TABLE I.—RADII OF CURVES.

Deg.	8°		9°		10°		11°		Deg.
Min.	Radius.	Log R	Radius.	Log R	Radius.	Log R	Radius.	Log R	Min.
0	716.78	2.85538	637.27	2.80432	573.69	2.75867	521.67	2.71739	0
1	715.29	.85448	636.10	.80352	572.73	.75795	520.88	.71674	1
2	713.81	.85358	634.93	.80272	571.78	.75723	520.10	.71608	2
3	712.34	.85268	633.76	.80192	570.84	.75651	519.32	.71543	3
4	710.87	.85178	632.60	.80113	569.90	.75579	518.54	.71478	4
5	709.40	.85089	631.44	.80033	568.96	.75508	517.76	.71413	5
6	707.95	2.85000	630.29	2.79954	568.02	2.75436	516.99	2.71348	6
7	706.49	.84911	629.14	.79874	567.09	.75365	516.21	.71283	7
8	705.05	.84822	627.99	.79795	566.16	.75293	515.44	.71218	8
9	703.61	.84733	626.85	.79716	565.23	.75222	514.68	.71153	9
10	702.17	.84644	625.71	.79637	564.31	.75151	513.91	.71088	10
11	700.75	2.84556	624.58	2.79558	563.38	2.75080	513.15	2.71024	11
12	699.33	.84468	623.45	.79480	562.47	.75009	512.38	.70959	12
13	697.91	.84380	622.32	.79401	561.55	.74939	511.63	.70895	13
14	696.50	.84292	621.20	.79323	560.64	.74868	510.87	.70831	14
15	695.09	.84204	620.09	.79245	559.73	.74798	510.11	.70767	15
16	693.70	2.84117	618.97	2.79167	558.82	2.74727	509.36	2.70702	16
17	692.30	.84029	617.87	.79089	557.92	.74657	508.61	.70638	17
18	690.91	.83942	616.76	.79011	557.02	.74587	507.86	.70575	18
19	689.53	.83855	615.66	.78934	556.12	.74517	507.12	.70511	19
20	688.16	.83768	614.56	.78856	555.23	.74447	506.38	.70447	20
21	686.78	2.83682	613.47	2.78779	554.34	2.74377	505.64	2.70383	21
22	685.42	.83595	612.38	.78702	553.45	.74307	504.90	.70320	22
23	684.06	.83509	611.30	.78625	552.56	.74238	504.16	.70257	23
24	682.70	.83423	610.21	.78548	551.68	.74168	503.42	.70193	24
25	681.35	.83337	609.14	.78471	550.80	.74099	502.69	.70130	25
26	680.01	2.83251	608.06	2.78395	549.92	2.74030	501.96	2.70067	26
27	678.67	.83166	606.99	.78318	549.05	.73961	501.23	.70004	27
28	677.34	.83080	605.93	.78242	548.17	.73892	500.51	.69941	28
29	676.01	.82995	604.86	.78165	547.30	.73823	499.78	.69878	29
30	674.69	.82910	603.80	.78089	546.44	.73754	499.06	.69815	30
31	673.37	2.82825	602.75	2.78013	545.57	2.73685	498.34	2.69752	31
32	672.06	.82740	601.70	.77938	544.71	.73617	497.62	.69690	32
33	670.75	.82656	600.65	.77862	543.86	.73548	496.91	.69627	33
34	669.45	.82571	599.61	.77786	543.00	.73480	496.19	.69565	34
35	668.15	.82487	598.57	.77711	542.15	.73412	495.48	.69503	35
36	666.86	2.82403	597.53	2.77636	541.30	2.73343	494.77	2.69440	36
37	665.57	.82319	596.50	.77561	540.45	.73275	494.07	.69378	37
38	664.29	.82235	595.47	.77486	539.61	.73207	493.36	.69316	38
39	663.01	.82152	594.44	.77411	538.76	.73140	492.66	.69254	39
40	661.74	.82068	593.42	.77336	537.92	.73072	491.96	.69192	40
41	660.47	2.81985	592.40	2.77261	537.09	2.73004	491.26	2.69131	41
42	659.21	.81902	591.38	.77187	536.25	.72937	490.56	.69069	42
43	657.95	.81819	590.37	.77112	535.42	.72869	489.86	.69007	43
44	656.69	.81736	589.36	.77038	534.59	.72802	489.17	.68946	44
45	655.45	.81653	588.36	.76964	533.77	.72735	488.48	.68884	45
46	654.20	2.81571	587.36	2.76890	532.94	2.72668	487.79	2.68823	46
47	652.96	.81489	586.36	.76816	532.12	.72601	487.10	.68762	47
48	651.73	.81406	585.36	.76742	531.30	.72534	486.42	.68701	48
49	650.50	.81324	584.37	.76669	530.49	.72467	485.73	.68640	49
50	649.27	.81243	583.38	.76595	529.67	.72401	485.05	.68579	50
51	648.05	2.81161	582.40	2.76522	528.86	2.72334	484.37	2.68518	51
52	646.84	.81079	581.42	.76449	528.05	.72267	483.69	.68457	52
53	645.63	.80998	580.44	.76376	527.25	.72201	483.02	.68396	53
54	644.42	.80917	579.47	.76303	526.44	.72135	482.34	.68335	54
55	643.22	.80836	578.49	.76230	525.64	.72069	481.67	.68275	55
56	642.02	2.80755	577.53	2.76157	524.84	2.72003	481.00	2.68214	56
57	640.83	.80674	576.56	.76084	524.05	.71937	480.33	.68154	57
58	639.64	.80593	575.60	.76012	523.25	.71871	479.67	.68094	58
59	638.45	.80513	574.64	.75939	522.46	.71805	479.00	.68033	59
60	637.27	.80432	573.69	.75867	521.67	.71739	478.34	.67973	60

TABLE I.—RADII OF CURVES.

Deg.	Radius.	Log R	Deg.	Radius.	Log R	Deg.	Radius.	Log R	Deg.	Radius.	Log R
12°	478.34	2.67973	14°	410.28	2.61307	16°	359.26	2.55541	21°	274.37	2.43833
2	477.02	.67853	2	409.31	.61205	5	357.42	.55317	10	272.23	.43494
4	475.71	.67734	4	408.31	.61102	10	355.59	.55094	20	270.13	.43157
6	474.40	.67614	6	407.38	.61000	15	353.77	.54872	30	268.06	.42823
8	473.10	.67495	8	406.42	.60898	20	351.98	.54652	40	266.02	.42492
10	471.81	2.67376	10	405.47	2.60796	25	350.21	.54432	50	264.02	.42163
12	470.53	.67258	12	404.53	.60694	30	348.45	2.54214	22°	262.04	2.41837
14	469.25	.67140	14	403.58	.60593	35	346.71	.53997	10	260.10	.41513
16	467.98	.67022	16	402.65	.60492	40	344.99	.53780	20	258.18	.41192
18	466.72	.66905	18	401.71	.60391	45	343.29	.53565	30	256.29	.40873
20	465.46	2.66788	20	400.78	2.60291	50	341.60	.53351	40	254.43	.40557
22	464.21	.66671	22	399.86	.60190	55	339.93	.53138	50	252.60	.40243
24	462.97	.66555	24	398.94	.60090	17°	338.27	2.52927	23°	250.79	2.39931
26	461.73	.66439	26	398.02	.59990	5	336.64	.52716	10	249.01	.39622
28	460.50	.66323	28	397.11	.59891	10	335.01	.52506	20	247.26	.39315
30	459.28	2.66207	30	396.20	2.59791	15	333.41	.52297	30	245.53	.39010
32	458.06	.66092	32	395.30	.59692	20	331.82	.52090	40	243.82	.38707
34	456.85	.65977	34	394.40	.59593	25	330.24	.51883	50	242.14	.38407
36	455.65	.65863	36	393.50	.59494	30	328.68	2.51677	24°	240.49	2.38109
38	454.45	.65748	38	392.61	.59396	35	327.13	.51472	10	238.85	.37813
40	453.26	2.65634	40	391.72	2.59298	40	325.60	.51269	20	237.24	.37519
42	452.07	.65521	42	390.84	.59199	45	324.09	.51066	30	235.65	.37227
44	450.89	.65407	44	389.96	.59102	50	322.59	.50864	40	234.08	.36937
46	449.72	.65294	46	389.08	.59004	55	321.10	.50663	50	232.54	.36649
48	448.56	.65181	48	388.21	.58907	18°	319.62	2.50464	25°	231.01	2.36363
50	447.40	2.65069	50	387.34	2.58809	5	318.16	.50265	30	226.55	.35517
52	446.24	.64957	52	386.48	.58713	10	316.71	.50067	26°	222.27	.34688
54	445.09	.64845	54	385.62	.58616	15	315.28	.49869	30	218.15	.33875
56	443.95	.64733	56	384.77	.58519	20	313.86	.49673	27°	214.18	2.33078
58	442.81	.64622	58	383.91	.58423	25	312.45	.49478	30	210.36	.32296
13°	441.68	2.64511	15°	383.06	2.58327	30	311.06	2.49284	28°	206.68	.31529
2	440.56	.64400	2	382.22	.58231	35	309.67	.49090	30	203.13	.30776
4	439.44	.64290	4	381.38	.58135	40	308.30	.48898	29°	199.70	2.30037
6	438.33	.64180	6	380.54	.58040	45	306.95	.48706	30	196.38	.29310
8	437.22	.64070	8	379.71	.57945	50	305.60	.48515	30°	193.19	.28597
10	436.12	2.63960	10	378.88	2.57850	55	304.27	.48325	30	190.09	.27896
12	435.02	.63851	12	378.05	.57755	19°	302.94	2.48136	31°	187.10	2.27207
14	433.93	.63742	14	377.23	.57661	5	301.63	.47948	32	181.40	.25863
16	432.84	.63633	16	376.41	.57566	10	300.33	.47760	33	176.05	.24563
18	431.76	.63524	18	375.60	.57472	15	299.04	.47573	34	171.02	.23303
20	430.68	2.63416	20	374.79	2.57378	20	297.77	.47388	35	166.28	.22083
22	429.62	.63308	22	373.98	.57284	25	296.50	.47203	36	161.80	2.20899
24	428.56	.63201	24	373.17	.57191	30	295.25	2.47018	37	157.58	.19749
26	427.50	.63093	26	372.37	.57097	35	294.00	.46835	38	153.58	.18633
28	426.44	.62986	28	371.57	.57004	40	292.77	.46652	39	149.79	.17547
30	425.40	2.62879	30	370.78	2.56911	45	291.55	.46471	40	146.19	.16492
32	424.35	.62773	32	369.99	.56819	50	290.33	.46289	41	142.77	2.15464
34	423.32	.62666	34	369.20	.56726	55	289.13	.46109	42	139.52	.14464
36	422.28	.62560	36	368.42	.56634	20°	287.94	2.45930	43	136.43	.13489
38	421.26	.62454	38	367.64	.56542	5	286.76	.45751	44	133.47	.12539
40	420.23	2.62349	40	366.86	2.56450	10	285.58	.45573	45	130.66	.11613
42	419.22	.62243	42	366.09	.56358	15	284.42	.45396	46	127.97	2.10709
44	418.20	.62138	44	365.31	.56266	20	283.27	.45219	47	125.39	.09827
46	417.19	.62034	46	364.55	.56175	25	282.12	.45044	48	122.93	.08965
48	416.19	.61929	48	363.78	.56084	30	280.99	2.44869	49	120.57	.08124
50	415.19	2.61825	50	363.02	2.55993	35	279.86	.44694	50	118.31	.07302
52	414.20	.61721	52	362.26	.55902	40	278.75	.44521	52	114.06	2.05713
54	413.21	.61617	54	361.51	.55812	45	277.64	.44348	54	110.13	.04192
56	412.23	.61514	56	360.76	.55721	50	276.54	.44176	56	106.50	.02736
58	411.25	.61410	58	360.01	.55631	55	275.45	.44004	58	103.13	.01340
14°	410.28	2.61307	16°	359.26	2.55541	21°	274.37	2.43833	60	100.00	2.00000

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TABLE II.—TANGENTS, EXTERNAL DISTANCES, AND LONG CHORDS FOR A 1° CURVE.

Δ	Tangent T.	Ext. Dist. E.	LongCh'd LC.	Δ	Tangent T.	Ext. Dist. E.	LongCh'd LC.	Δ	Tangent T.	Ext. Dist. E.	LongCh'd LC.
1°	50.00	0.218	100.00	11°	551.70	26.500	1098.3	21°	1061.9	97.58	2088.3
10'	58.34	0.297	116.67	10	560.11	27.313	1114.9	10	1070.6	99.15	2104.7
20	66.67	0.388	133.33	20	568.53	28.137	1131.5	20	1079.2	100.75	2121.1
30	75.01	0.491	150.00	30	576.95	28.974	1148.1	30	1087.8	102.35	2137.4
40	83.34	0.606	166.66	40	585.36	29.824	1164.7	40	1096.4	103.97	2153.8
50	91.68	0.733	183.33	50	593.79	30.686	1181.2	50	1105.1	105.60	2170.2
2°	100.01	0.873	199.99	12°	602.21	31.561	1197.8	22°	1113.7	107.24	2186.5
10	108.35	1.024	216.66	10	610.64	32.447	1214.4	10	1122.4	108.90	2202.9
20	116.68	1.188	233.32	20	619.07	33.347	1231.0	20	1131.0	110.57	2219.2
30	125.02	1.364	249.98	30	627.50	34.259	1247.5	30	1139.7	112.25	2235.6
40	133.36	1.552	266.65	40	635.93	35.183	1264.1	40	1148.4	113.95	2251.9
50	141.70	1.752	283.31	50	644.37	36.120	1280.7	50	1157.0	115.66	2268.3
3°	150.04	1.964	299.97	13°	652.81	37.069	1297.2	23°	1165.7	117.38	2284.6
10	158.38	2.188	316.63	10	661.25	38.031	1313.8	10	1174.4	119.12	2301.0
20	166.72	2.425	333.29	20	669.70	39.006	1330.3	20	1183.1	120.87	2317.3
30	175.06	2.674	349.95	30	678.15	39.993	1346.9	30	1191.8	122.63	2333.6
40	183.40	2.934	366.61	40	686.60	40.992	1363.4	40	1200.5	124.41	2349.9
50	191.74	3.207	383.27	50	695.06	42.004	1380.0	50	1209.2	126.20	2366.2
4°	200.08	3.492	399.92	14°	703.51	43.029	1396.5	24°	1217.9	128.00	2382.5
10	208.43	3.790	416.58	10	711.97	44.066	1413.1	10	1226.6	129.82	2398.8
20	216.77	4.099	433.24	20	720.44	45.116	1429.6	20	1235.3	131.65	2415.1
30	225.12	4.421	449.89	30	728.90	46.178	1446.2	30	1244.0	133.50	2431.4
40	233.47	4.755	466.54	40	737.37	47.253	1462.7	40	1252.8	135.36	2447.7
50	241.81	5.100	483.20	50	745.85	48.341	1479.2	50	1261.5	137.23	2464.0
5°	250.16	5.459	499.85	15°	754.32	49.441	1495.7	25°	1270.2	139.11	2480.2
10	258.51	5.829	516.50	10	762.80	50.554	1512.3	10	1279.0	141.01	2496.5
20	266.86	6.211	533.15	20	771.29	51.679	1528.8	20	1287.7	142.93	2512.8
30	275.21	6.606	549.80	30	779.77	52.818	1545.3	30	1296.5	144.85	2529.0
40	283.57	7.013	566.44	40	788.26	53.969	1561.8	40	1305.3	146.79	2545.3
50	291.92	7.432	583.09	50	796.75	55.132	1578.3	50	1314.0	148.75	2561.5
6°	300.28	7.863	599.73	16°	805.25	56.309	1594.8	26°	1322.8	150.71	2577.8
10	308.64	8.307	616.38	10	813.75	57.498	1611.3	10	1331.6	152.69	2594.0
20	316.99	8.762	633.02	20	822.25	58.699	1627.8	20	1340.4	154.69	2610.3
30	325.35	9.230	649.66	30	830.76	59.914	1644.3	30	1349.2	156.70	2626.5
40	333.71	9.710	666.30	40	839.27	61.141	1660.8	40	1358.0	158.72	2642.7
50	342.08	10.202	682.94	50	847.78	62.381	1677.3	50	1366.8	160.76	2658.9
7°	350.44	10.707	699.57	17°	856.30	63.634	1693.8	27°	1375.6	162.81	2675.1
10	358.81	11.224	716.21	10	864.82	64.900	1710.3	10	1384.4	164.87	2691.3
20	367.17	11.753	732.84	20	873.35	66.178	1726.8	20	1393.2	166.95	2707.5
30	375.54	12.294	749.47	30	881.88	67.470	1743.2	30	1402.0	169.04	2723.7
40	383.91	12.847	766.10	40	890.41	68.774	1759.7	40	1410.9	171.15	2739.9
50	392.28	13.413	782.73	50	898.95	70.091	1776.2	50	1419.7	173.27	2756.1
8°	400.66	13.991	799.36	18°	907.49	71.421	1792.6	28°	1428.6	175.41	2772.3
10	409.03	14.582	815.99	10	916.03	72.764	1809.1	10	1437.4	177.55	2788.4
20	417.41	15.184	832.61	20	924.58	74.119	1825.5	20	1446.3	179.72	2804.6
30	425.79	15.799	849.23	30	933.13	75.488	1842.0	30	1455.1	181.89	2820.7
40	434.17	16.426	865.85	40	941.69	76.869	1858.4	40	1464.0	184.08	2836.9
50	442.55	17.066	882.47	50	950.25	78.264	1874.9	50	1472.9	186.29	2853.0
9°	450.93	17.717	899.09	19°	958.81	79.671	1891.3	29°	1481.8	188.51	2869.2
10	459.32	18.381	915.70	10	967.38	81.092	1907.8	10	1490.7	190.74	2885.3
20	467.71	19.058	932.31	20	975.96	82.525	1924.2	20	1499.6	192.99	2901.4
30	476.10	19.746	948.92	30	984.53	83.972	1940.6	30	1508.5	195.25	2917.6
40	484.49	20.447	965.53	40	993.12	85.431	1957.1	40	1517.4	197.53	2933.7
50	492.88	21.161	982.14	50	1001.70	86.904	1973.5	50	1526.3	199.82	2949.8
10°	501.28	21.886	998.74	20°	1010.29	88.389	1989.9	30°	1535.3	202.12	2965.9
10	509.68	22.624	1015.35	10	1018.89	89.888	2006.3	10	1544.2	204.44	2982.0
20	518.08	23.375	1031.95	20	1027.49	91.399	2022.7	20	1553.1	206.77	2998.1
30	526.48	24.138	1048.54	30	1036.09	92.924	2039.1	30	1562.1	209.12	3014.2
40	534.89	24.913	1065.14	40	1044.70	94.462	2055.5	40	1571.0	211.48	3030.2
50	543.29	25.700	1081.73	50	1053.31	96.013	2071.9	50	1580.0	213.86	3046.3
11°	551.70	26.500	1098.33	21°	1061.93	97.577	2088.3	31°	1589.0	216.25	3062.4

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TABLE II.—TANGENTS, EXTERNAL DISTANCES, AND LONG CHORDS FOR A 1° CURVE.

Δ	Tangent T.	Ext. Dist. E.	LongCh'd LC.	Δ	Tangent T.	Ext.Dist. E.	LongCh'd LC.	Δ	Tangent T.	Ext.Dist. E.	LongCh'd LC.
31°	1589.0	216.25	3062.4	41°	2142.2	387.38	4013.1	51°	2732.9	618.39	4933.4
10'	1598.0	218.66	3078.4	10	2151.7	390.71	4028.7	10	2743.1	622.81	4948.4
20	1606.9	221.08	3094.5	20	2161.2	394.06	4044.3	20	2753.4	627.24	4963.4
30	1615.9	223.51	3110.5	30	2170.8	397.43	4059.9	30	2763.7	631.69	4978.4
40	1624.9	225.96	3126.6	40	2180.3	400.82	4075.5	40	2773.9	636.16	4993.4
50	1633.9	228.42	3142.6	50	2189.9	404.22	4091.1	50	2784.2	640.66	5008.4
32°	1643.0	230.90	3158.6	42°	2199.4	407.64	4106.6	52°	2794.5	645.17	5023.4
10	1652.0	233.39	3174.6	10	2209.0	411.07	4122.2	10	2804.9	649.70	5038.4
20	1661.0	235.90	3190.6	20	2218.6	414.52	4137.7	20	2815.2	654.25	5053.4
30	1670.0	238.43	3206.6	30	2228.1	417.99	4153.3	30	2825.6	658.83	5068.3
40	1679.1	240.96	3222.6	40	2237.7	421.48	4168.8	40	2835.9	663.42	5083.3
50	1688.1	243.52	3238.6	50	2247.3	424.98	4184.3	50	2846.3	668.03	5098.2
33°	1697.2	246.08	3254.6	43°	2257.0	428.50	4199.8	53°	2856.7	672.66	5113.1
10	1706.3	248.66	3270.6	10	2266.6	432.04	4215.3	10	2867.1	677.32	5128.0
20	1715.3	251.26	3286.6	20	2276.2	435.59	4230.8	20	2877.5	681.99	5142.9
30	1724.4	253.87	3302.5	30	2285.9	439.16	4246.3	30	2888.0	686.68	5157.8
40	1733.5	256.50	3318.5	40	2295.6	442.75	4261.8	40	2898.4	691.40	5172.7
50	1742.6	259.14	3334.4	50	2305.2	446.35	4277.3	50	2908.9	696.13	5187.6
34°	1751.7	261.80	3350.4	44°	2314.9	449.98	4292.7	54°	2919.4	700.89	5202.4
10	1760.8	264.47	3366.3	10	2324.6	453.62	4308.2	10	2929.9	705.66	5217.3
20	1770.0	267.16	3382.2	20	2334.3	457.27	4323.6	20	2940.4	710.46	5232.1
30	1779.1	269.86	3398.2	30	2344.1	460.95	4339.0	30	2951.0	715.28	5246.9
40	1788.2	272.58	3414.1	40	2353.8	464.64	4354.5	40	2961.5	720.11	5261.7
50	1797.4	275.31	3430.0	50	2363.5	468.35	4369.9	50	2972.1	724.97	5276.5
35°	1806.6	278.05	3445.9	45°	2373.3	472.08	4385.3	55°	2982.7	729.85	5291.3
10	1815.7	280.82	3461.8	10	2383.1	475.82	4400.7	10	2993.3	734.76	5306.1
20	1824.9	283.60	3477.7	20	2392.8	479.59	4416.1	20	3003.9	739.68	5320.9
30	1834.1	286.39	3493.5	30	2402.6	483.37	4431.4	30	3014.5	744.62	5335.6
40	1843.3	289.20	3509.4	40	2412.4	487.16	4446.8	40	3025.2	749.59	5350.4
50	1852.5	292.02	3525.3	50	2422.3	490.98	4462.2	50	3035.8	754.57	5365.1
36°	1861.7	294.86	3541.1	46°	2432.1	494.82	4477.5	56°	3046.5	759.58	5379.8
10	1870.9	297.72	3557.0	10	2441.9	498.67	4492.8	10	3057.2	764.61	5394.5
20	1880.1	300.59	3572.8	20	2451.8	502.54	4508.2	20	3067.9	769.66	5409.2
30	1889.4	303.47	3588.6	30	2461.7	506.42	4523.5	30	3078.7	774.73	5423.9
40	1898.6	306.37	3604.5	40	2471.5	510.33	4538.8	40	3089.4	779.83	5438.6
50	1907.9	309.29	3620.3	50	2481.4	514.25	4554.1	50	3100.2	784.94	5453.3
37°	1917.1	312.22	3636.1	47°	2491.3	518.20	4569.4	57°	3110.9	790.08	5467.9
10	1926.4	315.17	3651.9	10	2501.2	522.16	4584.7	10	3121.7	795.24	5482.5
20	1935.7	318.13	3667.7	20	2511.2	526.13	4599.9	20	3132.6	800.42	5497.2
30	1945.0	321.11	3683.5	30	2521.1	530.13	4615.2	30	3143.4	805.62	5511.8
40	1954.3	324.11	3699.3	40	2531.1	534.15	4630.4	40	3154.2	810.85	5526.4
50	1963.6	327.12	3715.0	50	2541.0	538.18	4645.7	50	3165.1	816.10	5541.0
38°	1972.9	330.15	3730.8	48°	2551.0	542.23	4660.9	58°	3176.0	821.37	5555.6
10	1982.2	333.19	3746.5	10	2561.0	546.30	4676.1	10	3186.9	826.66	5570.2
20	1991.5	336.25	3762.3	20	2571.0	550.39	4691.3	20	3197.8	831.98	5584.7
30	2000.9	339.32	3778.0	30	2581.0	554.50	4706.5	30	3208.8	837.31	5599.3
40	2010.2	342.41	3793.8	40	2591.1	558.63	4721.7	40	3219.7	842.67	5613.8
50	2019.6	345.52	3809.5	50	2601.1	562.77	4736.9	50	3230.7	848.06	5628.3
39°	2029.0	348.64	3825.2	49°	2611.2	566.94	4752.1	59°	3241.7	853.46	5642.8
10	2038.4	351.78	3840.9	10	2621.2	571.12	4767.3	10	3252.7	858.89	5657.3
20	2047.8	354.94	3856.6	20	2631.3	575.32	4782.4	20	3263.7	864.34	5671.8
30	2057.2	358.11	3872.3	30	2641.4	579.54	4797.5	30	3274.8	869.82	5686.3
40	2066.6	361.29	3888.0	40	2651.5	583.78	4812.7	40	3285.8	875.32	5700.8
50	2076.0	364.50	3903.6	50	2661.6	588.04	4827.8	50	3296.9	880.84	5715.2
40°	2085.4	367.72	3919.3	50°	2671.8	592.32	4842.9	60°	3308.0	886.38	5729.7
10	2094.9	370.95	3935.0	10	2681.9	596.62	4858.0	10	3319.1	891.95	5744.1
20	2104.3	374.20	3950.6	20	2692.1	600.93	4873.1	20	3330.3	897.54	5758.5
30	2113.8	377.47	3966.3	30	2702.3	605.27	4888.2	30	3341.4	903.15	5772.9
40	2123.3	380.76	3981.9	40	2712.5	609.62	4903.2	40	3352.6	908.79	5787.3
50	2132.7	384.06	3997.5	50	2722.7	614.00	4918.3	50	3363.8	914.45	5801.7
41°	2142.2	387.38	4013.1	51°	2732.9	618.39	4933.4	61°	3375.0	920.14	5816.0

TABLE II.—TANGENTS, EXTERNAL DISTANCES, AND LONG CHORDS FOR A 1° CURVE.

Δ	Tangent T.	Ext. Dist. E.	LongCh'd LC.	Δ	Tangent T.	Ext. Dist. E.	LongCh'd LC.	Δ	Tangent T.	Ext. Dist. E.	LongCh'd LC.
61°	3375.0	920.14	5816.0	71°	4086.9	1308.2	6654.4	81°	4893.6	1805.3	7442.2
10'	3386.3	925.85	5830.4	10	4099.5	1315.5	6668.0	10	4908.0	1814.7	7454.9
20	3397.5	931.58	5844.7	20	4112.1	1322.9	6681.6	20	4922.5	1824.1	7467.5
30	3408.8	937.34	5859.1	30	4124.8	1330.3	6695.1	30	4937.0	1833.6	7480.2
40	3420.1	943.12	5873.4	40	4137.4	1337.7	6708.6	40	4951.5	1843.1	7492.8
50	3431.4	948.92	5887.7	50	4150.1	1345.2	6722.1	50	4966.1	1852.6	7505.4
62°	3442.7	954.75	5902.0	72°	4162.8	1352.6	6735.6	82°	4980.7	1862.2	7518.0
10	3454.1	960.60	5916.3	10	4175.6	1360.1	6749.1	10	4995.4	1871.8	7530.5
20	3465.4	966.48	5930.5	20	4188.4	1367.6	6762.5	20	5010.0	1881.5	7543.1
30	3476.8	972.39	5944.8	30	4201.2	1375.2	6776.0	30	5024.8	1891.2	7555.6
40	3488.2	978.31	5959.0	40	4214.0	1382.8	6789.4	40	5039.5	1900.9	7568.2
50	3499.7	984.27	5973.3	50	4226.8	1390.4	6802.8	50	5054.3	1910.7	7580.7
63°	3511.1	990.24	5987.5	73°	4239.7	1398.0	6816.3	83°	5069.2	1920.5	7593.2
10	3522.6	996.24	6001.7	10	4252.6	1405.7	6829.6	10	5084.0	1930.4	7605.6
20	3534.1	1002.3	6015.9	20	4265.6	1413.5	6843.0	20	5099.0	1940.3	7618.1
30	3545.6	1008.3	6030.0	30	4278.5	1421.2	6856.4	30	5113.9	1950.2	7630.5
40	3557.2	1014.4	6044.2	40	4291.5	1429.0	6869.7	40	5128.9	1960.2	7643.0
50	3568.7	1020.5	6058.4	50	4304.6	1436.8	6883.1	50	5143.9	1970.3	7655.4
64°	3580.3	1026.6	6072.5	74°	4317.6	1444.6	6896.4	84°	5159.0	1980.4	7667.8
10	3591.9	1032.8	6086.6	10	4330.7	1452.5	6909.7	10	5174.1	1990.5	7680.1
20	3603.5	1039.0	6100.7	20	4343.8	1460.4	6923.0	20	5189.3	2000.6	7692.5
30	3615.1	1045.2	6114.8	30	4356.9	1468.4	6936.2	30	5204.4	2010.8	7704.9
40	3626.8	1051.4	6128.9	40	4370.1	1476.4	6949.5	40	5219.7	2021.1	7717.2
50	3638.5	1057.7	6143.0	50	4383.3	1484.4	6962.8	50	5234.9	2031.4	7729.5
65°	3650.2	1063.9	6157.1	75°	4396.5	1492.4	6976.0	85°	5250.3	2041.7	7741.8
10	3661.9	1070.2	6171.1	10	4409.8	1500.5	6989.2	10	5265.6	2052.1	7754.1
20	3673.7	1076.6	6185.2	20	4423.1	1508.6	7002.4	20	5281.0	2062.5	7766.3
30	3685.4	1082.9	6199.2	30	4436.4	1516.7	7015.6	30	5296.4	2073.0	7778.6
40	3697.2	1089.3	6213.2	40	4449.7	1524.9	7028.8	40	5311.9	2083.5	7790.8
50	3709.0	1095.7	6227.2	50	4463.1	1533.1	7041.9	50	5327.4	2094.1	7803.0
66°	3720.9	1102.2	6241.2	76°	4476.5	1541.4	7055.0	86°	5343.0	2104.7	7815.2
10	3732.7	1108.6	6255.2	10	4489.7	1549.7	7068.2	10	5358.6	2115.3	7827.4
20	3744.6	1115.1	6269.1	20	4503.4	1558.0	7081.3	20	5374.2	2126.0	7839.6
30	3756.5	1121.7	6283.1	30	4516.9	1566.3	7094.4	30	5389.9	2136.7	7851.7
40	3768.5	1128.2	6297.0	40	4530.4	1574.7	7107.5	40	5405.6	2147.5	7863.8
50	3780.4	1134.8	6310.9	50	4544.0	1583.1	7120.5	50	5421.4	2158.4	7876.0
67°	3792.4	1141.4	6324.8	77°	4557.6	1591.6	7133.6	87°	5437.2	2169.2	7888.1
10	3804.4	1148.0	6338.7	10	4571.2	1600.1	7146.6	10	5453.1	2180.2	7900.1
20	3816.4	1154.7	6352.6	20	4584.8	1608.6	7159.6	20	5469.0	2191.1	7912.2
30	3828.4	1161.3	6366.4	30	4598.5	1617.1	7172.6	30	5484.9	2202.2	7924.3
40	3840.5	1168.1	6380.3	40	4612.2	1625.7	7185.6	40	5500.9	2213.2	7936.3
50	3852.6	1174.8	6394.1	50	4626.0	1634.4	7198.5	50	5517.0	2224.3	7948.3
68°	3864.7	1181.6	6408.0	78°	4639.8	1643.0	7211.6	88°	5533.1	2235.5	7960.3
10	3876.8	1188.4	6421.8	10	4653.6	1651.7	7224.5	10	5549.2	2246.7	7972.3
20	3889.0	1195.2	6435.6	20	4667.4	1660.4	7237.4	20	5565.4	2258.0	7984.2
30	3901.2	1202.0	6449.4	30	4681.3	1669.2	7250.4	30	5581.6	2269.3	7996.2
40	3913.4	1208.9	6463.1	40	4695.2	1678.1	7263.3	40	5597.8	2280.6	8008.1
50	3925.6	1215.8	6476.9	50	4709.2	1686.9	7276.1	50	5614.2	2292.0	8020.0
69°	3937.9	1222.7	6490.6	79°	4723.2	1695.8	7289.0	89°	5630.5	2303.5	8031.9
10	3950.2	1229.7	6504.4	10	4737.2	1704.7	7301.9	10	5646.9	2315.0	8043.8
20	3962.5	1236.7	6518.1	20	4751.2	1713.7	7314.7	20	5663.4	2326.6	8055.7
30	3974.8	1243.7	6531.8	30	4765.3	1722.7	7327.5	30	5679.9	2338.2	8067.5
40	3987.2	1250.8	6545.5	40	4779.4	1731.7	7340.3	40	5696.4	2349.8	8079.3
50	3999.5	1257.9	6559.1	50	4793.6	1740.8	7353.1	50	5713.0	2361.5	8091.2
70°	4011.9	1265.0	6572.8	80°	4808.7	1749.9	7365.9	90°	5729.7	2373.3	8103.0
10	4024.4	1272.1	6586.4	10	4822.0	1759.0	7378.7	10	5746.3	2385.1	8114.7
20	4036.8	1279.3	6600.1	20	4836.2	1768.2	7391.4	20	5763.1	2397.0	8126.5
30	4049.3	1286.5	6613.7	30	4850.5	1777.4	7404.1	30	5779.9	2408.9	8138.2
40	4061.8	1293.7	6627.3	40	4864.8	1786.7	7416.8	40	5796.7	2420.9	8150.0
50	4074.4	1300.9	6640.9	50	4879.2	1796.0	7429.5	50	5813.6	2432.9	8161.7
71°	4086.9	1308.2	6654.4	81°	4893.6	1805.3	7442.2	91°	5830.5	2444.9	8173.4

TABLE III.—SWITCH LEADS AND DISTANCES.

LEAD-RAILS CIRCULAR THROUGHOUT; GAUGE 4' 8½". See § 262.

Frog Number (n).	Frog Angle (F)			Lead (L) (Eq. 79).	Chord (QT) (Eq. 77).	Radius of Lead Rails (r, Eq. 78).	Log r.	Degree of Curve (d).		Frog Number (n).
4	14°	15′	00″	37.67	37.38	150.67	2.17801̂	38°	46′	4
4.5	12	40	59	42.37	42.12	190.69	.28032	30	24	4.5
5	11	25	16	47.08	46.85	235.42	.37183̂	24	32	5
5.5	10	23	20	51.79	51.58	284.85	.45462	20	13	5 5
6	9	31	38	56.50	56.30	339.00	.53020	16	58	6
6.5	8	47	51	61.21	61.03	397.85	.59972	14	26	6.5
7	8	10	16	65.92	65.75	461.42	.66409̂	12	26	7
7.5	7	37	41	70.62	70.47	529.69	.72402	10	50	7.5
8	7	09	10	75.33	75.19	602.67	.78007̂	9	31	8
8.5	6	43	59	80.04	79.90	680.36	.83273̂	8	26	8.5
9	6	21	35	84.75	84.62	762.75	.88238	7	31	9
9.5	6	01	32	89.46	89.33	849.85	.92934̂	6	45	9.5
10	5	43	29	94.17	94.05	941.67	2.98364	6	05	10
10.5	5	27	09	98.87	98.76	1038.19	3.01627	5	32	10.5
11	5	12	18	103.58	103.47	1139.42	.05668̂	5	02	11
11.5	4	58	45	108.29	108.19	1245.36	.09529	4	36	11.5
12	4	46	19	113.00	112.90	1356.00	3.13226	4	14	12

TURNOUTS WITH STRAIGHT POINT-RAILS AND STRAIGHT FROG-RAILS; GAUGE 4' 8½". See § 265.

Frog Number (n).	Switch Point Angle (a).		Length of Switch Point (DN).	Length of Straight Frog-rail (f).	Lead (L) (Eq. 90).	Chord (ST) (Eq. 88).	Radius of Lead-rails (r. Eq.87).	Log r.	Degree of Curve (d).		Frog Number (n).
4	3°	40′	7.5	1.50	32.20	23.09	125.21	2.09764	47°	05′	4
4.5	3	40	7.5	1.69	34.29	25.03	159.25	.20208	36	36	4.5
5	2	45	10.0	1.87	41.85	29.88	197.65	.29589̂	29	22	5
5.5	2	45	10.0	2.06	44.16	32.03	240.44	.38100̂	24	00	5.5
6	1	50	15.0	2.25	56.00	38.66	288.09	.45953	19	59	6
6.5	1	50	15.0	2.44	58.84	41.34	340.19	.53172	16	54	6.5
7	1	50	15.0	2.62	61.65	43.98	397.65	.59950	14	27	7
7.5	1	50	15.0	2.81	64.36	46.50	460.00	.66276	12	29	7.5
8	1	50	15.0	3.00	67.04	48.99	527.91	.72256	10	52	8
8.5	1	50	15.0	3.19	69.60	51.38	600.94	.77883	9	33	8.5
9	1	50	15.0	3.37	72.20	53.80	681.16	.83325	8	25	9
9.5	1	50	15.0	3.56	74.70	56.11	767.11	.88486	7	28	9.5
10	1	50	15.0	3.75	77.04	58.28	858.14	.93356	6	41	10
10.5	1	50	15.0	3.94	79.51	60.57	959.00	2.98182	5	59	10.5
11	1	50	15.0	4.12	81.82	62.69	1065.52	3.02756	5	23	11
11.5	1	50	15.0	4.31	84.09	64.78	1180.16	3.07194	4	51	11.5
12	1	50	15.0	4.50	86.16	66.67	1299.93	3.11392	4	24	12

TRIGONOMETRICAL FUNCTIONS OF THE FROG ANGLES (F).

Frog Number (n).	Frog Angle (F).			Nat. sin F.	Nat. cos F.	Log sin F.	Log cos F.	Log cot F.	Log vers F.	Frog Number (n).
4	14°	15′	00″	.24615	.96923	9.39120̂	9.98642̂	10.59522	8.48811̂	4
4.5	12	40	49	.21951	.97561	.34145̂	.98927	.64782	.38721̂	4.5
5	11	25	16	.19802	.98020	.29670̂	.99131̂	.69461	.29670̂	5
5.5	10	23	20	.18033	.98360	.25606̂	.99282	.73675̂	.21467	5.5
6	9	31	38	.16552	.98621	.21884	.99397	.77513	.13966	6
6.5	8	47	51	.15294	.98823	.18453	.99486	.81033	.07058̂	6.5
7	8	10	16	.14213	.98985	.15268̂	.99557	.84288̂	8.00655̂	7
7.5	7	37	41	.13274	.99115	.12301	.99614	.87313	7.94691̂	7.5
8	7	09	10	.12452	.99222	.09522̂	.99660̂	.90138̂	.89110̂	8
8.5	6	43	59	.11724	.99310	.06909	.99699̂	.92790̂	.83864	8.5
9	6	21	35	.11077	.99385	.04442̂	.99732	.95289̂	.78915	9
9.5	6	01	32	.10497	.99448	9.02107	.99759̂	.97652	.74232	9.5
10	5	43	29	.09975	.99501	8.98891	.99783	10.99892	.69788	10
10.5	5	27	09	.09502	.99548	.97781̂	.99803	11.02021̂	.65560	10.5
11	5	12	18	.09072	.99588	.95770̂	.99820̂	.04050̂	.61528	11
11.5	4	58	45	.08679	.99623	.93848̂	.99836	.05987	.57676	11.5
12	4	46	19	.08319	.99653	8.92007	9.98849	11.07842̂	7.53986	12

TABLE IV.—ELEMENTS OF TRANSITION CURVES.

Point.	Total Central Angle φ	Nat. sin φ	Nat. cos φ	Log sin φ	Log cos φ	Log vers φ	x	Log x	y
1	0° 07' 30"	.0022	one	7.33878	10.00000	4.37654	0.027	8.43568	25.000
2	0 22 30	.0065	one	7.81596	9.99999	5.33978	0.136	9.13465	50.000
3	0 45 00	.0131	.9999	8.11692	9.99996	5.93284	0.382	9.58181	74.999
4	1 15 00	.0218	.9997	8.33875	9.99989	6.37653	0.818	9.91286	99.995
5	1 52 30	.0327	.9994	8.51480	9.99976	6.72869	1.500	0.17602	124.985
6	2 37 30	.0458	.9989	8.66085	9.99954	7.02091	2.481	0.39467	149.966
7	3 30 00	.0616	.9981	8.78567	9.99919	7.27072	3.817	0.58171	174.930
8	4 30 00	.0784	.9969	8.89464	9.99866	7.48892	5.561	0.74514	199.870
9	5 37 30	.0980	.9952	8.99130	9.99796	7.68262	7.792	0.89164	224.772
10	6 52 30	.1197	.9928	9.07816	9.99686	7.85675	10.489	1.02061	249.623

0° 30'-per-25-feet spiral.

Deflections from the tangent at the point occupied when the instrument is at—

— sighting at —	Q	1	2	3	4	5	6	7	8	9	10
Q	0° 0' 0"	0° 3' 45"	0° 13' 07"	0° 27' 30"	0° 46' 52"	1° 11' 15"	1° 40' 37"	2° 15' 00"	2° 54' 22"	3° 38' 45"	4° 28' 10"
1	0 3 45	0 0 00	0 07 30	0 20 37	0 38 45	1 01 52	1 30 00	2 03 07	2 41 15	3 24 22	4 12 22
2	0 9 22	0 7 30	0 00 00	0 11 15	0 28 07	0 50 00	1 16 52	1 48 45	2 25 37	3 07 30	3 54 45
3	0 17 30	0 16 52	0 11 15	0 00 00	0 15 00	0 35 37	1 01 15	1 31 52	2 07 30	2 48 07	3 33 37
4	0 28 07	0 28 45	0 24 22	0 15 00	0 00 00	0 18 45	0 43 07	1 12 30	1 46 52	2 26 15	3 10 00
5	0 41 15	0 43 07	0 40 00	0 31 52	0 18 45	0 00 00	0 22 30	0 50 37	1 23 45	2 01 52	2 45 52
6	0 56 52	1 00 00	0 58 07	0 51 15	0 39 22	0 22 30	0 00 00	0 26 15	0 58 07	1 35 00	2 16 15
7	1 15 00	1 19 22	1 18 45	1 13 07	1 02 30	0 46 52	0 26 15	0 00 00	0 30 00	1 05 37	1 46 07
8	1 35 37	1 41 15	1 41 52	1 37 30	1 28 07	1 13 45	0 54 22	0 30 00	0 00 00	0 33 45	1 13 30
9	1 58 45	2 05 37	2 07 30	2 04 22	1 56 15	1 43 07	1 25 00	1 01 52	0 33 45	0 00 00	0 37 30
10	2 24 20	2 32 30	2 35 37	2 33 45	2 26 52	2 15 00	1 58 07	1 36 15	1 09 22	0 37 30	0 00 00

TABLE IV.—ELEMENTS OF TRANSITION CURVES.

Point.	Total Central Angle φ	Nat. sin φ	Nat. cos φ	Log sin φ	Log cos φ	Log vers φ	x	Log x	y
1	0° 15′	.0043	one	7.63981	9.99999	4.97866	.055	8.73672	25.000
2	0 45	.0131	.9999	8.11692	9.99996	5.93284	.273	9.43616	49.999
3	1 30	.0262	.9996	8.41792	9.99985	6.54488	.763	9.88252	74.994
4	2 30	.0436	.9990	8.63968	9.99958	6.97853	1.636	0.21378	99.979
5	3 45	.0654	.9978	8.81560	9.99907	7.33063	2.999	0.47697	124.942
6	5 15	.0915	.9958	8.96143	9.99817	7.62274	4.960	0.69548	149.865
7	7 00	.1218	.9925	9.08589	9.99675	7.87238	7.628	0.88241	174.722
8	9 00	.1564	.9877	9.19433	9.99462	8.09031	11.107	1.04559	199.479
9	11 15	.1951	.9808	9.29023	9.99157	8.28363	15.502	1.19039	224.090
10	13 45	.2377	.9713	9.37660	9.98737	8.45724	20.913	1.32041	248.497

1°-per-25-feet spiral.

Deflections from the tangent at the point occupied when the instrument is at—

sighting at	Q	1	2	3	4	5	6	7	8	9	10
Q	0° 00′ 00″	0° 07′ 30″	0° 26′ 15″	0° 55′ 0″	1° 33′ 45″	2° 22′ 30″	3° 21′ 15″	4° 30′ 00″	5° 48′ 47″	7° 17′ 34″	8° 56′ 22″
1	0 07 30	0 00 00	0 15 00	0 41 15	1 17 30	2 03 45	3 00 00	4 06 15	5 22 30	6 48 47	8 25 05
2	0 18 45	0 15 00	0 00 00	0 22 30	0 56 15	1 40 00	2 33 45	3 37 30	4 51 15	6 15 02	7 48 49
3	0 35 00	0 33 45	0 22 30	0 00 00	0 30 00	1 11 15	2 02 30	3 03 45	4 15 00	5 36 15	7 07 32
4	0 56 15	0 57 30	0 48 45	0 30 00	0 00 00	0 37 30	1 26 15	2 25 00	3 33 45	4 52 30	6 21 17
5	1 22 30	1 26 15	1 20 00	1 03 45	0 37 30	0 00 00	0 45 00	1 41 15	2 47 30	4 03 45	5 30 00
6	1 53 45	2 00 00	1 56 15	1 42 30	1 18 45	0 45 00	0 00 00	0 52 30	1 56 15	3 10 00	4 33 45
7	2 30 00	2 38 45	2 37 30	2 26 15	2 05 00	1 33 45	0 52 30	0 00 00	1 00 00	2 11 15	3 32 30
8	3 11 13	3 22 30	3 23 45	3 15 00	2 56 15	2 27 30	1 48 45	1 00 00	0 00 00	1 07 30	2 26 15
9	3 57 26	4 11 13	4 14 58	4 08 45	3 52 30	3 26 15	2 50 00	2 03 45	1 07 30	0 00 00	1 15 00
10	4 48 38	5 04 55	5 11 11	5 07 28	4 53 43	4 30 00	3 56 15	3 12 30	2 18 45	1 15 00	0 00 00

TABLE IV.—ELEMENTS OF TRANSITION CURVES.

Point	Total Central Angle φ	Nat. sin φ	Nat. cos φ	Log sin φ	Log cos φ	Log vers φ	x	Log x	y
1	0° 30'	.0087	.9999	7.94084	9.99998	5.58066	0.109	9.03774	25.000
2	1 30	.0262	.9996	8.41792	9.99985	6.53488	0.545	9.73676	49.996
3	3 00	.0523	.9986	8.71880	9.99946	7.13687	1.527	0.18388	74.977
4	5 00	.0871	.9962	8.94029	9.99834	7.58039	3.271	0.51468	99.916
5	7 30	.1305	.9914	9.11570	9.99627	7.93222	5.992	0.77762	124.767
6	10 30	.1822	.9832	9.26663	9.99266	8.22389	9.903	0.99579	149.459
7	14 00	.2419	.9703	9.38367	9.98690	8.47282	15.208	1.18207	173.890
8	18 00	.3090	.9516	9.48998	9.97826	8.68969	22.099	1.34437	197.922
9	22 30	.3827	.9239	9.58284	9.9656?	8.88150	30.752	1.48787	221.376
10	27 30	.4617	.8870	9.66446	9.94793	9.05303	41.317	1.61613	244.034

2°-per-25-feet spiral.

Deflections from the tangent at the point occupied when the instrument is at—

sighting at	O	1	2	3	4	5	6	7	8	9	10
Q	0° 00' 00''	0° 15' 00''	0° 52' 30''	1° 50' 00''	3° 07' 30''	4° 45' 00''	6° 42' 30''	9° 00' 05''	11° 37' 45''	14° 35' 30''	17° 53' 25''
1	0 15 00	0 00 00	0 30 00	1 22 30	2 35 00	4 07 30	6 00 00	8 12 34	10 45 10	13 37 52	16 50 42
2	0 37 30	0 30 00	0 00 00	0 45 00	1 52 30	3 20 00	5 07 30	7 15 02	9 42 36	12 30 15	15 38 00
3	1 10 00	1 07 30	0 45 00	0 00 00	1 00 00	2 22 30	4 05 00	6 07 30	8 30 04	11 12 39	14 15 20
4	1 52 30	1 55 00	1 37 30	1 00 00	0 00 00	1 15 00	2 52 30	4 50 00	7 07 30	9 45 05	12 42 42
5	2 45 00	2 52 30	2 40 00	2 07 30	1 15 00	0 00 00	1 30 00	3 22 30	5 35 00	8 07 30	11 00 06
6	3 47 28	3 59 59	3 52 30	3 25 00	2 37 30	1 30 00	0 00 00	1 45 00	3 52 30	6 20 00	9 07 30
7	4 59 53	5 17 26	5 14 58	4 52 30	4 10 00	3 07 30	1 45 00	0 00 00	2 00 00	4 22 30	7 05 00
8	6 22 15	6 44 50	6 47 24	6 29 56	5 52 30	4 55 00	3 37 30	2 00 00	0 00 00	2 15 00	4 52 30
9	7 54 30	8 22 08	8 29 45	8 17 21	7 44 55	6 52 30	5 40 00	4 07 30	2 15 00	0 00 00	2 30 00
10	9 36 35	10 09 17	10 22 45	10 14 40	9 47 18	8 59 54	7 52 30	6 25 00	4 37 30	2 30 00	0 00 00

TABLE V.—LOGARITHMS OF NUMBERS.

N.	0	1	2	3	4	5	6	7	8	9
100	00 000	043	087	130	173	216	260	303	346	389
101	432	475	518	561	604	646	689	732	775	817
102	860	902	945	987	*030	*072	*114	*157	*199	*241
103	01 283	326	368	410	452	494	536	578	619	661
104	703	745	787	828	870	911	953	994	*036	*077
105	02 119	160	201	243	284	325	366	407	448	489
106	530	571	612	653	694	735	775	816	857	898
107	938	979	*019	*060	*100	*141	*181	*221	*262	*302
108	03 342	382	422	463	503	543	583	623	663	703
109	742	782	822	862	901	941	981	*020	*060	*100
110	04 139	178	218	257	297	336	375	415	454	493
111	532	571	610	649	688	727	766	805	844	883
112	922	960	999	*038	*076	*115	*154	*192	*231	*269
113	05 308	346	384	423	461	499	538	576	614	652
114	690	728	766	804	842	880	918	956	994	*032
115	06 070	107	145	183	220	258	296	333	371	408
116	446	483	520	558	595	632	670	707	744	781
117	818	855	893	930	967	*004	*040	*077	*114	*151
118	07 188	225	261	298	335	372	408	445	481	518
119	554	591	627	664	700	737	773	809	845	882
120	918	954	990	*026	*062	*098	*134	*170	*206	*242
121	08 278	314	350	386	422	457	493	529	564	600
122	636	671	707	742	778	813	849	884	920	955
123	990	*026	*061	*096	*131	*166	*202	*237	*272	*307
124	09 342	377	412	447	482	517	552	586	621	656
125	691	725	760	795	830	864	899	933	968	*002
126	10 037	071	106	140	174	209	243	277	312	346
127	380	414	448	483	517	551	585	619	653	687
128	721	755	789	822	856	890	924	958	991	*025
129	11 059	092	126	160	193	227	260	294	327	361
130	394	427	461	494	528	561	594	627	661	694
131	727	760	793	826	859	892	925	958	991	*024
132	12 057	090	123	156	189	221	254	287	320	352
133	385	418	450	483	515	548	580	613	645	678
134	710	743	775	807	840	872	904	937	969	*001
135	13 033	065	097	130	162	194	226	258	290	322
136	354	386	417	449	481	513	545	577	608	640
137	672	703	735	767	798	830	862	893	925	956
138	988	*019	*051	*082	*113	*145	*176	*207	*239	*270
139	14 301	332	364	395	426	457	488	519	550	582
140	613	644	675	706	736	767	798	829	860	891
141	922	952	983	*014	*045	*075	*106	*137	*167	*198
142	15 229	259	290	320	351	381	412	442	473	503
143	533	564	594	624	655	685	715	745	776	806
144	836	866	896	926	956	987	*017	*047	*077	*107
145	16 137	166	196	226	256	286	316	346	376	405
146	435	465	494	524	554	584	613	643	672	702
147	731	761	791	820	849	879	908	938	967	997
148	17 026	055	085	114	143	172	202	231	260	289
149	318	348	377	406	435	464	493	522	551	580
150	609	638	667	696	725	753	782	811	840	869
N.	0	1	2	3	4	5	6	7	8	9

P.P.

	43	43	42	41
.1	4.3	4.3	4.2	4.1
.2	8.7	8.6	8.4	8.2
.3	13.0	12.9	12.6	12.3
.4	17.4	17.2	16.8	16.4
.5	21.7	21.5	21.0	20.5
.6	26.1	25.8	25.2	24.6
.7	30.4	30.1	29.4	28.7
.8	34.8	34.4	33.6	32.8
.9	39.1	38.7	37.8	36.9

	46	40	39	38
.1	4.6	4.0	3.9	3.8
.2	8.1	8.0	7.8	7.6
.3	12.1	12.0	11.7	11.4
.4	16.2	16.0	15.6	15.2
.5	20.2	20.0	19.5	19.0
.6	24.3	24.0	23.4	22.8
.7	28.3	28.0	27.3	26.6
.8	32.4	32.0	31.2	30.4
.9	36.4	36.0	35.1	34.2

	37	37	36	35
.1	3.7	3.7	3.6	3.5
.2	7.5	7.4	7.2	7.0
.3	11.2	11.1	10.8	10.5
.4	15.0	14.8	14.4	14.0
.5	18.7	18.5	18.0	17.5
.6	22.5	22.2	21.6	21.0
.7	26.2	25.9	25.2	24.5
.8	30.0	29.6	28.8	28.0
.9	33.7	33.3	32.4	31.5

	34	34	33	32
.1	3.4	3.4	3.3	3.2
.2	6.9	6.8	6.6	6.4
.3	10.3	10.2	9.9	9.6
.4	13.8	13.6	13.2	12.8
.5	17.2	17.0	16.5	16.0
.6	20.7	20.4	19.8	19.2
.7	24.1	23.8	23.1	22.4
.8	27.6	27.2	26.4	25.6
.9	31.0	30.6	29.7	28.8

	31	31	30	29
.1	3.1	3.1	3.0	2.9
.2	6.3	6.2	6.0	5.8
.3	9.4	9.3	9.0	8.7
.4	12.6	12.4	12.0	11.6
.5	15.7	15.5	15.0	14.5
.6	18.9	18.6	18.0	17.4
.7	22.0	21.7	21.0	20.3
.8	25.2	24.8	24.0	23.2
.9	28.3	27.9	27.0	26.1

TABLE V.—LOGARITHMS OF NUMBERS.

N.	0	1	2	3	4	5	6	7	8	9
150	17 609	638	667	696	725	753̂	782̂	811̂	840	869
151	897̂	926̂	955	984	*012̂	*041̂	*070	*098̂	*127	*156̂
152	18 184̂	213	241̂	270	298̂	327	355̂	384	412̂	440̂
153	469	497̂	526	554	582̂	611	639	667̂	695̂	724
154	752	780̂	808̂	836̂	864	893	921	949	977	*005
155	19 033	061	089	117	145	173	201	229	256̂	284̂
156	312̂	340̂	368	396	423̂	451̂	479	507	534̂	562̂
157	590	617̂	645̂	673	700̂	728	755̂	783	810̂	838
158	865̂	893	920̂	948	975̂	*003	*030̂	*057̂	*085̂	*112̂
159	20 139̂	167	194̂	221̂	249	276	303̂	330̂	357̂	385
160	412	439	466̂	493̂	520	547̂	574̂	601̂	628̂	655̂
161	682̂	709̂	736̂	763̂	790̂	817̂	844	871	898	924̂
162	951̂	978̂	*005	*032̂	*058̂	*085̂	*112	*139̂	*165̂	*192̂
163	21 219	245̂	272	298̂	325	352̂	378	405	431̂	458
164	484̂	511	537	564	590	616̂	643	669̂	695̂	722
165	748	774̂	801	827	853̂	880	906	932̂	958̂	984̂
166	22 011	037	063	089	115̂	141̂	167̂	193̂	219	245̂
167	271̂	297̂	323̂	349̂	375̂	401̂	427̂	453̂	479	505
168	531	557	582̂	608̂	634	660	686	711̂	737	763
169	788̂	814̂	840	865̂	891̂	917	942̂	968	994	*019̂
170	23 045	070̂	096	121̂	147	172̂	198	223̂	249	274
171	299̂	325	350̂	375̂	401	426̂	451̂	477	502̂	527̂
172	553	578	603̂	628̂	653	679	704	729	754̂	779̂
173	804̂	829̂	855	880	905	930̂	955	980̂	*005	*030̂
174	24 055	080	105	129̂	154	179̂	204	229̂	254	279
175	304	328̂	353̂	378	403	427̂	452̂	477	502	526̂
176	551̂	576	600̂	625	650	674̂	699	723̂	748	773
177	797̂	822	846̂	871	895̂	920	944̂	968̂	993	*017̂
178	25 042	066̂	091	115	139̂	164	188	212̂	237	261
179	285̂	309̂	334	358	382	406̂	430̂	455	479	503
180	527̂	551̂	575̂	599̂	623̂	647̂	672	696	720	744
181	768	792	816	840	863̂	887	911̂	935̂	959̂	983̂
182	26 007	031	055	078̂	102̂	126̂	150	174	197̂	221̂
183	245	269	292̂	316	340	363̂	387̂	411	434̂	458
184	482	505̂	529	552̂	576	599̂	623	646̂	670	693̂
185	717	740̂	764	787̂	811	834̂	858	881	904̂	928
186	951̂	974̂	998	*021̂	*044̂	*068	*091	*114̂	*137̂	*161
187	27 184	207̂	230̂	254	277	300	323̂	346̂	369̂	392̂
188	416	439	462	485	508	531	554	577	600̂	623
189	646	669	692	715	738	761	784	806̂	829̂	852̂
190	875̂	898	921	944	966̂	989̂	*012̂	*035	*058	*080̂
191	28 103̂	126	149	171̂	194	217	239̂	262	285	307̂
192	330	352̂	375̂	398	420̂	443	465̂	488	510̂	533
193	555̂	578	600̂	623	645̂	668	690̂	713	735̂	758
194	780	802̂	825	847̂	869̂	892	914̂	936̂	959	981
195	29 003̂	025̂	048	070	092̂	114̂	137	159̂	181̂	203̂
196	225̂	248	270	292	314	336̂	358	380̂	402̂	424̂
197	446̂	468̂	490̂	512̂	534̂	556̂	578	600̂	622̂	644̂
198	666̂	688̂	710̂	732̂	754	776	798	820	841̂	863̂
199	885̂	907	929	950̂	972̂	994̂	*016	*038̂	*059̂	*081̂
200	30 103	124̂	146̂	168	190	211̂	233	254̂	276̂	298
N.	0	1	2	3	4	5	6	7	8	9

P. P.

	29	28	27
.1	2.9	2.8	2.7
.2	5.8	5.6	5.4
.3	8.7	8.4	8.1
.4	11.6	11.2	10.8
.5	14.5	14.0	13.5
.6	17.4	16.8	16.2
.7	20.3	19.6	18.9
.8	23.2	22.4	21.6
.9	26.1	25.2	24.3

	2̂6	26
.1	2.6̂	2.6
.2	5.3	5.2
.3	7.9	7.8
.4	10.6	10.4
.5	13.2̂	13.0
.6	15.9	15.6
.7	18.5̂	18.2
.8	21.2	20.8
.9	23.8̂	23.4

	2̂5	25	24
.1	2.5̂	2.5	2.4
.2	5.1	5.0	4.8
.3	7.6̂	7.5	7.2
.4	10.2	10.0	9.6
.5	12.7̂	12.5	12.0
.6	15.3	15.0	14.4
.7	17.8̂	17.5	16.8
.8	20.4	20.0	19.2
.9	22.9̂	22.5	21.6

	2̂3	23
.1	2.3̂	2.3
.2	4.7	4.6
.3	7.0̂	6.9
.4	9.4	9.2
.5	11.7̂	11.5
.6	14.1	13.8
.7	16.4̂	16.1
.8	18.8	18.4
.9	21.1̂	20.7

	2̂2	22	2̂1
.1	2.2̂	2.2	2.1̂
.2	4.5	4.4	4.3
.3	6.7	6.6	6.4
.4	9.0	8.8	8.6
.5	11.2̂	11.0	10.7̂
.6	13.5	13.2	12.9
.7	15.7̂	15.4	15.0̂
.8	18.0	17.6	17.2
.9	20.2̂	19.8	19.3

TABLE V.—LOGARITHMS OF NUMBERS.

N.	0	1	2	3	4	5	6	7	8	9
200	30 103	124̂	146̂	168	190	21î	233	254̂	276̂	298
201	319̂	341	363	38â	406	427̂	449	470̂	492	513̂
202	535	556̂	578	599̂	621	642̂	664	685̂	707	728
203	749̂	771	792̂	813̂	835	856̂	878	899	920̂	94î
204	963	984̂	*005̂	*027	*048̂	*069̂	*090̂	*112	*133̂	*154
205	31 175̂	196̂	217̂	239	260	281	302̂	323̂	344̂	365̂
206	386̂	408	429	450	471	492	513	534	555	576̂
207	597	618	639	660	681	702	722̂	743̂	764̂	785̂
208	806̂	827	848	869	890	910̂	931̂	952	973	994
209	32 014̂	035̂	056	077	097̂	118̂	139	160	180̂	201
210	222	242̂	263̂	284	304̂	325	346	366̂	387	407̂
211	428	449	469̂	490	510̂	531	551̂	572	592̂	613
212	633̂	654	674̂	695	715̂	736	756̂	776̂	797	817̂
213	838	858̂	878̂	899	919̂	940	960	980̂	*001	*021
214	33 041̂	061̂	082	102	122̂	142̂	163	183	203̂	223̂
215	244	264	284	304̂	324̂	344̂	365	385	405	425̂
216	445̂	465̂	485̂	505̂	525̂	546	566	586	606	626
217	646	666	686	706	726	746	766	786	806	825̂
218	845̂	865̂	885̂	905̂	925̂	945	965	985	*004̂	*024̂
219	34 044̂	064	084	104	123̂	143̂	163	183	203	222̂
220	242̂	262	28î	30î	321	341	360̂	380	400	419̂
221	439	459	478̂	498	518	537	557	576̂	596	615̂
222	635̂	655	674̂	694	713̂	733	752̂	772	79î	811
223	830̂	850	869̂	889	908̂	928	947	966̂	986	*005̂
224	35 025	044	063̂	083	102̂	12î	141	160̂	179̂	199
225	218̂	237̂	257	276	295̂	314̂	334	353	372̂	39î
226	411	430	449̂	468̂	487̂	507	526	545	564̂	583̂
227	602̂	62î	641	660	679	698	717	736̂	755̂	774̂
228	793̂	812̂	83î	850̂	869̂	888̂	907̂	926̂	945	964̂
229	983̂	*002̂	*021̂	*040̂	*059̂	*078̂	*097	*116	*135	*154
230	36 173	19î	210̂	229̂	248	267	286	305	323̂	342̂
231	361	380	399	417̂	436̂	455	474	492̂	51î	530
232	549	567̂	586	605	623̂	642̂	661	679̂	698̂	717
233	735̂	754	773	79î	810	828̂	847̂	866	884̂	903
234	92î	940	958̂	977	996	*014̂	*033	*051̂	*070	*088̂
235	37 107	125̂	143̂	162	180̂	199	217̂	236	254̂	273
236	291	309̂	328	346̂	364̂	383	40î	420	438̂	456̂
237	475	493	51î	530	548	566̂	584̂	603	621	639̂
238	657̂	676	694	712̂	730̂	749	767	785	803̂	82î
239	840	858	876	894̂	912̂	930̂	948	967	985	*003
240	38 021	039	057̂	075̂	093̂	11î	129̂	147̂	165̂	183̂
241	20î	219̂	237̂	255̂	273̂	29î	309̂	327̂	345̂	363̂
242	38î	399̂	417̂	435̂	453̂	471	489	507	525	543
243	560̂	578̂	596̂	614	632	650	667̂	685̂	703̂	721
244	739	757	774̂	792̂	810	828	845̂	863̂	881	899
245	916̂	934̂	952	970	987̂	*005	*023	*040̂	*058̂	*076̂
246	39 093̂	111	129	146̂	164	18î	199̂	217	234̂	252
247	269̂	287̂	305	322̂	340	357̂	375	392̂	410	427̂
248	445	462̂	480	497̂	515	532̂	550	567̂	585	602̂
249	620	637̂	655	672	689̂	707	724̂	742	759	776̂
250	794	81î	828̂	846	863̂	881	898	915̂	933	950
N.	**0**	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**

P. P.

	22	21
.1	2.2	2.1
.2	4.4	4.2
.3	6.6	6.3
.4	8.8	8.4
.5	11.0	10.5
.6	13.2	12.6
.7	15.4	14.7
.8	17.6	16.8
.9	19.8	18.9

	20̂	20
.1	2.0̂	2.0
.2	4.1	4.0
.3	6.1̂	6.0
.4	8.2	8.0
.5	10.2̂	10.0
.6	12.3	12.0
.7	14.3̂	14.0
.8	16.4	16.0
.9	18.4̂	18.0

	19̂	19
.1	1.9̂	1.9
.2	3.9	3.8
.3	5.8	5.7
.4	7.8	7.6
.5	9.7̂	9.5
.6	11.7	11.4
.7	13.6	13.3
.8	15.6	15.2
.9	17.5̂	17.1

	18̂	18
.1	1.8̂	1.8
.2	3.7	3.6
.3	5.5	5.4
.4	7.4	7.2
.5	9.2̂	9.0
.6	11.1	10.8
.7	12.9̂	12.6
.8	14.8	14.4
.9	16.6	16.2

	17̂	17
.1	1.7̂	1.7
.2	3.5	3.4
.3	5.2̂	5.1
.4	7.0	6.8
.5	8.7̂	8.5
.6	10.5	10.2
.7	12.2̂	11.9
.8	14.0	13.6
.9	15.7̂	15.3

TABLE V.—LOGARITHMS OF NUMBERS.

N.	0	1	2	3	4	5	6	7	8	9
250	39 794	811	828	846	863	881	898	915	933	950
251	967	984	*002	*019	*036	*054	*071	*088	*105	*123
252	40 140	157	174	191	209	226	243	260	277	295
253	312	329	346	363	380	398	415	432	449	466
254	483	500	517	534	551	569	586	603	620	637
255	654	671	688	705	722	739	756	773	790	807
256	824	841	858	875	892	908	925	942	959	976
257	993	*010	*027	*044	*061	*077	*094	*111	*128	*145
258	41 162	179	195	212	229	246	263	279	296	313
259	330	346	363	380	397	413	430	447	464	480
260	497	514	530	547	564	581	597	614	631	647
261	664	680	697	714	730	747	764	780	797	813
262	830	846	863	880	896	913	929	946	962	979
263	995	*012	*028	*045	*061	*078	*094	*111	*127	*144
264	42 166	177	193	209	226	242	259	275	292	308
265	324	341	357	373	390	406	423	439	455	472
266	488	504	521	537	553	569	586	602	618	635
267	651	667	683	700	716	732	748	765	781	797
268	813	829	846	862	878	894	910	927	943	959
269	975	991	*007	*023	*040	*056	*072	*088	*104	*120
270	43 136	152	168	184	200	216	233	249	265	281
271	297	313	329	345	361	377	393	409	425	441
272	457	473	489	505	520	536	552	568	584	600
273	616	632	648	664	680	695	711	727	743	759
274	775	791	806	822	838	854	870	886	901	917
275	933	949	965	980	996	*012	*028	*043	*059	*075
276	44 091	106	122	138	154	169	185	201	216	232
277	248	263	279	295	310	326	342	357	373	389
278	404	420	435	451	467	482	498	513	529	545
279	560	576	591	607	622	638	653	669	685	700
280	716	731	747	762	778	793	809	824	839	855
281	870	886	901	917	932	948	963	978	994	*009
282	45 025	040	055	071	086	102	117	132	148	163
283	178	194	209	224	240	255	270	286	301	316
284	332	347	362	377	393	408	423	438	454	469
285	484	499	515	530	545	560	576	591	606	621
286	636	652	667	682	697	712	727	743	758	773
287	788	803	818	833	848	864	879	894	909	924
288	939	954	969	984	999	*014	*029	*044	*059	*075
289	46 090	105	120	135	150	165	180	195	210	225
290	240	255	269	284	299	314	329	344	359	374
291	389	404	419	434	449	464	479	493	508	523
292	538	553	568	583	597	612	627	642	657	672
293	687	701	716	731	746	761	775	790	805	820
294	834	849	864	879	894	908	923	938	952	967
295	982	997	*011	*026	*041	*055	*070	*085	*100	*114
296	47 129	144	158	173	188	202	217	232	246	261
297	275	290	305	319	334	348	363	378	392	407
298	421	436	451	465	480	494	509	523	538	552
299	567	581	596	610	625	639	654	668	683	697
300	712	726	741	755	770	784	799	813	828	842
N.	0	1	2	3	4	5	6	7	8	9

P. P.

	17	17		16	16		15	15		14	14
.1	1.7	1.7	.1	1.6	1.6	.1	1.5	1.5	.1	1.4	1.4
.2	3.5	3.4	.2	3.3	3.2	.2	3.1	3.0	.2	2.9	2.8
.3	5.2	5.1	.3	4.9	4.8	.3	4.6	4.5	.3	4.3	4.2
.4	7.0	6.8	.4	6.6	6.4	.4	6.2	6.0	.4	5.8	5.6
.5	8.7	8.5	.5	8.2	8.0	.5	7.7	7.5	.5	7.2	7.0
.6	10.5	10.2	.6	9.9	9.6	.6	9.3	9.0	.6	8.7	8.4
.7	12.2	11.9	.7	11.5	11.2	.7	10.8	10.5	.7	10.1	9.8
.8	14.0	13.6	.8	13.2	12.8	.8	12.4	12.0	.8	11.6	11.2
.9	15.7	15.3	.9	14.8	14.4	.9	13.9	13.5	.9	13.0	12.6

TABLE V.—LOGARITHMS OF NUMBERS.

N.	0	1	2	3	4	5	6	7	8	9	P. P.	
300	47 712	726	741	755	770	784	799	813	828	842		
301	856	871	885	900	914	928	943	957	972	986		
302	48 000	015	029	044	058	072	087	101	115	130		
303	144	158	173	187	201	216	230	244	259	273		
304	287	301	316	330	344	358	373	387	401	415		
305	430	444	458	472	487	501	515	529	543	558		**14**	**14**
306	572	586	600	614	629	643	657	671	685	699		
307	714	728	742	756	770	784	798	812	827	841	.1 1.4	1.4
308	855	869	883	897	911	925	939	953	967	982	.2 2.9	2.8
309	996	*010	*024	*038	*052	*066	*080	*094	*108	*122	.3 4.3	4.2
310	49 136	150	164	178	192	206	220	234	248	262	.4 5.8 .5 7.2	5.6 7.0
311	276	290	304	318	332	346	359	373	387	401	.6 8.7	8.4
312	415	429	443	457	471	485	499	513	526	540		
313	554	568	582	596	610	624	637	651	665	679	.7 10.1	9.8
314	693	707	720	734	748	762	776	789	803	817	.8 11.6	11.2
315	831	845	858	872	886	900	913	927	941	955	.9 13.0	12.6
316	968	982	996	*010	*023	*037	*051	*065	*078	*092		
317	50 106	119	133	147	160	174	188	201	215	229		
318	242	256	270	283	297	311	324	338	352	365		
319	379	392	406	420	433	447	460	474	488	501		
320	515	528	542	555	569	583	596	610	623	637		**13**	**13**
321	650	664	677	691	704	718	731	745	758	772	.1 1.3	1.3
322	785	799	812	826	839	853	866	880	893	907	.2 2.7	2.6
323	920	933	947	960	974	987	*001	*014	*027	*041	.3 4.0	3.9
324	51 054	068	081	094	108	121	135	148	161	175		
325	188	201	215	228	242	255	268	282	295	308	.4 5.4 .5 6.7	5.2 6.5
326	322	335	348	361	375	388	401	415	428	441	.6 8.1	7.8
327	455	468	481	494	508	521	534	547	561	574		
328	587	600	614	627	640	653	667	680	693	706	.7 9.4	9.1
329	719	733	746	759	772	785	798	812	825	838	.8 10.8	10.4
330	851	864	877	891	904	917	930	943	956	969	.9 12.1	11.7
331	983	996	*009	*022	*035	*048	*061	*074	*087	*100		
332	52 114	127	140	153	166	179	192	205	218	231		
333	244	257	270	283	296	309	322	335	348	361		
334	374	387	400	413	426	439	452	465	478	491		
335	504	517	530	543	556	569	582	595	608	621		**12**	**12**
336	634	647	660	672	685	698	711	724	737	750		
337	763	776	789	801	814	827	840	853	866	879	.1 1.2	1.2
338	891	904	917	930	943	956	968	981	994	*007	.2 2.5	2.4
339	53 020	033	045	058	071	084	097	109	122	135	.3 3.7	3.6
340	148	160	173	186	199	211	224	237	250	262	.4 5.0 .5 6.2	4.8 6.0
341	275	288	301	313	326	339	352	364	377	390	.6 7.5	7.2
342	402	415	428	440	453	466	478	491	504	516		
343	529	542	554	567	580	592	605	618	630	643	.7 8.7	8.4
344	656	668	681	693	706	719	731	744	756	769	.8 10.0	9.6
345	782	794	807	819	832	845	857	870	882	895	.9 11.2	10.8
346	907	920	932	945	958	970	983	995	*008	*020		
347	54 033	045	058	070	083	095	108	120	133	145		
348	158	170	183	195	208	220	233	245	257	270		
349	282	295	307	320	332	344	357	369	382	394		
350	407	419	431	444	456	469	481	493	506	518		
N.	0	1	2	3	4	5	6	7	8	9	P. P.	

TABLE V.—LOGARITHMS OF NUMBERS.

N.	0	1	2	3	4	5	6	7	8	9
350	54 407	419	431	444	456	469	481	493	506	518
351	530	543	555	568	580	592	605	617	629	642
352	654	666	679	691	703	716	728	740	753	765
353	777	790	802	814	826	839	851	863	876	888
354	900	912	925	937	949	961	974	986	998	*010
355	55 023	035	047	059	071	084	096	108	120	133
356	145	157	169	181	194	206	218	230	242	254
357	267	279	291	303	315	327	340	352	364	376
358	388	400	412	424	437	449	461	473	485	497
359	509	521	533	545	558	570	582	594	606	618
360	630	642	654	666	678	690	702	714	726	738
361	750	762	775	787	799	811	823	835	847	859
362	871	883	895	907	919	931	943	955	966	978
363	990	*002	*014	*026	*038	*050	*062	*074	*086	*098
364	56 110	122	134	146	158	170	181	193	205	217
365	229	241	253	265	277	288	300	312	324	336
366	348	360	372	383	395	407	419	431	443	455
367	466	478	490	502	514	525	537	549	561	573
368	585	596	608	620	632	643	655	667	679	691
369	702	714	726	738	749	761	773	785	796	808
370	820	832	843	855	867	879	890	902	914	925
371	937	949	961	972	984	996	*007	*019	*031	*042
372	57 054	066	077	089	101	112	124	136	147	159
373	171	182	194	206	217	229	240	252	264	275
374	287	299	310	322	333	345	357	368	380	391
375	403	414	426	438	449	461	472	484	495	507
376	519	530	542	553	565	576	588	599	611	622
377	634	645	657	668	680	691	703	714	726	737
378	749	760	772	783	795	806	818	829	841	852
379	864	875	887	898	909	921	932	944	955	967
380	978	990	*001	*012	*024	*035	*047	*058	*069	*081
381	58 092	104	115	126	138	149	161	172	183	195
382	206	217	229	240	252	263	274	286	297	308
383	320	331	342	354	365	376	388	399	410	422
384	433	444	455	467	478	489	501	512	523	535
385	546	557	568	580	591	602	613	625	636	647
386	658	670	681	692	703	715	726	737	748	760
387	771	782	793	804	816	827	838	849	861	872
388	883	894	905	916	928	939	950	961	972	984
389	995	*006	*017	*028	*039	*050	*062	*073	*084	*095
390	59 106	117	128	140	151	162	173	184	195	206
391	217	229	240	251	262	273	284	295	306	317
392	328	339	351	362	373	384	395	406	417	428
393	439	450	461	472	483	494	505	516	527	538
394	549	560	571	582	593	604	615	626	637	648
395	659	670	681	692	703	714	725	736	747	758
396	769	780	791	802	813	824	835	846	857	868
397	879	890	901	912	923	933	944	955	966	977
398	988	999	*010	*021	*032	*043	*053	*064	*075	*086
399	60 097	108	119	130	141	151	162	173	184	195
400	206	217	227	238	249	260	271	282	293	303
N.	0	1	2	3	4	5	6	7	8	9

P. P.

12		12		11		11		10	
.1	1.2	.1	1.2	.1	1.1	.1	1.1	.1	1.0
.2	2.5	.2	2.4	.2	2.3	.2	2.2	.2	2.1
.3	3.7	.3	3.6	.3	3.4	.3	3.3	.3	3.1
.4	5.0	.4	4.8	.4	4.6	.4	4.4	.4	4.2
.5	6.2	.5	6.0	.5	5.7	.5	5.5	.5	5.2
.6	7.5	.6	7.2	.6	6.9	.6	6.6	.6	6.3
.7	8.7	.7	8.4	.7	8.0	.7	7.7	.7	7.3
.8	10.0	.8	9.6	.8	9.2	.8	8.8	.8	8.4
.9	11.2	.9	10.8	.9	10.3	.9	9.9	.9	9.4

TABLE V.—LOGARITHMS OF NUMBERS.

N.	0	1	2	3	4	5	6	7	8	9	P.P.	
400	60 206	217	227̂	238̂	249̂	260̂	271	282	293	303̂		
401	314̂	325̂	336̂	347	357̂	368̂	379̂	390	401	412		
402	422̂	433̂	444	455	466	476̂	487̂	498	509	519̂		
403	530̂	541̂	552	563	573̂	584̂	595	606	616̂	627̂		**11**
404	638	649	659̂	670̂	681	692	702̂	713̂	724	735	.1	1.1
405	745̂	756	767	777̂	788̂	799	810	820̂	831	842	.2	2.2
406	852̂	863̂	874	884̂	895̂	906	916̂	927̂	938	949	.3	3.3
407	959̂	970	981	991̂	*002	*013	*023̂	*034̂	*044̂	*055̂	.4	4.4
408	61 066	076̂	087̂	098	108̂	119	130	140̂	151	161̂	.5	5.5
409	172̂	183	193̂	204	215	225̂	236	246̂	257	268	.6	6.6
410	278̂	289	299̂	310	320̂	331̂	342	352̂	363	373̂	.7	7.7
411	384	394̂	405̂	416	426̂	437	447̂	458	468̂	479	.8	8.8
412	489̂	500̂	511	521̂	532	542̂	553	563̂	574	584̂	.9	9.9
413	595	605̂	616	626̂	637	647̂	658	668̂	679	689̂		
414	700	710̂	721	731̂	742	752̂	763	773̂	784	794̂		
415	805	815̂	825̂	836	846̂	857	867̂	878	888̂	899		**1ô**
416	909̂	920	930	940̂	951	961̂	972	982̂	993	*003	.1	1.ô
417	62 013̂	024	034̂	045	055	065̂	076	086̂	097	107	.2	2.1
418	117̂	128	138̂	149	159	169̂	180	190̂	200̂	211	.3	3.1̂
419	221̂	232	242	252̂	263	273	283̂	294	304	314̂	.4	4.2
420	325	335̂	345̂	356	366̂	376̂	387	397̂	407̂	418	.5	5.2̂
421	428	438̂	449	459	469̂	480	490	500̂	510̂	521	.6	6.3
422	531	541̂	552	562	572̂	582̂	593	603	613̂	624	.7	7.3̂
423	634	644̂	654̂	665	675	685̂	695̂	706	716	726̂	.8	8.4
424	736̂	747	757	767̂	777̂	788	798	808	818̂	828̂	.9	9.4̂
425	839	849	859̂	869̂	879̂	890	900	910̂	920̂	931		
426	941	951	961̂	971̂	981̂	992	*002	*012̂	*022̂	*032̂		
427	63 043	053	063	073̂	083̂	093̂	104	114	124	134		**10**
428	144̂	154̂	164̂	175	185̂	195	205	215̂	225̂	235̂	.1	1.0
429	245̂	256	266	276	286	296̂	306̂	316̂	326̂	336̂	.2	2.0
430	347	357	367	377	387̂	397	407̂	417̂	427̂	437̂	.3	3.0
431	447̂	458	468	478	488	498	508	518	528̂	538̂	.4	4.0
432	548̂	558	568̂	578̂	588̂	598	608̂	618̂	628̂	639	.5	5.0
433	649	659	669	679	689	699	709	719	729	739	.6	6.0
434	749	759	769	779	789	799	809	819	829	839	.7	7.0
435	849	859	869	879	889	899	909	919	928̂	938̂	.8	8.0
436	948̂	958̂	968̂	978̂	988̂	998	*008̂	*018̂	*028̂	*038̂	.9	9.0
437	64 048	058	068	078	088	098	107̂	117̂	127̂	137̂		
438	147̂	157̂	167	177	187	197	207	217	226̂	236̂		
439	246̂	256̂	266	276	286	296	306	315̂	325̂	335̂		**9̂**
440	345̂	355	365	375	384̂	394̂	404̂	414̂	424	434	.1	0.9̂
441	444	453̂	463̂	473̂	483	493	503	512̂	522̂	532̂	.2	1.9
442	542	552	562	571̂	581̂	591̂	601	611	621	630̂	.3	2.8̂
443	640̂	650	660	670	679̂	689̂	699	709	718̂	728̂	.4	3.8
444	738̂	748	758	767̂	777̂	787	797	806̂	816̂	826	.5	4.7̂
445	836	846	855̂	865̂	875	885	894̂	904̂	914	923̂	.6	5.7
446	933̂	943	953	962̂	972̂	982	992	*001̂	*011̂	*021	.7	6.6̂
447	65 031	040̂	050	060	069̂	079̂	089	098̂	108̂	118	.8	7.6
448	128	137̂	147	157	166̂	176	186	195̂	205̂	215	.9	8.5̂
449	224̂	234̂	244	253̂	263̂	273	282̂	292̂	302	311̂		
450	321̂	331	340̂	350	360	369̂	379	389̂	398̂	408		
N.	0	1	2	3	4	5	6	7	8	9	P.P.	

TABLE V.—LOGARITHMS OF NUMBERS.

N.	0	1	2	3	4	5	6	7	8	9
450	65 321	331	346	350	360	369	379	389	398	408
451	417	427	437	446	456	466	475	485	494	504
452	514	523	533	542	552	562	571	581	590	600
453	610	619	629	638	648	657	667	677	686	696
454	705	715	724	734	744	753	763	772	782	791
455	801	810	820	830	839	849	858	868	877	887
456	896	906	915	925	934	944	953	963	972	982
457	991	*001	*010	*020	*029	*039	*048	*058	*067	*077
458	66 086	096	105	115	124	134	143	153	162	172
459	181	190	200	209	219	228	238	247	257	266
460	276	285	294	304	313	323	332	342	351	360
461	370	379	389	398	408	417	426	436	445	455
462	464	473	483	492	502	511	520	530	539	548
463	558	567	577	586	595	605	614	623	633	642
464	652	661	670	680	689	698	708	717	726	736
465	745	754	764	773	782	792	801	810	820	829
466	838	848	857	866	876	885	894	904	913	922
467	931	941	950	959	969	978	987	996	*006	*015
468	67 024	034	043	052	061	071	080	089	099	108
469	117	126	136	145	154	163	173	182	191	200
470	210	219	228	237	246	256	265	274	283	293
471	302	311	320	329	339	348	357	366	376	385
472	394	403	412	422	431	440	449	458	467	477
473	486	495	504	513	523	532	541	550	559	568
474	578	587	596	605	614	623	633	642	651	660
475	669	678	687	697	706	715	724	733	742	751
476	760	770	779	788	797	806	815	824	833	842
477	852	861	870	879	888	897	906	915	924	933
478	943	952	961	970	979	988	997	*006	*015	*024
479	68 033	042	051	060	070	079	088	097	106	115
480	124	133	142	151	160	169	178	187	196	205
481	214	223	232	241	250	259	268	277	286	295
482	304	313	322	331	340	349	358	367	376	385
483	394	403	412	421	430	439	448	457	466	475
484	484	493	502	511	520	529	538	547	556	565
485	574	583	592	601	610	619	628	637	646	654
486	663	672	681	690	699	708	717	726	735	744
487	753	762	770	779	788	797	806	815	824	833
488	842	851	860	868	877	886	895	904	913	922
489	931	940	948	957	966	975	984	993	*002	*010
490	69 019	028	037	046	055	064	073	081	090	099
491	108	117	126	134	143	152	161	170	179	187
492	196	205	214	223	232	240	249	258	267	276
493	284	293	302	311	320	328	337	346	355	364
494	372	381	390	399	408	416	425	434	443	451
495	460	469	478	487	495	504	513	522	530	539
496	548	557	565	574	583	592	600	609	618	627
497	635	644	653	662	670	679	688	697	705	714
498	723	731	740	749	758	766	775	784	792	801
499	810	819	827	836	845	853	862	871	879	888
500	897	905	914	923	931	940	949	958	966	975
N.	0	1	2	3	4	5	6	7	8	9

P. P.

10
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.9 7.6

TABLE V.—LOGARITHMS OF NUMBERS.

N.	0	1	2	3	4	5	6	7	8	9
500	69 897	905̂	914̂	923	931̂	940̂	949	958	966̂	975
501	984	992̂	*001	*010	*018̂	*027	*036	*044̂	*053	*061̂
502	70 070̂	079	087̂	096̂	105	113̂	122	131	139̂	148
503	157	165̂	174	182̂	191	200	208̂	217	226	234̂
504	243	251̂	260̂	269	277̂	286	294̂	303̂	312	320̂
505	329	337̂	346̂	355	363̂	372	380̂	389	398	406̂
506	415	423̂	432	441	449̂	458	466̂	475	483̂	492
507	501	509̂	518	526̂	535	543̂	552	560̂	569̂	578
508	586̂	595	603̂	612	620̂	629	637̂	646	654̂	663
509	672	680̂	689	697̂	706	714̂	723	731̂	740	748
510	757	765̂	774	782̂	791	799̂	808	816̂	825	833̂
511	842	850̂	859	867̂	876	884̂	893	901̂	910	918̂
512	927	935̂	944	952̂	961	969̂	978	986̂	995	*003̂
513	71 011̂	020	028̂	037	045̂	054	062̂	071	079̂	088
514	096̂	105	113	121̂	130	138̂	147	155̂	164	172̂
515	180̂	189	197̂	206	214̂	223	231̂	239	248	256̂
516	265	273̂	282	290	298̂	307	315̂	324	332̂	340̂
517	349	357̂	366	374	382̂	391	399̂	408	416	424̂
518	433	441̂	449̂	458	466̂	475	483̂	491̂	500	508̂
519	516̂	525	533̂	542	550	558	567	575̂	583̂	592
520	600̂	608̂	617	625̂	633̂	642	650̂	659	667	675̂
521	684	692	700̂	709	717	725̂	734	742	750̂	758̂
522	767	775̂	783̂	792	800̂	808̂	817	825̂	833̂	842
523	850	858̂	867	875	883̂	891̂	900	908̂	916̂	925
524	933	941̂	949̂	958	966̂	974̂	983	991	999̂	*007̂
525	72 016	024	032̂	040̂	049	057	065̂	074	082	090̂
526	098	107	115	123̂	131̂	140	148	156̂	164̂	173
527	181	189̂	197	206	214	222	230̂	238̂	247	255
528	263̂	271̂	280	288	296̂	304̂	312̂	321	329	337̂
529	345̂	354	362	370	378	386̂	395	403	411	419̂
530	427̂	436	444	452	460̂	468̂	476̂	485	493	501̂
531	509̂	517̂	526	534	542	550̂	558̂	566̂	575	583
532	591	599̂	607	615̂	624	632	640	648	656̂	664̂
533	672̂	681	689	697	705̂	713̂	721̂	729̂	738	746
534	754	762̂	770̂	778̂	786̂	795	803	811	819	827̂
535	835̂	843̂	851̂	859̂	868	876	884	892	900̂	908̂
536	916̂	924̂	932̂	941	949	957	965	973	981̂	989̂
537	997̂	*005̂	*013̂	*021̂	*030	*038̂	*046̂	*054̂	*062	*070
538	73 078	086̂	094̂	102̂	110̂	118̂	126̂	134̂	143	151
539	159	167	175	183	191	199	207	215	223̂	231̂
540	239̂	247̂	255̂	263̂	271̂	279̂	287̂	295̂	303̂	311̂
541	319̂	328	336	344	352	360	368	376	384	392
542	400	408	416	424	432	440	448	456	464	472
543	480	488	496	504	512	520	528	536	544	552
544	560	568	576	584	592	600	608	615̂	623̂	631̂
545	639̂	647̂	655̂	663̂	671̂	679̂	687̂	695̂	703̂	711̂
546	719̂	727	735	743	751	759	767	775	783	791
547	798̂	806̂	814̂	822̂	830̂	838̂	846̂	854̂	862	870
548	878	886	894	902	909̂	917̂	925̂	933̂	941̂	949̂
549	957	965	973	981	989	997	*004̂	*012̂	*020̂	*028̂
550	74 036̂	044	052	060	068	075̂	083̂	091̂	099̂	107̂
N.	0	1	2	3	4	5	6	7	8	9

P. P.

9
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.9	8.1

8
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.2	1.7
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.4	3.4
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.9	7.6̂

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.4	3.2
.5	4.0
.6	4.8
.7	5.6
.8	6.4
.9	7.2

7
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.2	1.5
.3	2.2̂
.4	3.0
.5	3.7̂
.6	4.5
.7	5.2̂
.8	6.0
.9	6.7̂

TABLE V.—LOGARITHMS OF NUMBERS.

N.	0	1	2	3	4	5	6	7	8	9
550	74 036̂	044	052	060	068	075̂	083̂	091̂	099̂	107̂
551	115	123	131	139	146̂	154̂	162̂	170̂	178	186
552	194	202	209̂	217̂	225̂	233	241	249	257	264̂
553	272̂	280̂	288	296	304	312	319̂	327̂	335̂	343
554	351	359	366̂	374̂	382̂	390	398	406	413̂	421̂
555	429̂	437	445	453	460̂	468̂	476	484	492	499̂
556	507̂	515̂	523	531	538̂	546̂	554̂	562	570	577̂
557	585̂	593̂	601	609	616̂	624̂	632̂	640	648	655̂
558	663̂	671	679	687	694̂	702̂	710	718	725̂	733̂
559	741	749	756̂	764̂	772	780	788	795̂	803̂	811
560	819	826̂	834̂	842	850	857̂	865̂	873	881	888̂
561	896̂	904	912	919̂	927	935	942̂	950̂	958	966
562	973̂	981̂	989	997	*004̂	*012̂	*020	*027̂	*035̂	*043
563	75 051	058̂	066̂	074	081̂	089̂	097	105	112̂	120
564	128	135̂	143̂	151	158̂	166̂	174	182	189̂	197
565	205	212̂	220	228	235̂	243̂	251	258̂	266̂	274
566	281̂	289̂	297	304̂	312̂	320	327̂	335̂	343	350̂
567	358̂	366	373̂	381̂	389	396̂	404	412	419̂	427
568	435	442̂	450	458	465̂	473	480̂	488̂	496	503̂
569	511	519	526̂	534	541̂	549̂	557	564̂	572	580
570	587̂	595	602̂	610̂	618	625̂	633	641	648̂	656
571	663̂	671	679	686̂	694	701̂	709	717	724̂	732
572	739̂	747	755	762̂	770	777̂	785	792̂	800̂	808
573	815̂	823	830̂	838	846	853̂	861	868̂	876	883̂
574	891	899	906̂	914	921̂	929	936̂	944	951̂	959
575	967	974̂	982	989̂	997	*004̂	*012	*019̂	*027	*034̂
576	76 042	050	057̂	065	072̂	080	087̂	095	102̂	110
577	117̂	125	132̂	140	147̂	155	162̂	170	178	185̂
578	193	200̂	208	215̂	223	230̂	238	245̂	253	260̂
579	268	275̂	283	290̂	298	305̂	313	320̂	328	335̂
580	343	350̂	358	365̂	372̂	380	387	395	402̂	410
581	417̂	425	432̂	440	447̂	455	462̂	470	477̂	485
582	492̂	500	507	514̂	522	529̂	537	544̂	552	559̂
583	567	574̂	582	589	596̂	604	611̂	619	626̂	634
584	641̂	648̂	656	663̂	671	678̂	686	693̂	700̂	708
585	715̂	723	730̂	738	745̂	752̂	760	767̂	775	782̂
586	790	797	804̂	812	819̂	827	834	841̂	849	856̂
587	864	871	878̂	886	893̂	901	908	915̂	923	930̂
588	937̂	945	952̂	960	967̂	974̂	982	989̂	997	*004
589	77 011̂	019	026̂	033̂	041	048̂	055̂	063	070̂	078
590	085	092̂	100	107̂	114̂	122	129̂	136̂	144	151̂
591	158̂	166	173̂	181	188	195̂	203	210	217̂	225
592	232	239̂	247	254	261̂	269	276	283̂	291	298
593	305̂	313	320	327̂	335	342	349̂	356̂	364	371̂
594	378̂	386	393̂	400̂	408	415	422̂	430	437	444̂
595	451̂	459	466̂	473̂	481	488	495̂	503	510	517̂
596	524̂	532	539	546̂	554	561	568̂	575̂	583	590
597	597̂	604̂	612	619̂	626̂	634	641	648̂	655̂	663
598	670	677̂	684̂	692	699	706̂	713̂	721	728	735̂
599	742̂	750	757	764̂	771̂	779	786	793̂	800̂	808
600	815	822̂	829̂	837	844	851̂	858̂	866	873	880
N.	**0**	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**

P. P.

8
.1	0.8
.2	1.6
.3	2.4
.4	3.2
.5	4.0
.6	4.8
.7	5.6
.8	6.4
.9	7.2

7̂
.1	0.7̂
.2	1.5
.3	2.2̂
.4	3.0
.5	3.7̂
.6	4.5
.7	5.2̂
.8	6.0
.9	6.7̂

7
.1	0.7
.2	1.4
.3	2.1
.4	2.8
.5	3.5
.6	4.2
.7	4.9
.8	5.6
.9	6.3

TABLE V.—LOGARITHMS OF NUMBERS.

N.	0	1	2	3	4	5	6	7	8	9
600	77 815	82̂2	82̂9	837	844	85̂1	85̂8	866	873	880
601	88̂7	89̂4	902	909	91̂6	92̂3	931	938	945	95̂2
602	95̂9	967	974	98̂1	98̂8	99̂5	*003	*010	*01̂7	*02̂4
603	78 03̂1	039	046	05̂3	06̂0	067	075	082	08̂9	09̂6
604	10̂3	111	118	12̂5	13̂2	13̂9	147	154	161	16̂8
605	17̂5	18̂2	190	197	204	21̂1	21̂8	226	233	240
606	24̂7	25̂4	26̂1	269	276	283	290	29̂7	304	31̂1
607	319	326	333	34̂0	34̂7	35̂4	362	369	376	383
608	39̂0	397	40̂4	412	419	426	433	44̂0	44̂7	45̂4
609	46̂1	469	476	483	490	497	50̂4	51̂1	51̂8	526
610	533	540	547	55̂4	56̂1	568	575	58̂3	590	597
611	604	611	61̂8	62̂5	63̂2	639	64̂6	654	661	668
612	675	682	68̂9	69̂6	70̂3	71̂0	71̂7	725	732	739
613	746	753	760	76̂7	77̂4	78̂1	78̂8	79̂5	80̂2	810
614	817	824	831	838	845	852	85̂9	86̂6	87̂3	88̂0
615	88̂7	89̂4	90̂1	90̂8	91̂5	923	930	937	944	951
616	958	965	972	979	98̂6	99̂3	*00̂0	*00̂7	*01̂4	*02̂1
617	79 02̂8	03̂5	04̂2	049	056	06̂3	07̂0	078	085	092
618	099	106	113	120	127	134	141	148	155	162
619	169	176	183	190	197	204	211	218	225	232
620	239	246	253	260	267	274	281	288	295	302
621	309	316	323	330	337	344	35̂1	358	365	372
622	379	386	393	400	407	414	421	428	435	442
623	449	456	46̂2	46̂9	47̂6	48̂3	49̂0	497	504	51̂1
624	51̂8	52̂5	53̂2	53̂9	546	553	560	567	574	581
625	588	595	602	609	616	62̂2	62̂9	63̂6	64̂3	65̂0
626	65̂7	66̂4	67̂1	678	685	692	699	706	713	720
627	727	73̂3	74̂0	74̂7	75̂4	76̂1	76̂8	775	782	789
628	796	803	810	81̂6	82̂3	83̂0	837	84̂4	85̂1	858
629	865	872	879	886	89̂2	89̂9	90̂6	91̂3	92̂0	927
630	934	941	948	95̂4	96̂1	96̂8	97̂5	98̂2	989	996
631	80 00̂3	010	01̂6	02̂3	03̂0	03̂7	044	05̂1	058	065
632	07̂1	07̂8	08̂5	09̂2	099	106	113	120	12̂6	13̂3
633	14̂0	147	154	161	168	17̂4	18̂1	18̂8	195	202
634	209	216	22̂2	22̂9	23̂6	243	250	257	26̂3	27̂0
635	27̂7	284	291	298	30̂4	31̂1	31̂8	325	332	339
636	34̂5	35̂2	35̂9	366	373	380	38̂6	393	40̂0	407
637	414	421	42̂7	43̂4	441	448	455	46̂1	46̂8	47̂5
638	482	489	49̂5	50̂2	50̂9	516	523	52̂9	53̂6	543
639	550	557	56̂3	57̂0	57̂7	584	591	59̂7	60̂4	611
640	618	625	63̂1	63̂8	645	652	65̂8	66̂5	67̂2	679
641	686	69̂2	69̂9	706	713	71̂9	72̂6	733	740	74̂6
642	75̂3	76̂0	767	774	78̂0	78̂7	794	801	80̂7	81̂4
643	821	828	83̂4	84̂1	848	855	86̂1	86̂8	875	882
644	88̂8	89̂5	902	909	91̂5	92̂2	929	936	94̂2	949
645	956	96̂2	96̂9	976	983	98̂9	99̂6	*003	*010	*01̂6
646	81 02̂3	030	03̂6	04̂3	050	057	06̂3	07̂0	077	08̂3
647	09̂0	097	104	11̂0	11̂7	124	13̂0	13̂7	144	151
648	15̂7	164	171	17̂7	18̂4	191	19̂7	20̂4	211	218
649	22̂4	231	238	24̂4	251	258	26̂4	27̂1	278	28̂4
650	29̂1	298	30̂4	31̂1	318	32̂4	33̂1	338	345	35̂1
N.	0	1	2	3	4	5	6	7	8	9

P. P.

	7
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.9	6.7̂

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.4	2.8
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.7	4.9
.8	5.6
.9	6.3

	6
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.2	1.3
.3	1.9
.4	2.6
.5	3.2̂
.6	3.9
.7	4.5̂
.8	5.2
.9	5.8

TABLE V.—LOGARITHMS OF NUMBERS.

N.	0	1	2	3	4	5	6	7	8	9
650	81 29î	298	30â	31î	318	32â	33î	338	345	35î
651	358	365	37î	378	385	39î	398	405	41î	418
652	425	43î	438	44â	45î	458	46â	47î	478	48â
653	49î	498	50â	51î	518	52â	531	538	54â	551
654	558	56â	571	57î	584	591	59î	604	611	61î
655	624	631	63î	644	65ô	65î	664	67ô	677	684
656	69ô	697	70â	710	717	72â	730	736	74â	750
657	75ô	763	770	77ô	783	78ô	796	803	80ô	816
658	82â	829	836	84â	849	85î	862	869	87ô	882
659	88ô	895	90î	90ô	915	92î	928	93â	941	948
660	954	961	96î	974	98ô	98î	994	*00ô	*007	*01â
661	82 020	02ô	03â	040	04ô	053	05î	066	07â	079
662	086	09â	099	10ô	112	11ô	125	13î	138	145
663	15î	158	16â	171	17î	184	19ô	197	20â	21ô
664	217	22â	230	23ô	243	24ô	256	26â	269	27ô
665	282	28ô	295	302	30ô	315	32î	328	33â	341
666	34î	354	36ô	367	37â	380	38ô	393	39ô	406
667	41â	419	42ô	432	43ô	445	45î	458	46â	471
668	47ô	484	49ô	497	50â	510	51ô	523	52ô	536
669	54â	549	555	562	56ô	575	58î	588	59â	601
670	60ô	614	62ô	627	63â	640	64ô	653	65ô	666
671	67â	67ô	685	69î	698	70â	711	71ô	724	73ô
672	737	74â	750	75ô	763	769	77ô	782	78ô	795
673	80î	808	81â	821	82ô	834	840	84ô	853	85ô
674	866	87â	879	88ô	892	898	90â	911	91ô	924
675	93ô	937	943	949	956	96â	969	97ô	982	988
676	994	*001	*00ô	*01â	*02ô	*027	*03â	*03ô	*046	*052
677	83 059	06ô	07î	078	08â	091	09ô	10â	110	11ô
678	123	12ô	136	142	14ô	155	16î	168	174	18ô
679	187	19â	200	206	21â	219	22ô	23î	238	24â
680	251	25ô	26â	270	27ô	283	289	29ô	302	30ô
681	31â	321	32ô	334	340	34ô	353	35ô	36ô	372
682	37ô	385	391	39ô	404	41ô	41ô	423	42ô	43ô
683	442	448	455	461	467	474	480	48ô	493	49ô
684	50ô	512	51ô	52â	531	53ô	54â	550	55ô	56â
685	569	57ô	58î	588	59â	60ô	607	61â	61ô	626
686	63â	63ô	645	65î	657	664	67ô	676	683	68ô
687	69ô	702	70ô	71â	721	72ô	73â	740	746	75â
688	759	765	77î	778	784	79ô	796	803	80ô	81ô
689	822	828	83â	841	847	85â	85ô	866	87â	87ô
690	885	891	89ô	904	910	91ô	92â	929	935	94î
691	948	954	96ô	96ô	973	979	98ô	992	998	*00â
692	84 01ô	017	023	02ô	03ô	042	048	05â	061	067
693	07â	07ô	086	092	098	10â	111	117	12â	12ô
694	136	142	14ô	15â	161	167	17â	17ô	186	192
695	19ô	20â	211	217	22â	22ô	236	242	24ô	25â
696	261	267	27â	27ô	286	292	298	30â	311	317
697	32â	32ô	33ô	342	348	35â	36ô	367	373	37ô
698	38ô	392	398	404	41ô	41ô	423	429	43ô	44î
699	44ô	454	460	46ô	47â	479	485	491	49ô	50â
700	510	516	522	52ô	53â	541	547	553	55ô	56ô

P. P.

	7		8		6
.1	0.7	.1	0.6	.1	0.6
.2	1.4	.2	1.3	.2	1.2
.3	2.1	.3	1.9	.3	1.8
.4	2.8	.4	2.6	.4	2.4
.5	3.5	.5	3.2	.5	3.0
.6	4.2	.6	3.9	.6	3.6
.7	4.9	.7	4.5	.7	4.2
.8	5.6	.8	5.2	.8	4.8
.9	6.3	.9	5.8	.9	5.4

TABLE V.—LOGARITHMS OF NUMBERS.

N.	0	1	2	3	4	5	6	7	8	9
700	84 510	516	522	528̂	534̂	541	547	553	559̂	565̂
701	572	578	584	590̂	596	603	609	615	621̂	627̂
702	633̂	640	646	652̂	658̂	664̂	671	677	683	689
703	695̂	701̂	708	714	720	726̂	732̂	739	745	751
704	757̂	763̂	769̂	776	782	788	794̂	800̂	806̂	813
705	819	825	831	837̂	843̂	849	856	862	868	874̂
706	880̂	886̂	893	899	905	911	917̂	923̂	929̂	936
707	942	948	954	960̂	966̂	972̂	979	985	991	997
708	85 003̂	009̂	015	021̂	028	034	040	046	052̂	058̂
709	064̂	070̂	077	083	089	095	101̂	107	113̂	119̂
710	126	132	138	144	150̂	156̂	162̂	168̂	174̂	181
711	187	193	199	205̂	211̂	217̂	223̂	229̂	236	242
712	248	254	260	266̂	272̂	278	284̂	290̂	297	303
713	309	315	321	327	333	339	345̂	351̂	357̂	363̂
714	370	376	382	388	394	400	406̂	412̂	418̂	424̂
715	430̂	436̂	443	449	455	461	467	473	479	485
716	491̂	497̂	503̂	509̂	515̂	521̂	527̂	533	540	546
717	552	558	564	570	576	582	588	594	600̂	606̂
718	612̂	618̂	624̂	630̂	636̂	642̂	648̂	655	661	667
719	673	679	685	691	697	703	709	715	721	727
720	733	739̂	745̂	751̂	757̂	763̂	769̂	775̂	781̂	787̂
721	793̂	799̂	805̂	811̂	817̂	823̂	829̂	835̂	841̂	847̂
722	853̂	859̂	865̂	872	878	884	890	896	902	908
723	914	920	926	932	938	944	950	956	962	968
724	974	980	986	992	998	*004	*010	*016	*022	*028
725	86 034	040	046	052	058	063̂	069̂	075̂	081̂	087̂
726	093̂	099̂	105̂	111̂	117̂	123̂	129̂	135̂	141̂	147̂
727	153̂	159̂	165̂	171̂	177̂	183̂	189̂	195	201	207
728	213	219	225	231	237	243	249	255	261	267
729	273	278̂	284̂	290̂	296̂	302̂	308̂	314̂	320̂	326̂
730	332̂	338̂	344	350̂	356	362	368	374	380	386
731	391̂	397̂	403̂	409̂	415̂	421̂	427̂	433̂	439	445
732	451̂	457	463	469	475	481	486̂	492̂	498̂	504̂
733	510̂	516̂	522	528	534	540	546	552	558	563̂
734	569̂	575̂	581̂	587̂	593̂	599	605	611	617	623
735	628̂	634̂	640̂	646̂	652̂	658̂	664	670	676	682
736	688	693̂	699̂	705̂	711̂	717̂	723	729	735	741
737	746̂	752̂	758̂	764̂	770̂	776	782	788	794	800
738	805̂	811̂	817̂	823̂	829	835	841	847	852̂	858̂
739	864̂	870̂	876	882	888	894	899̂	905̂	911̂	917̂
740	923	929	935	941	946̂	952̂	958̂	964	970	976
741	982	987̂	993̂	999̂	*005̂	*011	*017	*023	*028̂	*034̂
742	87 040̂	046	052	058	064	069̂	075̂	081̂	087	093
743	099	104̂	110̂	116̂	122̂	128	134	140	145̂	151̂
744	157̂	163	169	175	180̂	186̂	192̂	198	204	210
745	215̂	221̂	227̂	233	239	245	250̂	256̂	262	268
746	274	279̂	285̂	291̂	297	303	309	314̂	320̂	326
747	332	338	343̂	349̂	355̂	361	367	372̂	378̂	384̂
748	390	396	402	407̂	413̂	419	425	431	436̂	442̂
749	448̂	454	460	465̂	471̂	477	483	489	494̂	500̂
750	506	512	517̂	523̂	529̂	535	541	546̂	552̂	558̂
N.	0	1	2	3	4	5	6	7	8	9

P. P.

δ
.1	0.6̂
.2	1.3
.3	1.9̂
.4	2.6
.5	3.2̂
.6	3.9
.7	4.5̂
.8	5.2
.9	5.8̂

6
.1	0.6
.2	1.2
.3	1.8
.4	2.4
.5	3.0
.6	3.6
.7	4.2
.8	4.8
.9	5.4

δ
.1	0.5̂
.2	1.1̂
.3	1.6̂
.4	2.2
.5	2.7̂
.6	3.3
.7	3.8̂
.8	4.4
.9	4.9̂

TABLE V.—LOGARITHMS OF NUMBERS.

N.	0	1	2	3	4	5	6	7	8	9	P. P.	
750	87 506	512	517̂	523̂	529̂	535	541	546̂	55̂2	558̂		
751	564	570	575̂	58î	587	593	598̂	604̂	610	616		
752	622	627̂	633̂	639	645	65ô	656̂	662	668	673̂		
753	679̂	685̂	691	697	70̂2	708̂	714	720	725̂	73î		
754	737	743	748̂	754̂	760	766	77î	777̂	783	789		
755	794̂	80ô	806	812	817̂	823̂	829	835	84ô	846̂		
756	852	858	863̂	869̂	875	881	886̂	89̂2	898	904		**6**
757	909̂	915̂	921	927	93̂2	938̂	944	949̂	955̂	961	.1	0.6
758	967	97̂2	978̂	984	990	995̂	*00î	*007	*01̂2	*018̂	.2	1.2
759	88 024	030	035̂	04î	047	053	058̂	064	070	075̂	.3	1.8
760	08î	087	093	098̂	104	110	115̂	12î	127	133	.4	2.4
761	138̂	144	150	155̂	16î	167	17̂2	178̂	184	190	.5	3.0
762	195̂	201	207	21̂2	218̂	224	229̂	235̂	241	247	.6	3.6
763	25̂2	258	264	269̂	275	281	286̂	29̂2	298	303̂	.7	4.2
764	309̂	315	32ô	32̂6	33̂2	337̂	343̂	349	355	36ô	.8	4.8
765	366	37̂2	377̂	383	389̂	39̂4	400	406	41î	417	.9	5.4
766	423	428̂	434	440	445̂	451	457	46̂2	468	474		
767	479̂	485	491	496̂	502	508	513̂	519	525	53ô		
768	536̂	542	547̂	553	558̂	564̂	570	575̂	58î	587		
769	59̂2	598	604	609̂	615	621	626̂	632	638	643̂		
770	649	654̂	66ô	666	67î	677̂	683	688̂	694	700		
771	705̂	711	716̂	72̂2	728	733̂	739	745	75ô	756		**5**
772	76î	767̂	773	778̂	784	790	795̂	801	806̂	81̂2	.1	0.5̂
773	818	823̂	829	835	84ô	846	85î	857	863	868̂	.2	1.1
774	874	879̂	885̂	891	896̂	902	907̂	913̂	919	924̂	.3	1.6̂
775	930	936	94î	947	95̂2	958	964	969̂	975	98ô	.4	2.2
776	986	992	997̂	*003	*008̂	*014	*019̂	*025̂	*031	*036̂	.5	2.7̂
777	89 042	047̂	053̂	059	064̂	070	075̂	081	087	09̂2	.6	3.3
778	098	103̂	109	114̂	12ô	126	13î	137	14̂2	148	.7	3.8̂
779	153̂	159̂	165	17ô	176	18î	187	193	198̂	204	.8	4.4
780	209̂	215	22ô	226	23î	237̂	243	248̂	254	259̂	.9	4.9
781	265	27ô	276	282	287̂	293	298̂	304	309̂	315		
782	32ô	326	332	337̂	343	348̂	354	359̂	365	37ô		
783	376	38î	387̂	393	398̂	404	409̂	415	42ô	426		
784	43î	437	44̂2	448	454	459̂	465	47ô	476	48î		
785	487	49̂2	498	503̂	509	514̂	520	525̂	531	536̂		
786	54̂2	548	553̂	559	564̂	570	575̂	581	586̂	592		**5**
787	597̂	603	608̂	614	619̂	625	63ô	636	64î	647	.1	0.5
788	65̂2	658	663̂	669	674̂	680	685̂	691	696̂	702	.2	1.0
789	70̂7	713	718̂	724	729̂	735	74ô	746	75î	757	.3	1.5
790	76̂2	768	773̂	779	784̂	790	795̂	801	80̂6	812	.4	2.0
791	817̂	823	828̂	834	839̂	845	85ô	856	86î	867	.5	2.5
792	87̂2	878	883̂	889	894̂	900	905̂	911	916̂	922	.6	3.0
793	92̂7	933	938̂	943̂	949	954̂	960	965̂	971	976̂	.7	3.5
794	982	987̂	993	998̂	*004	*00̂9	*015	*02ô	*026	*031	.8	4.0
795	90 036̂	042	047̂	053	058̂	064	069̂	075	08ô	086	.9	4.5
796	09î	097	102	107̂	113	118̂	124	129̂	135	14ô		
797	146	15î	156̂	162	167̂	173	178̂	184	189̂	195		
798	20ô	205̂	211	216̂	222	227̂	233	238̂	244	249		
799	254̂	260	265̂	271	276̂	282	287̂	29̂2	298	303̂		
800	309	314̂	320	325̂	33ô	336	34î	347	35̂2	358		
N.	0	1	2	3	4	5	6	7	8	9	P. P.	

TABLE V.—LOGARITHMS OF NUMBERS.

N.	0	1	2	3	4	5	6	7	8	9	P. P.	
800	90 309	31â	320	32ŝ	33ô	336	34î	347	35â	358		
801	36ŝ	36ŝ	374	37ŷ	385	39ô	396	401	40ŝ	412		
802	41ŷ	423	42ŝ	43ŝ	439	44â	450	45ŝ	46ô	466		
803	47î	477	48â	488	493	49ŝ	504	50ŷ	515	520		
804	52ŝ	531	53ô	542	547	55â	55ŝ	56ŝ	569	574		
805	57ŷ	585	59ô	596	601	60ŝ	612	61ŷ	62â	628		
806	63ŝ	639	64â	64ŷ	655	66ô	666	671	67ŝ	682		
807	68ŷ	69â	698	70ŝ	709	71â	71ŷ	725	73ô	736		
808	741	74ŝ	752	75ŷ	76â	768	77ŝ	77ŝ	784	78ŷ		
809	795	800	80ŝ	811	81ŝ	82î	827	83â	838	843		
810	84ŝ	854	859	86â	870	87ŝ	88ô	886	89î	89ô		ŝ
811	902	90ŷ	913	918	92ŝ	929	934	93ŷ	945	95ô	.1	0.ŝ
812	95ŝ	961	96ŝ	97î	977	98â	98ŷ	993	998	*00ŝ	.2	1.1
813	91 009	01â	01ŷ	025	03ô	036	041	04ŝ	052	057	.3	1.ŝ
814	06â	068	073	07ŝ	084	089	09â	100	105	11ô	.4	2.2
815	116	121	12ŝ	13î	137	14â	14ŷ	153	15ŝ	16ŝ	.5	2.ŷ
816	169	17â	17ŷ	185	19ô	19ŝ	201	20ŝ	21î	217	.6	3.3
817	222	22ŷ	233	238	24ŝ	249	254	25ŷ	26â	270	.7	3.ŝ
818	27ŝ	28ô	286	29î	29ŝ	302	307	31â	318	323	.8	4.4
819	32ŝ	333	339	344	349	355	360	36ŝ	371	376	.9	4.ŷ
820	38î	38ŝ	392	39ŷ	40â	408	413	41ŝ	42ŝ	429		
821	43â	43ŷ	445	45ô	45ŝ	461	466	47î	47ŝ	482		
822	487	49â	49ŷ	503	50ŝ	51ŝ	519	524	52ŷ	53â		
823	540	54ŝ	55ô	55ŝ	561	56ŝ	57î	577	582	58ŷ		
824	59â	598	60ŝ	60ŝ	614	619	62â	62ŷ	635	640		
825	64ŝ	65ô	656	661	66ŝ	67î	677	682	68ŷ	69â		
826	698	70ŝ	70ŝ	714	719	72â	72ŷ	735	740	74ŝ		
827	75ô	756	761	76ŝ	77î	777	782	78ŷ	79â	798		
828	803	80ŝ	81ŝ	819	824	82ŷ	83â	83ŷ	845	850		
829	85ŝ	86ô	866	871	876	88î	887	892	89ŷ	90â		
830	908	913	91ŝ	92ŝ	92ŝ	934	939	94â	94ŷ	955		5
831	960	96ŝ	97ô	976	981	986	99î	996	*002	*007	.1	0.5
832	92 01â	01ŷ	023	028	033	03ŝ	04ŝ	049	054	05ŷ	.2	1.0
833	06â	06ŷ	075	080	08ŝ	09ô	096	101	106	11î	.3	1.5
834	11ŝ	122	127	132	13ŷ	14â	148	153	158	16ŝ	.4	2.0
835	16ŝ	174	179	184	18ŷ	19â	200	205	210	21ŝ	.5	2.5
836	22ô	226	231	236	24î	246	252	257	262	26ŷ	.6	3.0
837	27â	27ŷ	283	288	29ŝ	29ŝ	30ŝ	309	314	319	.7	3.5
838	32â	32ŷ	335	340	345	35ô	35ŝ	36ô	366	371	.8	4.0
839	376	38î	38ŝ	39î	397	402	407	41â	41ŷ	423	.9	4.5
840	428	433	43ŝ	44ŝ	448	454	459	464	46ŷ	47â		
841	47ŷ	485	490	495	50ô	50ŝ	51ô	51ŝ	521	526		
842	531	53ô	54î	54ô	552	557	562	56ŷ	57â	577		
843	583	588	593	598	60ŝ	60ŝ	61ŝ	619	624	629		
844	634	63ŷ	64â	64ŷ	655	660	665	670	67ŝ	68ô		
845	68ŝ	691	696	701	706	71î	71ŝ	72î	727	73²		
846	737	742	74ŷ	75â	75ŷ	76â	768	773	77ŝ	783		
847	78ŝ	793	79ŝ	80ŝ	809	814	819	824	82ŷ	83â		
848	83ŷ	84â	850	855	860	865	87ô	87ŝ	88ô	88ŝ		
849	891	896	901	906	911	91ŝ	92î	92ŝ	93î	937		
850	942	947	95²	957	96â	967	97²	97ŷ	98â	988		
N.	0	1	2	3	4	5	6	7	8	9	P. P.	

TABLE V.—LOGARITHMS OF NUMBERS.

N.	0	1	2	3	4	5	6	7	8	9
850	92 942	947	952	957	962̂	967̂	972̂	977̂	982̂	988
851	993	998	*003	*008̂	*013̂	*018̂	*023̂	*028̂	*034	*039
852	93 044	049	054	059	064̂	069̂	074̂	079̂	084̂	090
853	095	100	105	110	115̂	120̂	125̂	130̂	135̂	140̂
854	146	151	156	161	166	171	176̂	181̂	186̂	191̂
855	196̂	201̂	207	212	217	222	227	232	237	242̂
856	247̂	252̂	257̂	262̂	267̂	272̂	278	283	288	293
857	298	303	308	313̂	318̂	323̂	328̂	333̂	338̂	343̂
858	348̂	354	359	364	369	374	379	384	389	394̂
859	399̂	404̂	409̂	414̂	419̂	424̂	429̂	434	439̂	445
860	450	455	460	465	470	475	480	485	490	495̂
861	500̂	505̂	510̂	515̂	520̂	525̂	530̂	535̂	540̂	545̂
862	550̂	556	561	566	571	576	581	586	591	596
863	601	606	611	616	621	626	631̂	636̂	641̂	646̂
864	651̂	656̂	661̂	666̂	671̂	676̂	681̂	686̂	691̂	696̂
865	701̂	706̂	711̂	716̂	721̂	726̂	731̂	736̂	742	747
866	752	757	762	767	772	777	782	787	792	797
867	802	807	812	817	822	827	832	837	842	847
868	852	857	862	867	872	877	882	887	892	897
869	902	907	912	917	922	927	932	937	942	947
870	952	957	962	967	972	977	982	987	992	997
871	94 002	007	012	017	022	026̂	031̂	036̂	041̂	046̂
872	051̂	056̂	061̂	066̂	071̂	076̂	081̂	086̂	091̂	096̂
873	101̂	106̂	111̂	116̂	121̂	126̂	131̂	136	141	146
874	151	156	161	166	171	176	181	186	191	196
875	201	206	210̂	215̂	220̂	225̂	230̂	235̂	240̂	245̂
876	250̂	255̂	260̂	265̂	270	275	280	285	290	295
877	300	305	310	315	320	324̂	329̂	334̂	339̂	344̂
878	349̂	354̂	359̂	364̂	369	374	379	384	389	394
879	399	404	409	413̂	418̂	423̂	428̂	433̂	438̂	443̂
880	448̂	453	458	463	468	473	478	483	487̂	492̂
881	497̂	502̂	507̂	512̂	517̂	522	527	532	537	542
882	547	552	556̂	561̂	566̂	571̂	576̂	581̂	586	591
883	596	601	606	611	615̂	620̂	625̂	630̂	635̂	640̂
884	645	650	655	660	665	670	674̂	679̂	684̂	689̂
885	694̂	699	704	709	714	719	724	728̂	733̂	738̂
886	743̂	748̂	753	758̂	763	768	773	777̂	782̂	787̂
887	792̂	797̂	802	807	812	817	821̂	826̂	831̂	836̂
888	841̂	846	851	856	861	865̂	870̂	875̂	880̂	885̂
889	890	895	900	905	909̂	914̂	919̂	924̂	929	934
890	939	944	949	953̂	958	963̂	968̂	973	978	983
891	988	992̂	997̂	*002̂	*007̂	*012	*017	*022	*026̂	031̂
892	95 036̂	041̂	046	051	056	061	065̂	070̂	075̂	080̂
893	085	090	095	099̂	104̂	109̂	114̂	119	124	129
894	134	138̂	143̂	148̂	153	158	163	167̂	172̂	177̂
895	182̂	187	192	197	201̂	206̂	211̂	216̂	221	226
896	231	235̂	240̂	245̂	250	255	260	264̂	269̂	274̂
897	279	284	289	294	298	303̂	308̂	313	318	323
898	327̂	332̂	337̂	342	347	352	356̂	361̂	366̂	371
899	376̂	381	385̂	390̂	395̂	400	405	410	414̂	419̂
900	424̂	429	434	438̂	443̂	448̂	453	458̂	463	467̂
N.	0	1	2	3	4	5	6	7	8	9

P. P.

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.1	0.5̂
.2	1.1
.3	1.6̂
.4	2.2
.5	2.7̂
.6	3.3
.7	3.8̂
.8	4.4
.9	4.9̂

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.2	1.0
.3	1.5
.4	2.0
.5	2.5
.6	3.0
.7	3.5
.8	4.0
.9	4.5

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.1	0.4̂
.2	0.9
.3	1.3̂
.4	1.8
.5	2.2̂
.6	2.7
.7	3.1̂
.8	3.6
.9	4.0̂

TABLE V.—LOGARITHMS OF NUMBERS.

N.	0	1	2	3	4	5	6	7	8	9
900	95 424̂	429	434	438̂	443̂	448̂	453	458̂	463	467̂
901	47̂2̂	477̂	482	487	492	496̂	50î	506	511	516
902	52ô	525̂	53ô	535	540	544̂	549̂	554̂	559	564
903	569	573̂	578̂	583	588̂	593	597̂	602̂	607	612
904	617	62î	626̂	63î	636	641	645̂	650̂	655̂	660
905	665	669̂	674̂	679̂	684	689	693̂	698̂	703	708
906	713	717̂	722̂	727	732	737	74î	746	751	756
907	760̂	765̂	77ô	775	780	784̂	789̂	794	799	804
908	808̂	813̂	818	823	827̂	832̂	837̂	842	847	85î
909	856̂	861	866	87ô	875̂	880̂	885	890	894̂	899̂
910	904	909	913̂	918̂	923	928	933	937̂	942̂	947
911	952	956̂	96î	966	97î	975̂	980̂	985	990	994̂
912	999̂	*004	*009	*014	*018̂	*023̂	*028	*033	*037̂	*042̂
913	96 047	052	056̂	06î	066	071	075̂	080̂	085	090
914	094̂	099̂	104	109	113̂	118̂	123	128	132̂	137̂
915	142	147	15î	156̂	161	166	170̂	175̂	180	185
916	189̂	194̂	199	204	208̂	213	218	222̂	227̂	232
917	237	24î	246̂	251	256	260̂	265̂	270	275	279̂
918	284̂	289	293̂	298	303	308	312̂	317̂	322	327
919	33î	336̂	341	345̂	35ô	355	360	364̂	369̂	374
920	379	383̂	388	393	397̂	402̂	407	412	416̂	421
921	426	430̂	435̂	440	445	449̂	454	459	463̂	468̂
922	473	478	482̂	487	492	496̂	50î	506	511	515̂
923	520	525	529̂	534̂	539	543̂	548̂	553	558	562̂
924	567	572	576̂	58î	586	590̂	595̂	600	605	609̂
925	614	619	623̂	628̂	633	637̂	642̂	647	65î	656̂
926	661	666	670̂	675	680	684̂	689	694	698̂	703̂
927	708	712̂	717̂	722	726̂	73î	736	741	745̂	750
928	755	759̂	764	769	773̂	778	783	787̂	792	797
929	80î	806	811	815̂	820̂	825	829̂	834̂	839	843̂
930	848̂	853	857̂	862̂	867	87î	876̂	881	885̂	890̂
931	895	899̂	904̂	909	913̂	918̂	923	927̂	932̂	937
932	94î	946̂	951	955̂	960	965	969̂	974	979	983̂
933	988	993	997̂	*002	*007	*011̂	*016	*020̂	*025̂	*030
934	97 034̂	039̂	044	048̂	053̂	058	062̂	067	072	076̂
935	081	086	090̂	095	099̂	104̂	109	113̂	118̂	123
936	127̂	132	137	141̂	146	151	155̂	160	164̂	169̂
937	174	178̂	183	188	192̂	197	202	206̂	211	215̂
938	220̂	225	229̂	234	239	243̂	248	252̂	257̂	262
939	266̂	271	276	280̂	285	289̂	294̂	299	303̂	308
940	313	317̂	322	326̂	33î	336	340̂	345	349̂	354̂
941	359	363̂	368	373	377̂	382	386̂	39î	396	400̂
942	405	409̂	414̂	419	423̂	428	432̂	437̂	442	446̂
943	451	456	460̂	465	469̂	474	479	483̂	488	492̂
944	497	502	506̂	511	515̂	520	525	529̂	534	538̂
945	543	548	552̂	557	561̂	566	570̂	575̂	580	584̂
946	589	593̂	598̂	603	607̂	612	616̂	621	626	630̂
947	635	639̂	644	649	653̂	658	662̂	667	67î	676̂
948	681	685̂	690	694̂	699	703̂	708̂	713	717̂	722
949	726̂	731	736	740̂	745	749̂	754	758̂	763	768
950	772̂	777	78î	786	790̂	795	800	804̂	809	813̂
N.	0	1	2	3	4	5	6	7	8	9

P. P.

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.1	0.5
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.1	0.4̂
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.3	1.3̂
.4	1.8
.5	2.2̂
.6	2.7
.7	3.î
.8	3.6
.9	4.ô

TABLE V.—LOGARITHMS OF NUMBERS.

N.	0	1	2	3	4	5	6	7	8	9
950	97 77̂2	777	78̂1	786	79̂6	795	800	80̂4	809	81̂3
951	818	82̂2	827	83̂1	83̂6	841	84̂5	850	85̂4	859
952	86̂3	86̂8	873	87̂7	882	88̂6	891	89̂5	900	90̂4
953	90̂9	914	91̂8	923	92̂7	932	936	941	94̂5	95̂6
954	955	95̂9	964	968	973	97̂7	982	98̂6	991	996
955	98 00̂0	005	00̂9	014	01̂8	023	02̂7	032	03̂6	041
956	046	05̂0	055	05̂9	064	06̂8	073	07̂7	082	08̂6
957	091	09̂5	10̂0	105	10̂9	114	11̂8	123	12̂7	132
958	13̂6	141	14̂5	150	15̂4	159	16̂3	168	173	17̂7
959	182	18̂6	191	19̂5	200	20̂4	209	21̂3	218	22̂2
960	227	23̂1	236	24̂0	245	24̂9	25̂4	259	26̂3	268
961	27̂2	277	28̂1	286	29̂0	295	29̂9	304	30̂8	313
962	31̂7	322	32̂6	331	33̂5	340	34̂4	349	35̂3	358
963	36̂2	367	37̂1	376	38̂0	385	38̂9	394	39̂8	403
964	40̂7	412	41̂6	421	42̂5	430	43̂4	439	44̂3	448
965	45̂2	457	46̂1	466	47̂0	475	47̂9	484	48̂8	493
966	49̂7	502	50̂6	511	51̂5	520	52̂4	529	53̂3	538
967	54̂2	547	55̂1	556	56̂0	565	56̂9	574	57̂8	583
968	58̂7	592	59̂6	601	60̂5	610	61̂4	619	62̂3	628
969	63̂2	637	64̂1	646	65̂0	655	65̂9	663	668	67̂2
970	677	68̂1	686	69̂0	695	69̂9	704	70̂8	713	71̂7
971	722	72̂6	731	73̂5	740	74̂4	749	753	75̂7	762
972	76̂6	771	77̂5	780	78̂4	789	79̂3	798	80̂2	807
973	81̂1	815	820	82̂4	829	83̂3	838	84̂2	847	85̂1
974	856	86̂0	865	86̂9	87̂3	878	88̂2	887	89̂1	896
975	90̂0	905	90̂9	914	91̂8	92̂2	927	93̂1	936	94̂0
976	945	94̂9	954	958	963	967	97̂1	976	98̂0	985
977	98̂9	994	99̂8	*003	*007	*01̂1	*016	*02̂0	*025	*02̂9
978	99 034	03̂8	043	047	05̂1	056	06̂0	065	06̂9	074
979	07̂8	08̂2	087	09̂1	096	10̂0	105	10̂9	11̂3	118
980	12̂2	127	13̂1	136	14̂0	145	149	15̂3	158	16̂2
981	167	17̂1	176	180	18̂4	189	19̂3	198	20̂2	20̂6
982	211	21̂5	220	22̂4	229	23̂3	23̂7	242	24̂6	251
983	25̂5	260	264	26̂8	273	27̂7	282	28̂6	29̂0	295
984	29̂9	304	30̂8	31̂2	317	32̂1	326	33̂0	335	339
985	34̂3	348	35̂2	357	36̂1	36̂5	370	37̂4	379	38̂3
986	38̂7	392	396	40̂1	405	40̂9	414	41̂8	423	42̂7
987	43̂1	436	44̂0	445	44̂9	45̂3	458	46̂2	467	47̂1
988	47̂5	480	48̂4	489	49̂3	497	502	50̂6	511	515
989	51̂9	524	52̂8	533	537	54̂1	546	55̂0	55̂4	559
990	56̂3	568	57̂2	576	581	58̂5	590	594	59̂8	603
991	60̂7	61̂1	616	62̂0	625	62̂9	63̂3	638	64̂2	647
992	651	65̂5	660	66̂4	66̂8	673	67̂7	682	686	69̂0
993	695	69̂9	70̂3	708	71̂2	717	721	72̂5	730	73̂4
994	73̂8	743	74̂7	75̂1	756	76̂0	765	769	77̂3	778
995	78̂2	78̂6	791	79̂5	800	804	80̂8	813	817	82̂1
996	826	83̂0	83̂4	839	84̂3	847	852	856	861	865
997	86̂9	874	878	88̂2	887	89̂1	89̂5	900	90̂4	90̂8
998	913	91̂7	922	926	93̂0	935	939	94̂3	948	952
999	95̂6	961	965	96̂9	974	978	98̂2	987	99̂1	99̂5
1000	00 000	00̂4	00̂8	013	01̂7	02̂1	026	03̂0	03̂4	039
N.	**0**	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**

P. P.

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.1	0.5
.2	1.0
.3	1.5
.4	2.0
.5	2.5
.6	3.0
.7	3.5
.8	4.0
.9	4.5

	4̂
.1	0.4̂
.2	0.9
.3	1.3̂
.4	1.8
.5	2.2̂
.6	2.7
.7	3.1̂
.8	3.6
.9	4.0̂

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.1	0.4
.2	0.8
.3	1.2
.4	1.6
.5	2.0
.6	2.4
.7	2.8
.8	3.2
.9	3.6

TABLE V.—LOGARITHMS OF NUMBERS.

N.	0	1	2	3	4	5	6	7	8	9	P.P.
1000	000 000	043̂	087	130̂	173̂	217	260̂	304	347̂	390̂	
01	434	477̂	521	564	607̂	651	694̂	737̂	781	824̂	
02	867̂	911	954̂	997̂	*041	*084̂	*127̂	*171	*214̂	*257̂	
03	001 301	344	387̂	431	474	517̂	560̂	604	647	690̂	
04	733̂	777	820	863̂	906̂	950	993	*036̂	*079̂	*123	
05	002 166	209̂	252̂	295̂	339	382	425̂	468̂	511̂	555	
06	598	641	684̂	727̂	770̂	814	857	900	943	986̂	
07	003 029̂	072̂	115̂	159	202	245	288	331̂	374̂	417̂	43̂ 43
08	460̂	503̂	546̂	590	633	676	719	762	805	848	.1 4.3 4.3
09	891	934	977	*020̂	*063̂	*106̂	*149̂	*192̂	*235̂	*278̂	.2 8.7 8.6
1010	004 321̂	364̂	407̂	450̂	493̂	536̂	579̂	622̂	665	708	.3 13.0 12.9
11	751	794	837	880	923	966	*009	*051̂	*094̂	*137̂	.4 17.4 17.2 .5 21.7 21.5 .6 26.1 25.8
12	005 180̂	223̂	266̂	309	352	395	438	481	523̂	566̂	
13	609̂	652̂	695	738	781	824	866̂	909̂	952̂	995	.7 30.4 30.1 .8 34.8 34.4
14	006 038	081	123̂	166̂	209	252	295	337̂	380̂	423̂	.9 39.1 38.7
15	466	509	551̂	594̂	637	680	722̂	765̂	808	851	
16	893̂	936	979	*022̂	*064̂	*107̂	*150̂	*193̂	*235̂	*278̂	
17	007 321	363̂	406̂	449	491̂	534̂	577	620	662̂	705	
18	748	790̂	833	875̂	918̂	961	*003̂	*046̂	*089̂	*131̂	
19	008 174	217	259̂	302	344̂	387	430	472̂	515	557̂	
1020	600	642̂	685̂	728	770̂	813	855̂	898	940̂	983	
21	009 025̂	068̂	111	153̂	196	238̂	281	323̂	366	408̂	
22	451	493̂	536	578̂	621	663̂	706	748̂	790̂	833	42̂ 42
23	875̂	918	960̂	*003̂	*045̂	*088̂	*130̂	*172̂	*215	*257̂	.1 4.2 4.2 .2 8.5 8.4 .3 12.7 12.6
24	010 300	342̂	385	427	469̂	512	554̂	596̂	639	681̂	
25	724	766	808̂	851	893̂	935̂	978	*020̂	*062̂	*105	.4 17.0 16.8 .5 21.2 21.0 .6 25.5 25.2
26	011 147̂	189̂	232	274̂	316̂	359	401̂	443̂	486	528	
27	570̂	612̂	655	697̂	739̂	782	824	866̂	908̂	951	.7 29.7 29.4 .8 34.0 33.6
28	993	*035̂	*077̂	*120̂	*162̂	*204̂	*246̂	*288̂	*331	*373̂	.9 38.2 37.8
29	012 415̂	457̂	500	542	584	626̂	668̂	710̂	753	795	
1030	837	879̂	921̂	963̂	*006	*048̂	*090	*132̂	174̂	216̂	
31	013 258̂	301	343	385	427	469	511̂	553̂	595̂	637̂	
32	679̂	722	764	806	848	890	932	974	*016̂	*058̂	
33	014 100̂	142̂	184̂	226̂	268̂	310̂	352̂	394̂	436̂	478	
34	520̂	562̂	604̂	646̂	688̂	730̂	772̂	814̂	856̂	898̂	
35	940̂	982̂	*024̂	*066̂	*108̂	*150̂	*192̂	*234̂	*276̂	*318̂	
36	015 360	401̂	443̂	485̂	527̂	569̂	611	653	695	737	41̂ 41
37	779	820̂	862̂	904̂	946	988	*030̂	*072̂	*113̂	*155̂	.1 4.1 4.1 .2 8.3 8.2 .3 12.4 12.3
38	016 197̂	239̂	281	323	364̂	406̂	448̂	490	532	573̂	
39	615̂	657̂	699	741	782̂	824̂	866̂	908	950	991̂	.4 16.6 16.4 .5 20.7 20.5 .6 24.9 24.6
1040	017 033̂	075	117	158̂	200̂	242	284	325̂	367̂	409	
41	450̂	492̂	534	576	617̂	659̂	701	742̂	784̂	826	.7 29.0 28.7 .8 33.2 32.8
42	867̂	909̂	951	992̂	*034̂	*076̂	*117̂	*159̂	*201	*242̂	.9 37.3 36.9
43	018 284̂	326	367̂	409	451	492̂	534	575̂	617̂	659	
44	700̂	742	783̂	825̂	867	908̂	950	991̂	*033	*074̂	
45	019 116̂	158	199̂	241	282̂	324	365̂	407	448̂	490	
46	53î	573	614̂	656	697̂	739	780̂	822	863̂	905	
47	946̂	988	*029̂	*071	*112̂	*154	*195̂	*237̂	*278̂	*320	
48	020 361̂	402̂	444	485̂	527	568̂	610	651̂	692̂	734	
49	775̂	817	858̂	899̂	941	982̂	*024	*065̂	*106̂	*148	
1050	021 189̂	230̂	272	313̂	354̂	396	437̂	478̂	520	561̂	
N.	0	1	2	3	4	5	6	7	8	9	P.P.

TABLE V.—LOGARITHMS OF NUMBERS.

N.	0	1	2	3	4	5	6	7	8	9
1050	021 189	230	272	313	354	396	437	478	520	561
51	602	644	685	726	768	809	850	892	933	974
52	022 015	057	098	139	181	222	263	304	346	387
53	428	469	511	552	593	634	676	717	758	799
54	846	882	923	964	*005	*046	*088	*129	*170	*211
55	023 252	293	335	376	417	458	499	540	581	623
56	664	705	746	787	828	869	910	951	993	*034
57	024 075	116	157	198	239	280	321	362	403	444
58	485	526	568	609	650	691	732	773	814	855
59	896	937	978	*019	*060	*101	*142	*183	*224	*265
1060	025 306	347	388	429	469	510	551	592	633	674
61	715	756	797	838	879	920	961	*002	*042	*083
62	026 124	165	206	247	288	329	370	410	451	492
63	533	574	615	656	696	737	778	819	860	901
64	941	982	*023	*064	*105	*145	*186	*227	*268	*309
65	027 349	390	431	472	512	553	594	635	675	716
66	757	798	838	879	920	961	*001	*042	*083	*123
67	028 164	205	246	286	327	368	408	449	490	530
68	571	612	652	693	734	774	815	856	896	937
69	977	*018	*059	*099	*140	*181	*221	*262	*302	*343
1070	029 384	424	465	505	546	586	627	668	708	749
71	789	830	870	911	951	992	*032	*073	*114	*154
72	030 195	235	276	316	357	397	438	478	519	559
73	599	640	680	721	761	802	842	883	923	964
74	031 004	044	085	125	166	206	247	287	327	368
75	408	449	489	529	570	610	651	691	731	772
76	812	852	893	933	973	*014	*054	*094	*135	*175
77	032 215	256	296	336	377	417	457	498	538	578
78	619	659	699	739	780	820	860	900	941	981
79	033 021	061	102	142	182	222	263	303	343	383
1080	424	464	504	544	584	625	665	705	745	785
81	825	866	906	946	986	*026	*066	*107	147	187
82	034 227	267	307	347	388	428	468	508	548	588
83	628	668	708	748	789	829	869	909	949	989
84	035 029	069	109	149	189	229	269	309	349	389
85	429	470	510	550	590	630	670	710	750	790
86	830	870	910	950	990	*029	*069	*109	*149	*189
87	036 229	269	309	349	389	429	469	509	549	589
88	629	669	708	748	788	828	868	908	948	988
89	037 028	068	107	147	187	227	267	307	347	386
1090	426	466	506	546	586	625	665	705	745	785
91	825	864	904	944	984	*023	*063	*103	143	183
92	038 222	262	302	342	381	421	461	501	540	580
93	620	660	699	739	779	819	858	898	938	977
94	039 017	057	096	136	176	216	255	295	335	374
95	414	454	493	533	572	612	652	691	731	771
96	810	850	890	929	969	*008	*048	*088	*127	*167
97	040 206	246	286	325	365	404	444	483	523	563
98	602	642	681	721	760	800	839	879	918	958
99	997	*037	*076	*116	*155	*195	*234	*274	*313	*353
1100	041 392	432	471	511	550	590	629	669	708	748
N.	0	1	2	3	4	5	6	7	8	9

P. P.

41
.1 4.1
.2 8.3
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.4 16.6
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.3 12.0
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.6 24.0
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.1 3.9
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.3 11.8
.4 15.8
.5 19.7
.6 23.7
.7 27.9
.8 31.6
.9 35.5

TABLE VI.—LOGARITHMIC SINES AND TANGENTS OF SMALL ANGLES.

log sin ϕ = log ϕ'' + S. log tan ϕ = log ϕ'' + T.				**0°**	log ϕ'' = log sin ϕ + S'. log ϕ'' = log tan ϕ + T''.		
″	′	S	T	Log. Sin.	S′	T′	Log. Tan.
0	**0**	4.685 57̂	57̂	− ∞	5.314 4â	4â	− ∞
60	1	57̂	57̂	6.46 37â	4â	4â	6.46 37â
120	2	57̂	57̂	.76 47ŝ	4â	4â	.76 47ŝ
180	3	57̂	57̂	.94 08â	4â	4â	.94 08â
240	4	57̂	57̂	7.06 578	4â	4â	7.06 578
300	**5**	4.685 57̂	57̂	7.16 269	5.314 4â	4â	7.16 269
360	6	57̂	57̂	.24 187	4â	4â	.24 188
420	7	57̂	57̂	.30 88â	4â	4â	.30 882
480	8	57̂	57̂	.36 681	4â	4â	.36 681
540	9	57̂	57̂	.41 797	4â	4â	.41 797
600	**10**	4.685 57̂	57̂	7.46 37â	5.314 4â	4â	7.46 37â
660	11	57̂	57̂	.50 512	4â	4â	.50 512
720	12	57̂	57̂	.54 29ô	4â	4â	.54 291
780	13	57̂	57̂	.57 767	4â	4â	.57 767
840	14	57̂	57̂	.60 98ŝ	4â	4â	.60 98ŝ
900	**15**	4.685 57̂	58	7.63 98î	5.314 4â	42	7.63 982
960	16	57̂	58	.66 78ŝ	4â	42	.66 785
1020	17	57̂	58	.69 41ŷ	4â	42	.69 418
1080	18	57̂	58	.71 89ŷ	4â	42	.71 90ô
1140	19	57̂	58	.74.248	4â	42	.74 248
1200	**20**	4.685 57	58	7.76 47ŝ	5.314 43	42	7.76 476
1260	21	57	58	.78 59â	43	42	.78 595
1320	22	57	58	.80 61â	43	42	.80 61ŝ
1380	23	57	58	.82 545	43	42	.82 546
1440	24	57	58	.84 39ŝ	43	42	.84 39â
1500	25	4.685 57	5ŝ	7.86 166	5.314 43	4î	7.86 16ŷ
1560	26	57	58	.87 86ŷ	43	4î	.87 871
1620	27	57	58	.89 50ŝ	43	4î	.89 510
1680	28	57	5ŝ	.91 088	43	4î	.91 08ŷ
1740	29	57	58	.92 612	43	4î	.92 61ŝ
1800	**30**	4.685 57	58	7.94 084	5.314 43	4î	7.94 086
1860	31	57	5ŝ	.95 508	43	4î	.95 510
1920	32	57	58	.96 887	43	4î	.96 889
1980	33	57	59	.98 22ŝ	43	41	.98 22ŝ
2040	34	57	59	.99 520	43	41	.99 522
2100	35	4.685 5ŝ	59	8.00 77ŝ	5.314 4ŝ	41	8.00 781
2160	36	5ŝ	59	.02 002	4ŝ	41	.02 00â
2220	37	5ŝ	59	.03 192	4ŝ	41	.03 19â
2280	38	5ŝ	5ŷ	.04 350	4ŝ	4ô	.04 35â
2340	39	5ŝ	5ŷ	.05 478	4ŝ	4ô	.05 481
2400	**40**	4.685 5ŝ	5ŷ	8.06 57ŷ	5.314 4ŝ	4ô	8.06 58ô
2460	41	5ŝ	5ŷ	.07 650	4ŝ	4ô	.07 653
2520	42	5ŝ	5ŷ	.08 696	4ŝ	4ô	.08 69ŷ
2580	43	5ŝ	60	.09 71ŝ	4ŝ	40	.09 72î
2640	44	5ŝ	60	.10 71ŝ	4ŝ	40	.10 72ô
2700	45	4.685 56	60	8.11 69â	5.314 44	40	8.11 696
2760	46	56	60	.12 647	44	40	.12 651
2820	47	56	60	.13 581	44	40	.13 585
2880	48	56	6ô	.14 49ŝ	44	3ŷ	.14 499
2940	49	56	6ô	.15 39ô	44	3ŷ	.15 395
3000	**50**	4.685 56	6ô	8.16 268	5.314 44	3ŷ	8.16 27â
3060	51	56	6ô	.17 128	44	3ŷ	.17 133
3120	52	56	61	.17 97î	44	39	.17 976
3180	53	56	61	.18 798	44	39	.18 80ŝ
3240	54	5ŝ	61	.19 610	4â	39	.19 61ŝ
3300	55	4.685 5ŝ	61	8.20 407	5.314 4â	39	8.20 41â
3360	56	5ŝ	6î	.21 18ŷ	4â	3ŝ	.21 19ŝ
3420	57	5ŝ	6î	.21 958	4â	3ŝ	.21 964
3480	58	5ŝ	6î	.22 71ŝ	4â	3ŝ	.22 71ŷ
3540	59	5ŝ	62	.23 45ŝ	4â	3ŝ	.23 462

TABLE VI.—LOGARITHMIC SINES AND TANGENTS OF SMALL ANGLES.

log sin ϕ = log ϕ'' + S. log tan ϕ = log ϕ'' + T.		1°			log ϕ'' = log sin ϕ + S'. log ϕ'' = log tan ϕ + T'.		
''	'	S	T	Log. Sin.	S'	T'	Log. Tan.
3600	0	4.685 55	62	8.24 185	5.314 44	38	8.24 192
3660	1	55	62	.24 903	45	38	.24 910
3720	2	55	62	.25 609	45	38	.25 616
3780	3	55	62	.26 304	45	37	.26 311
3840	4	55	62	.26 988	45	37	.26 995
3900	5	4.685 55	62	8.27 661	5.314 45	37	8.27 669
3960	6	55	63	.28 324	45	37	.28 332
4020	7	54	63	.28 977	45	37	.28 985
4080	8	54	63	.29 620	45	37	.29 629
4140	9	54	63	.30 254	45	36	.30 263
4200	10	4.685 54	63	8.30 879	5.314 45	36	8.30 888
4260	11	54	63	.31 495	45	36	.31 504
4320	12	54	64	.32 102	45	36	.32 112
4380	13	54	64	.32 701	46	36	.32 711
4440	14	54	64	.33 292	46	36	.33 302
4500	15	4.685 54	64	8.33 875	5.314 46	35	8.33 885
4560	16	54	64	.34 450	46	35	.34 461
4620	17	54	65	.35 018	46	35	.35 029
4680	18	54	65	.35 578	46	35	.35 589
4740	19	53	65	.36 131	46	35	.36 143
4800	20	4.685 53	65	8.36 677	5.314 46	34	8.36 689
4860	21	53	65	.37 217	46	34	.37 229
4920	22	53	65	.37 750	46	34	.37 762
4980	23	53	66	.38 276	46	34	.38 289
5040	24	53	66	.38 796	47	34	.38 809
5100	25	4.685 53	66	8.39 310	5.314 47	33	8.39 323
5160	26	53	66	.39 818	47	33	.39 831
5220	27	53	67	.40 320	47	33	.40 334
5280	28	52	67	.40 816	47	33	.40 830
5340	29	52	67	.41 307	47	33	.41 321
5400	30	4.685 52	67	8.41 792	5.314 47	32	8.41 807
5460	31	52	67	.42 271	47	32	.42 287
5520	32	52	68	.42 746	47	32	.42 762
5580	33	52	68	.43 215	48	32	.43 231
5640	34	52	68	.43 680	48	31	.43 696
5700	35	4.685 52	68	8.44 139	5.314 48	31	8.44 156
5760	36	52	69	.44 594	48	31	.44 611
5820	37	51	69	.45 044	48	31	.45 061
5880	38	51	69	.45 489	48	30	.45 507
5940	39	51	69	.45 930	48	30	.45 948
6000	40	4.685 51	69	8.46 366	5.314 48	30	8.46 385
6060	41	51	70	.46 798	49	30	.46 817
6120	42	51	70	.47 226	49	30	.47 245
6180	43	51	70	.47 650	49	29	.47 669
6240	44	51	70	.48 069	49	29	.48 089
6300	45	4.685 50	71	8.48 485	5.314 49	29	8.48 505
6360	46	50	71	.48 896	49	28	.48 917
6420	47	50	71	.49 304	49	28	.49 325
6480	48	50	72	.49 708	49	28	.49 729
6540	49	50	72	.50 108	50	28	.50 130
6600	50	4.685 50	72	8.50 504	5.314 50	27	8.50 526
6660	51	50	72	.50 897	50	27	.50 920
6720	52	50	73	.51 286	50	27	.51 310
6780	53	49	73	.51 672	50	27	.51 696
6840	54	49	73	.52 055	50	26	.52 079
6900	55	4.685 49	73	8.52 434	5.314 50	26	8.52 458
6960	56	49	74	.52 810	51	26	.52 835
7020	57	49	74	.53 183	51	25	.53 208
7080	58	49	74	.53 552	51	25	.53 578
7140	59	49	75	.53 918	51	25	.53 944

TABLE VI.—LOGARITHMIC SINES AND TANGENTS OF SMALL ANGLES.

log sin ϕ = log ϕ'' + S_1
log tan ϕ = log ϕ'' + T.

2°

log ϕ'' = log sin ϕ + S'.
log ϕ'' = log tan ϕ + T'.

"	'	S	T	Log. Sin.	S'	T'	Log. Tan.
7200	**0**	4.685 48̂	75	8.54 282	5.314 5î	25	8.54 308̂
7260	I	48̂	75̂	.54 642	5î	24̂	.54 669
7320	2	48̂	75̂	.54 999̂	5î	24̂	.55 027
7380	3	48̂	76	.55 354	52	24	.55 381̂
7440	4	48̂	76̂	.55 705̂	52	23̂	.55 733̂
7500	5	4.685 48̂	76̂	8.56 054	5.314 52	23̂	8.56 083̂
7560	6	48̂	77	.56 400	52	23	.56 429
7620	7	47̂	77̂	.56 743	5â	22̂	.56 772̂
7680	8	47̂	77̂	.57 083̂	5â	22̂	.57 113̂
7740	9	47̂	78	.57 421̂	5â	22	.57 452
7800	**10**	4.685 47	78	8.57 756̂	5.314 53	22	8.57 787̂
7860	11	47	78̂	.58 089	53	2î	.58 121
7920	12	47	79	.58 419̂	53	21	.58 451̂
7980	13	46	79	.58 747	53̂	21	.58 779̂
8040	14	46	79̂	.59 072	53̂	2ô	.59 105
8100	15	4.685 46̂	80	8.59 395	5.314 53̂	20	8.59 428̂
8160	16	46	80	.59 715	54	20	.59 749
8220	17	46	8ô	.60 033	54	1ŷ	.60 067̂
8280	18	46	81	.60 349	54	19	.60 384
8340	19	45	81	.60 662	5â	19	.60 698
8400	**20**	4.685 45̂	8î	8.60 973̂	5.314 5â	18	8.61 009̂
8460	21	45	82	.61 282̂	5â	18	.61 319
8520	22	45	82	.61 589	55	18	.61 626
8580	23	45	8â	.61 893̂	55	1ŷ	.61 931̂
8640	24	45	83	.62 196	55	17	.62 234̂
8700	25	4.685 44̂	83̂	8.62 496̂	5.314 55̂	1ô	8.62 535
8760	26	44̂	83̂	.62 795	55̂	1ô	.62 834
8820	27	44̂	84	.63 091	55̂	16	.63 131
8880	28	44	84̂	.63 385̂	56	1ŝ	.63 425̂
8940	29	44	84̂	.63 677	56	1ŝ	.63 718̂
9000	**30**	4.685 43̂	85	8.63 968	5.314 56	15	8.64 009̂
9060	31	43̂	8ŝ	.64 256̂	56	1â	.64 298
9120	32	43̂	86	.64 543	56	14	.64 585̂
9180	33	43	86	.64 827	57	14	.64 870̂
9240	34	43	8ô	.65 110	57	1â	.65 153̂
9300	35	4.685 43	87	8.65 391	5.314 57	13	8.65 435
9360	36	4â	8ŷ	.65 670	5ŷ	1â	.65 715
9420	37	4â	8ŷ	.65 947	5ŷ	1â	.65 993
9480	38	42	88	.66 223	58	12	.66 269
9540	39	42	8â	.66 497	58	1î	.66 543̂
9600	**40**	4.685 42	89	8.66 769	5.314 58	11	8.66 816
9660	41	4î	8ŷ	.67 039̂	58	1ô	.67 087
9720	42	4î	8ŷ	.67 308	58	1ô	.67 356̂
9780	43	41	90	.67 575	59	10	.67 624
9840	44	41	9ô	.67 840̂	59	0ŷ	.67 890
9900	45	4.685 41	91	8.68 104̂	5.314 59	09	8.68 154̂
9960	46	4ô	9î	.68 365	5ŷ	0ŷ	.68 417
10020	47	4ô	9î	.68 627	5ŷ	0ŷ	.68 678̂
10080	48	40	92	.68 888̂	60	08	.68 938
10140	49	40	9â	.69 144	60	07	.69 196̂
10200	**50**	4.685 40	93	8.69 400	5.314 60	07	8.69 453
10260	51	3ŷ	9â	.69 654̂	6ô	0ô	.69 708
10320	52	3ŷ	9â	.69 907	6ô	0ô	.69 961̂
10380	53	39	94	.70 159	61	06	.70 214
10440	54	39	9â	.70 409	61	0ŝ	.70 464̂
10500	55	4.685 38̂	95	8.70 657̂	5.314 6î	05	8.70 714
10560	56	38̂	9ŝ	.70 905	6î	0â	.70 962
10620	57	38̂	96	.71 150̂	6î	04	.71 208̂
10680	58	38	9ô	.71 395	62	0ŝ	.71 453̂
10740	59	38	97	.71 638̂	62	03	.71 697

TABLE VII.—LOGARITHMIC SINES, COSINES, TANGENTS, AND COTANGENTS.

0°

′	Log. Sin.	D	Log. Tan.	Com. D.	Log. Cot.	Log. Cos.	
0	− ∞		− ∞		+ ∞	0.00 000	60
1	6.46 372		6.46 372		3.53 627	0.00 000	59
2	6.76 475	30103	6.76 475	30103	3.23 524	0.00 000	58
3	6.94 084	17609	6.94 084	17609	3.05 915	0.00 000	57
4	7.06 578	12494	7.06 578	12494	2.93 421	0.00 000	56
5	7.16 269	9691	7.16 269	9691	2.83 730	0.00 000	55
6	7.24 187	7918	7.24 188	7918	2.75 812	0.00 000	54
7	7.30 882	6695	7.30 882	6694	2.69 117	0.00 000	53
8	7.36 681	5799	7.36 681	5799	2.63 318	0.00 000	52
9	7.41 797	5115	7.41 797	5115	2.58 203	0.00 000	51
10	7.46 372	4575	7.46 372	4575	2.53 627	0.00 000	50
11	7.50 512	4139	7.50 512	4139	2.49 488	0.00 000	49
12	7.54 291	3778	7.54 291	3779	2.45 709	9.99 999	48
13	7.57 767	3476	7.57 767	3476	2.42 233	9.99 999	47
14	7.60 985	3218	7.60 985	3218	2.39 014	9.99 999	46
15	7.63 981	2996	7.63 982	2996	2.36 018	9.99 999	45
16	7.66 784	2803	7.66 785	2803	2.33 215	9.99 999	44
17	7.69 417	2633	7.69 418	2633	2.30 582	9.99 999	43
18	7.71 896	2482	7.71 900	2482	2.28 099	9.99 999	42
19	7.74 248	2348	7.74 248	2348	2.25 751	9.99 999	41
20	7.76 475	2227	7.76 476	2227	2.23 524	9.99 999	40
21	7.78 594	2119	7.78 595	2119	2.21 405	9.99 999	39
22	7.80 614	2020	7.80 615	2020	2.19 384	9.99 999	38
23	7.82 545	1936	7.82 546	1936	2.17 454	9.99 999	37
24	7.84 393	1848	7.84 394	1848	2.15 605	9.99 999	36
25	7.86 166	1772	7.86 167	1773	2.13 832	9.99 999	35
26	7.87 869	1703	7.87 871	1703	2.12 129	9.99 999	34
27	7.89 508	1639	7.89 510	1639	2.10 490	9.99 998	33
28	7.91 088	1579	7.91 089	1579	2.08 910	9.99 998	32
29	7.92 612	1524	7.92 613	1524	2.07 386	9.99 998	31
30	7.94 084	1472	7.94 086	1472	2.05 914	9.99 998	30
31	7.95 508	1424	7.95 510	1424	2.04 490	9.99 998	29
32	7.96 887	1379	7.96 889	1379	2.03 111	9.99 998	28
33	7.98 223	1336	7.98 225	1336	2.01 774	9.99 998	27
34	7.99 520	1296	7.99 522	1296	2.00 478	9.99 998	26
35	8.00 778	1258	8.00 781	1259	1.99 219	9.99 997	25
36	8.02 002	1223	8.02 004	1223	1.97 995	9.99 997	24
37	8.03 192	1190	8.03 194	1190	1.96 805	9.99 997	23
38	8.04 350	1158	8.04 352	1158	1.95 647	9.99 997	22
39	8.05 478	1128	8.05 481	1128	1.94 519	9.99 997	21
40	8.06 577	1099	8.06 586	1072	1.93 419	9.99 997	20
41	8.07 650	1072	8.07 653	1046	1.92 347	9.99 997	19
42	8.08 696	1046	8.08 699	1022	1.91 300	9.99 997	18
43	8.09 718	1022	8.09 721		1.90 278	9.99 996	17
44	8.10 716	998	8.10 720	999	1.89 279	9.99 996	16
45	8.11 692	976	8.11 696	976	1.88 303	9.99 996	15
46	8.12 647	954	8.12 651	954	1.87 349	9.99 996	14
47	8.13 581	934	8.13 585	934	1.86 415	9.99 996	13
48	8.14 495	914	8.14 499	914	1.85 500	9.99 996	12
49	8.15 390	895	8.15 395	895	1.84 605	9.99 995	11
50	8.16 268	877	8.16 272	877	1.83 727	9.99 995	10
51	8.17 128	860	8.17 133	860	1.82 867	9.99 995	9
52	8.17 971	843	8.17 976	843	1.82 023	9.99 995	8
53	8.18 798	827	8.18 803	827	1.81 196	9.99 995	7
54	8.19 610	811	8.19 615	812	1.80 384	9.99 994	6
55	8.20 407	797	8.20 412	797	1.79 587	9.99 994	5
56	8.21 189	782	8.21 195	783	1.78 804	9.99 994	4
57	8.21 958	768	8.21 964	768	1.78 036	9.99 994	3
58	8.22 713	755	8.22 719	755	1.77 280	9.99 994	2
59	8.23 455	742	8.23 462	742	1.76 538	9.99 993	1
60	8.24 185	730	8.24 192	730	1.75 808	9.99 993	0
	Log. Cos.	D	Log. Cot.	Com. D.	Log. Tan.	Log. Sin.	′

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TABLE VII.—LOGARITHMIC SINES, COSINES, TANGENTS, AND COTANGENTS.

1°

'	Log. Sin.	D	Log. Tan.	Com. D.	Log. Cot.	Log. Cos.	'
0	8.24 185	718	8.24 192	718	1.75 808	9.99 993	60
1	8.24 903	706	8.24 910	706	1.75 090	9.99 993	59
2	8.25 609	694	8.25 616	695	1.74 383	9.99 993	58
3	8.26 304	684	8.26 311	684	1.73 688	9.99 992	57
4	8.26 988	673	8.26 995	673	1.73 004	9.99 992	56
5	8.27 661	663	8.27 669	663	1.72 331	9.99 992	55
6	8.28 324	653	8.28 332	653	1.71 667	9.99 992	54
7	8.28 977	643	8.28 985	643	1.71 014	9.99 992	53
8	8.29 620	634	8.29 629	634	1.70 371	9.99 991	52
9	8.30 254	625	8.30 263	625	1.69 736	9.99 991	51
10	8.30 879	616	8.30 888	616	1.69 111	9.99 991	50
11	8.31 495	607	8.31 504	607	1.68 495	9.99 990	49
12	8.32 102	599	8.32 112	599	1.67 888	9.99 990	48
13	8.32 701	591	8.32 711	591	1.67 288	9.99 990	47
14	8.33 292	583	8.33 302	583	1.66 697	9.99 990	46
15	8.33 875	575	8.33 885	575	1.66 114	9.99 989	45
16	8.34 450	567	8.34 461	568	1.65 539	9.99 989	44
17	8.35 018	560	8.35 029	566	1.64 971	9.99 989	43
18	8.35 578	553	8.35 589	553	1.64 410	9.99 989	42
19	8.36 131	546	8.36 143	546	1.63 857	9.99 988	41
20	8.36 677	539	8.36 689	539	1.63 310	9.99 988	40
21	8.37 217	533	8.37 229	533	1.62 771	9.99 988	39
22	8.37 750	526	8.37 762	527	1.62 238	9.99 987	38
23	8.38 276	520	8.38 289	520	1.61 711	9.99 987	37
24	8.38 796	514	8.38 809	514	1.61 191	9.99 987	36
25	8.39 310	508	8.39 323	508	1.60 676	9.99 986	35
26	8.39 818	502	8.39 831	502	1.60 168	9.99 986	34
27	8.40 320	496	8.40 334	496	1.59 666	9.99 986	33
28	8.40 816	491	8.40 830	491	1.59 169	9.99 986	32
29	8.41 307	485	8.41 321	483	1.58 678	9.99 985	31
30	8.41 792	479	8.41 807	480	1.58 193	9.99 985	30
31	8.42 271	474	8.42 287	475	1.57 713	9.99 985	29
32	8.42 746	469	8.42 762	469	1.57 238	9.99 984	28
33	8 43 215	464	8.43 231	464	1.56 768	9.99 984	27
34	8.43 680	459	8.43 696	460	1.56 304	9.99 984	26
35	8.44 139	454	8.44 156	455	1.55 844	9.99 983	25
36	8.44 594	450	8.44 611	450	1.55 389	9.99 983	24
37	8.45 044	445	8.45 061	445	1.54 938	9.99 983	23
38	8.45 489	440	8.45 507	441	1.54 493	9.99 982	22
39	8.45 930	436	8.45 948	437	1.54 052	9.99 982	21
40	8.46 366	432	8.46 385	432	1.53 615	9.99 981	20
41	8.46 798	428	8.46 817	428	1.53 183	9.99 981	19
42	8.47 226	423	8.47 245	424	1.52 754	9.99 981	18
43	8.47 650	419	8.47 669	419	1.52 330	9.99 980	17
44	8.48 069	415	8.48 089	416	1.51 911	9.99 980	16
45	8.48 485	411	8.48 505	412	1.51 495	9.99 979	15
46	8.48 896	407	8.48 917	408	1.51 083	9.99 979	14
47	8.49 304	404	8.49 325	404	1.50 675	9.99 979	13
48	8.49 708	400	8.49 729	400	1.50 270	9.99 978	12
49	8.50 108	396	8.50 130	396	1.49 870	9.99 978	11
50	8.50 504	393	8.50 526	393	1.49 473	9.99 978	10
51	8.50 897	389	8.50 920	390	1.49 080	9.99 977	9
52	8.51 286	386	8.51 310	386	1.48 690	9.99 977	8
53	8.51 672	382	8.51 696	383	1.48 304	9.99 976	7
54	8.52 055	379	8.52 079	379	1.47 921	9.99 976	6
55	8.52 434	375	8.52 458	376	1.47 541	9.99 975	5
56	8.52 810	373	8.52 835	373	1.47 165	9.99 975	4
57	8.53 183	369	8.53 208	370	1.46 792	9.99 975	3
58	8.53 552	366	8.53 578	366	1.46 422	9.99 974	2
59	8.53 918	363	8.53 944	364	1.46 055	9.99 974	1
60	8.54 282		8.54 308		1.45 691	9.99 973	0
	Log. Cos.	D	Log. Cot.	Com. D.	Log. Tan.	Log. Sin.	'

TABLE VII.—LOGARITHMIC SINES, COSINES, TANGENTS, AND COTANGENTS.

2°

′	Log. Sin.	D	Log. Tan.	Com. D.	Log. Cot.	Log. Cos.	
0	8.54 282		8.54 308		1.45 691	9.99 973	60
1	8.54 642	360	8.54 669	366	1.45 331	9.99 973	59
2	8.54 999	357	8.55 027	358	1.44 973	9.99 972	58
3	8.55 354	354	8.55 381	354	1.44 618	9.99 972	57
4	8.55 705	351	8.55 733	352	1.44 266	9.99 971	56
5	8.56 054	348	8.56 083	349	1.43 917	9.99 971	55
6	8.56 400	346	8.56 429	346	1.43 571	9.99 971	54
7	8.56 743	343	8.56 772	343	1.43 227	9.99 970	53
8	8.57 083	346	8.57 113	341	1.42 886	9.99 970	52
9	8.57 421	338	8.57 452	338	1.42 548	9.99 969	51
10	8.57 756	335	8.57 787	335	1.42 212	9.99 969	50
11	8.58 089	332	8.58 121	333	1.41 879	9.99 968	49
12	8.58 419	330	8.58 451	330	1.41 548	9.99 968	48
13	8.58 747	327	8.58 779	328	1.41 220	9.99 967	47
14	8.59 072	325	8.59 105	325	1.40 895	9.99 967	46
15	8.59 395	323	8.59 428	323	1.40 571	9.99 966	45
16	8.59 715	320	8.59 749	320	1.40 251	9.99 966	44
17	8.60 033	318	8.60 067	318	1.39 932	9.99 965	43
18	8.60 349	316	8.60 384	316	1.39 616	9.99 965	42
19	8.60 662	313	8.60 698	314	1.39 302	9.99 964	41
20	8.60 973	311	8.61 009	311	1.38 990	9.99 964	40
21	8.61 282	309	8.61 319	309	1.38 681	9.99 963	39
22	8.61 589	306	8.61 626	307	1.38 374	9.99 963	38
23	8.61 893	304	8.61 931	305	1.38 068	9.99 962	37
24	8.62 196	302	8.62 234	303	1.37 765	9.99 962	36
25	8.62 496	300	8.62 535	300	1.37 465	9.99 961	35
26	8.62 795	298	8.62 834	299	1.37 166	9.99 961	34
27	8.63 091	296	8.63 131	297	1.36 869	9.99 960	33
28	8.63 385	294	8.63 425	294	1.36 574	9.99 959	32
29	8.63 677	292	8.63 718	293	1.36 281	9.99 959	31
30	8.63 968	290	8.64 009	291	1.35 990	9.99 958	30
31	8.64 256	288	8.64 298	288	1.35 702	9.99 958	29
32	8.64 543	286	8.64 585	287	1.35 414	9.99 957	28
33	8.64 827	284	8.64 870	285	1.35 129	9.99 957	27
34	8.65 110	282	8.65 153	283	1.34 846	9.99 956	26
35	8.65 391	281	8.65 435	281	1.34 565	9.99 956	25
36	8.65 670	279	8.65 715	280	1.34 285	9.99 955	24
37	8.65 947	277	8.65 993	278	1.34 007	9.99 954	23
38	8.66 223	275	8.66 269	276	1.33 731	9.99 954	22
39	8.66 497	274	8.66 543	274	1.33 456	9.99 953	21
40	8.66 769	272	8.66 816	272	1.33 184	9.99 953	20
41	8.67 039	270	8.67 087	271	1.32 913	9.99 952	19
42	8.67 308	268	8.67 356	269	1.32 643	9.99 952	18
43	8.67 575	267	8.67 624	267	1.32 376	9.99 951	17
44	8.67 840	265	8.67 890	266	1.32 110	9.99 950	16
45	8.68 104	264	8.68 154	264	1.31 845	9.99 950	15
46	8.68 366	262	8.68 417	262	1.31 583	9.99 949	14
47	8.68 627	260	8.68 678	261	1.31 321	9.99 948	13
48	8.68 886	259	8.68 938	259	1.31 062	9.99 948	12
49	8.69 144	257	8.69 196	258	1.30 803	9.99 947	11
50	8.69 400	256	8.69 453	256	1.30 547	9.99 947	10
51	8.69 654	254	8.69 708	255	1.30 292	9.99 946	9
52	8.69 907	253	8.69 961	253	1.30 038	9.99 945	8
53	8.70 159	251	8.70 214	252	1.29 786	9.99 945	7
54	8.70 409	250	8.70 464	250	1.29 535	9.99 944	6
55	8.70 657	248	8.70 714	249	1.29 286	9.99 943	5
56	8.70 905	247	8.70 962	248	1.29 038	9.99 943	4
57	8.71 150	245	8.71 208	246	1.28 791	9.99 942	3
58	8.71 395	244	8.71 453	245	1.28 546	9.99 942	2
57	8.71 638	243	8.71 697	243	1.28 303	9.99 941	1
60	8.71 880	241	8.71 939	242	1.28 060	9.99 940	0
	Log. Cos.	D	Log. Cot.	Com. D.	Log. Tan.	Log. Sin.	′

TABLE VII.—LOGARITHMIC SINES, COSINES, TANGENTS, AND COTANGENTS.

3°

′	Log. Sin.	d.	Log. Tan.	c. d.	Log. Cot.	Log. Cos.	′
0	8.71 880	246	8.71 939	241	1.28 066	9.99 946	60
1	8.72 126	239	8.72 180	240	1.27 819	9.99 940	59
2	8.72 359	237	8.72 420	238	1.27 579	9.99 939	58
3	8.72 597	236	8.72 659	237	1.27 341	9.99 938	57
4	8.72 833	235	8.72 896	235	1.27 104	9.99 938	56
5	8.73 069	233	8.73 131	235	1.26 868	9.99 937	55
6	8.73 302	233	8.73 366	233	1.26 633	9.99 936	54
7	8.73 535	231	8.73 599	232	1.26 400	9.99 935	53
8	8.73 766	230	8.73 831	231	1.26 168	9.99 935	52
9	8.73 997	229	8.74 062	229	1.25 937	9.99 934	51
10	8.74 226	227	8.74 292	228	1.25 708	9.99 933	50
11	8.74 453	226	8.74 520	227	1.25 479	9.99 933	49
12	8.74 680	225	8.74 748	226	1.25 252	9.99 932	48
13	8.74 905	224	8.74 974	225	1.25 026	9.99 931	47
14	8.75 129	223	8.75 199	223	1.24 801	9.99 931	46
15	8.75 353	221	8.75 422	223	1.24 577	9.99 930	45
16	8.75 574	221	8.75 645	221	1.24 354	9.99 929	44
17	8.75 795	219	8.75 867	220	1.24 133	9.99 928	43
18	8.76 015	218	8.76 087	219	1.23 913	9.99 928	42
19	8.76 233	217	8.76 306	218	1.23 693	9.99 927	41
20	8.76 451	216	8.76 524	217	1.23 475	9.99 926	40
21	8.76 667	215	8.76 741	216	1.23 258	9.99 925	39
22	8.76 883	214	8.76 958	214	1.23 042	9.99 925	38
23	8.77 097	213	8.77 172	214	1.22 827	9.99 924	37
24	8.77 310	212	8.77 386	213	1.22 613	9.99 923	36
25	8.77 522	211	8.77 599	212	1.22 400	9.99 922	35
26	8.77 733	210	8.77 811	210	1.22 188	9.99 922	34
27	8.77 943	209	8.78 022	210	1.21 978	9.99 921	33
28	8.78 152	208	8.78 232	209	1.21 768	9.99 920	32
29	8.78 360	207	8.78 441	207	1.21 559	9.99 919	31
30	8.78 567	206	8.78 648	207	1.21 351	9.99 919	30
31	8.78 773	205	8.78 855	206	1.21 144	9.99 918	29
32	8.78 978	204	8.79 061	204	1.20 938	9.99 917	28
33	8.79 183	203	8.79 266	204	1.20 734	9.99 916	27
34	8.79 386	202	8.79 470	203	1.20 530	9.99 916	26
35	8.79 588	201	8.79 673	202	1.20 327	9.99 915	25
36	8.79 789	200	8.79 875	201	1.20 125	9.99 914	24
37	8.79 989	199	8.80 076	200	1.19 923	9.99 913	23
38	8.80 189	198	8.80 276	199	1.19 723	9.99 912	22
39	8.80 387	197	8.80 476	198	1.19 524	9.99 912	21
40	8.80 585	197	8.80 674	197	1.19 326	9.99 911	20
41	8.80 782	195	8.80 871	197	1.19 128	9.99 910	19
42	8.80 977	195	8.81 068	195	1.18 931	9.99 909	18
43	8.81 172	194	8.81 264	195	1.18 736	9.99 908	17
44	8.81 366	193	8.81 459	194	1.18 541	9.99 907	16
45	8.81 560	192	8.81 653	193	1.18 347	9.99 907	15
46	8.81 752	191	8.81 846	192	1.18 154	9.99 906	14
47	8.81 943	191	8.82 038	191	1.17 961	9.99 905	13
48	8.82 134	189	8.82 230	190	1.17 770	9.99 904	12
49	8.82 324	189	8.82 420	190	1.17 579	9.99 903	11
50	8.82 513	188	8.82 610	188	1.17 389	9.99 902	10
51	8.82 701	187	8.82 799	188	1.17 201	9.99 902	9
52	8.82 888	186	8.82 987	187	1.17 012	9.99 901	8
53	8.83 075	185	8.83 175	186	1.16 825	9.99 900	7
54	8.83 260	185	8.83 361	185	1.16 638	9.99 899	6
55	8.83 445	184	8.83 547	185	1.16 453	9.99 898	5
56	8.83 629	183	8.83 732	184	1.16 268	9.99 897	4
57	8.83 813	182	8.83 916	183	1.16 083	9.99 896	3
58	8.83 995	182	8.84 100	182	1.15 900	9.99 896	2
59	8.84 177	181	8.84 282	182	1.15 717	9.99 895	1
60	8.84 358		8.84 464		1.15 535	9.99 894	0
	Log. Cos.	d.	Log. Cot.	c. d.	Log. Tan.	Log. Sin.	′

P. P.

	330	320	310	300
6	33.0	32.0	31.0	30.0
7	38.5	37.3	36.1	35.0
8	44.0	42.6	41.3	40.0
9	49.5	48.0	46.5	45.0
10	55.0	53.3	51.6	50.0
20	110.0	106.6	103.3	100.0
30	165.0	160.0	155.0	150.0
40	220.0	213.3	206.6	200.0
50	275.0	266.6	258.3	250.0

	290	280	270	260
6	29.0	28.0	27.0	26.0
7	33.8	32.6	31.5	30.3
8	38.6	37.3	36.0	34.6
9	43.5	42.0	40.5	39.0
10	48.3	46.6	45.0	43.3
20	96.6	93.3	90.0	86.6
30	145.0	140.0	135.0	130.0
40	193.3	186.6	180.0	173.3
50	241.6	233.3	225.0	216.6

	250	240	230	220
6	25.0	24.0	23.0	22.0
7	29.1	28.0	26.8	25.6
8	33.3	32.0	30.6	29.3
9	37.5	36.0	34.5	33.0
10	41.6	40.0	38.3	36.6
20	83.3	80.0	76.6	73.3
30	125.0	120.0	115.0	110.0
40	166.6	160.0	153.3	146.6
50	208.3	200.0	191.6	183.3

	210	200	190	180
6	21.0	20.0	19.0	18.0
7	24.5	23.3	22.1	21.0
8	28.0	26.6	25.3	24.0
9	31.5	30.0	28.5	27.0
10	35.0	33.3	31.6	30.0
20	70.0	66.6	63.3	60.0
30	105.0	100.0	95.0	90.0
40	140.0	133.3	126.6	120.0
50	175.0	166.6	158.3	150.0

	9̄	9	8	7	6	5
6	0.9	0.9	0.8	0.7	0.6	0.5
7	1.1	1.0	0.9	0.8	0.7	0.6
8	1.2	1.2	1.0	0.9	0.8	0.6
9	1.4	1.3	1.2	1.0	0.9	0.7
10	1.6	1.5	1.3	1.1	1.0	0.8
20	3.1	3.0	2.6	2.3	2.0	1.6
30	4.7	4.5	4.0	3.5	3.0	2.5
40	6.3	6.0	5.3	4.6	4.0	3.3
50	7.9	7.5	6.6	5.8	5.0	4.1

	4̄	4	3	2	1	0̄
6	0.4	0.4	0.3	0.2	0.1	0.0
7	0.5	0.4	0.3	0.2	0.1	0.0
8	0.6	0.5	0.4	0.2	0.1	0.0
9	0.6	0.6	0.4	0.3	0.1	0.0
10	0.7	0.6	0.5	0.3	0.1	0.1
20	1.6	1.3	1.0	0.6	0.3	0.1
30	2.2	2.0	1.5	1.0	0.5	0.2
40	3.0	2.6	2.0	1.3	0.6	0.3
50	3.7	3.3	2.5	1.6	0.8	0.4

P. P.

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TABLE VII.—LOGARITHMIC SINES, COSINES, TANGENTS, AND COTANGENTS.
4°

′	Log. Sin.	d.	Log. Tan.	c. d.	Log. Cot.	Log. Cos.	′
0	8.84 358^{8}		8.84 464^{4}		1.15 535^{5}	9.99 894^{4}	60
1	8.84 538^{8}	180	8.84 645^{5}	181	1.15 354	9.99 893	59
2	8.84 718^{8}	180	8.84 826	186	1.15 174	9.99 892^{2}	58
3	8.84 897^{7}	178	8.85 005^{5}	179	1.14 994^{4}	9.99 891^{1}	57
4	8.85 075^{7}	178	8.85 184^{4}	179	1.14 815^{5}	9.99 890^{0}	56
5	8.85 252^{2}	177	8.85 363	178	1.14 637	9.99 889^{9}	55
6	8.85 429^{9}	176	8.85 540^{0}	177	1.14 459^{9}	9.99 888^{8}	54
7	8.85 605^{5}	176	8.85 717^{7}	176	1.14 283	9.99 888	53
8	8.85 780^{0}	175	8.85 893^{3}	176	1.14 107	9.99 887	52
9	8.85 954^{4}	174	8.86 068^{8}	175	1.13 931^{1}	9.99 886	51
10	8.86 128^{8}	174	8.86 243^{3}	175	1.13 756^{6}	9.99 885	50
11	8.86 301^{1}	173	8.86 417^{7}	174	1.13 582^{2}	9.99 884	49
12	8.86 474^{4}	173	8.86 590^{0}	173	1.13 409^{9}	9.99 883	48
13	8.86 645^{5}	172	8.86 763^{3}	173	1.13 237	9.99 882^{2}	47
14	8.86 816^{6}	171	8.86 935	172	1.13 065	9.99 881^{1}	46
15	8.86 987^{7}	171	8.87 106^{6}	171	1.12 893^{3}	9.99 880^{0}	45
16	8.87 156^{6}	170	8.87 277	176	1.12 723	9.99 879^{9}	44
17	8.87 325^{5}	169	8.87 447^{7}	170	1.12 553^{3}	9.99 878^{8}	43
18	8.87 494^{4}	169	8.87 616^{6}	169	1.12 384	9.99 877^{7}	42
19	8.87 661^{1}	168	8.87 785	169	1.12 215	9.99 876^{6}	41
20	8.87 828^{8}	167	8.87 953^{3}	168	1.12 047	9.99 875^{5}	40
21	8.87 995^{5}	167	8.88 120^{0}	167	1.11 880^{0}	9.99 874^{4}	39
22	8.88 160^{0}	166	8.88 287	167	1.11 713^{3}	9.99 874	38
23	8.88 326^{6}	165	8.88 453^{3}	166	1.11 547	9.99 873	37
24	8.88 490^{0}	·155	8.88 618^{8}	165	1.11 381^{1}	9.99 872	36
25	8.88 654^{4}	164	8.88 783^{3}	165	1.11 216^{6}	9.99 871	35
26	8.88 817^{7}	163	8.88 947^{7}	164	1.11 052^{2}	9.99 870	34
27	8.88 980^{0}	163	8.89 111	163	1.10 889^{9}	9.99 869	33
28	8.89 142^{2}	162	8.89 274^{4}	163	1.10 726^{6}	9.99 868	32
29	8.89 303^{3}	162	8.89 436^{6}	162	1.10 563^{3}	9.99 867	31
30	8.89 464^{4}	161	8.89 598^{8}	162	1.10 401^{1}	9.99 866	30
31	8.89 624^{4}	161	8.89 759^{9}	161	1.10 240^{0}	9.99 865	29
32	8.89 784^{4}	160	8.89 920^{0}	161	1.10 079^{9}	9.99 864	28
33	8.89 943^{3}	159	8.90 080^{0}	160	1.09 919^{9}	9.99 863	27
34	8.90 101^{1}	159	8.90 240	160	1.09 760^{0}	9.99 862	26
35	8.90 259^{9}	158	8.90 398^{8}	158	1.09 601^{1}	9.99 861	25
36	8.90 417	158	8.90 557^{7}	159	1.09 443^{3}	9.99 860	24
37	8.90 573^{3}	157	8.90 714^{4}	158	1.09 285^{5}	9.99 859	23
38	8.90 729^{9}	156	8.90 872^{2}	157	1.09 128	9.99 858	22
39	8.90 885^{5}	156	8.91 028^{8}	156	1.08 971^{1}	9.99 857	21
40	8.91 040^{0}	156	8.91 184^{4}	156	1.08 815^{5}	9.99 856	20
41	8.91 195	155	8.91 340^{0}	155	1.08 660^{0}	9.99 855	19
42	8.91 349^{9}	154	8.91 495	155	1.08 505^{5}	9.99 853^{3}	18
43	8.91 502^{2}	154	8.91 649^{9}	154	1.08 350^{0}	9.99 852^{2}	17
44	8.91 655^{5}	153	8.91 803^{3}	154	1.08 196^{6}	9.99 851^{1}	16
45	8.91 807^{7}	153	8.91 957^{7}	153	1.08 043^{3}	9.99 849^{9}	15
46	8.91 959^{9}	152	8.92 110^{0}	152	1.07 890^{0}	9.99 849	14
47	8.92 110^{0}	151	8.92 262	152	1.07 738	9.99 848^{8}	13
48	8.92 261	151	8.92 413^{3}	151	1.07 586^{6}	9.99 847^{7}	12
49	8.92 411	150	8.92 565	151	1.07 435	9.99 846^{6}	11
50	8.92 561	150	8.92 715^{5}	150	1.07 284^{4}	9.99 845^{5}	10
51	8.92 710^{0}	149	8.92 866^{6}	150	1.07 134	9.99 844	9
52	8.92 858^{8}	148	8.93 015^{5}	149	1.06 984^{4}	9.99 843	8
53	8.93 007^{7}	148	8.93 164^{4}	149	1.06 835^{5}	9.99 842	7
54	8.93 154^{4}	147	8.93 313^{3}	149	1.06 686^{6}	9.99 841	6
55	8.93 301^{1}	147	8.93 461^{1}	148	1.06 538	9.99 840	5
56	8.93 448^{8}	146	8.93 609^{9}	148	1.06 390^{0}	9.99 839	4
57	8.93 594^{4}	146	8.93 756^{6}	147	1.06 243^{3}	9.99 837^{7}	3
58	8.93 740^{0}	146	8.93 903^{3}	147	1.06 097	9.99 836^{6}	2
59	8.93 885^{5}	145	8.94 049^{9}	146	1.05 950^{5}	9.99 835^{5}	1
60	8.94 029^{9}	144	8.94 195	145	1.05 805	9.99 834^{4}	0
	Log. Cos.	d.	Log. Cot.	c. d.	Log. Tan.	Log. Sin.	′

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	181	180	178	176
6	18.1	18.0	17.8	17.6
7	21.1	21.0	20.7	20.5
8	24.1	24.0	23.7	23.4
9	27.1	27.0	26.7	26.4
10	30.1	30.0	29.6	29.3
20	60.3	60.0	59.3	58.6
30	90.5	90.0	89.0	88.0
40	120.6	120.0	118.6	117.3
50	150.8	150.0	148.3	146.6

	174	172	170	168
6	17.4	17.2	17.0	16.8
7	20.3	20.0	19.8	19.6
8	23.2	22.6	22.6	22.4
9	26.1	25.8	25.5	25.2
10	29.0	28.6	28.3	28.0
20	58.0	57.3	56.6	56.0
30	87.0	86.0	85.0	84.0
40	116.0	114.6	113.3	112.0
50	145.0	143.3	141.6	140.0

	166	164	162	160
6	16.6	16.4	16.2	16.0
7	19.3	19.1	18.9	18.6
8	22.1	21.8	21.6	21.3
9	24.9	24.6	24.3	24.0
10	27.6	27.3	27.0	26.6
20	55.3	54.6	54.0	53.3
30	83.0	82.0	81.0	80.0
40	110.6	109.3	108.0	106.6
50	138.3	136.6	135.0	133.3

	158	156	154	152
6	15.8	15.6	15.4	15.2
7	18.4	18.2	17.9	17.7
8	21.0	20.8	20.5	20.2
9	23.7	23.4	23.1	22.8
10	26.3	26.0	25.6	25.3
20	52.6	52.0	51.3	50.6
30	79.0	78.0	77.0	76.0
40	105.3	104.0	102.6	101.3
50	131.6	130.0	128.3	126.6

	150	149	148	147
6	15.0	14.9	14.8	14.7
7	17.5	17.4	17.2	17.1
8	20.0	19.8	19.7	19.6
9	22.5	22.3	22.2	22.0
10	25.0	24.8	24.6	24.5
20	50.0	49.6	49.3	49.0
30	75.0	74.5	74.0	73.5
40	100.0	99.3	98.6	98.0
50	125.0	124.1	123.3	122.5

	146	145	1	1	0
6	14.6	14.5	0.1	0.1	0.0
7	17.0	16.9	0.2	0.1	0.0
8	19.4	19.3	0.2	0.1	0.0
9	21.9	21.7	0.2	0.1	0.0
10	24.3	24.1	0.2	0.1	0.1
20	48.6	48.3	0.5	0.3	0.1
30	73.0	72.5	0.7	0.5	0.3
40	97.3	96.6	1.0	0.6	0.3
50	121.6	120.8	1.2	0.8	0.4

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TABLE VII.—LOGARITHMIC SINES, COSINES, TANGENTS, AND COTANGENTS.
5°

′	Log. Sin.	d.	Log. Tan.	c. d.	Log. Cot.	Log. Cos.	′
0	8.94 029	144	8.94 195	145	1.05 805	9.99 834	60
1	8.94 174	143	8.94 340	144	1.05 659	9.99 833	59
2	8.94 317	143	8.94 485	144	1.05 515	9.99 832	58
3	8.94 460	143	8.94 629	144	1.05 370	9.99 831	57
4	8.94 603	142	8.94 773	144	1.05 226	9.99 830	56
5	8.94 745	142	8.94 917	143	1.05 083	9.99 829	55
6	8.94 887	141	8.95 059	142	1.04 940	9.99 827	54
7	8.95 028	141	8.95 202	142	1.04 798	9.99 826	53
8	8.95 169	141	8.95 344	142	1.04 656	9.99 825	52
9	8.95 310	140	8.95 485	141	1.04 514	9.99 824	51
10	8.95 450	140	8.95 626	141	1.04 373	9.99 823	50
11	8.95 589	139	8.95 767	141	1.04 232	9.99 822	49
12	8.95 728	139	8.95 907	140	1.04 092	9.99 821	48
13	8.95 867	138	8.96 047	140	1.03 952	9.99 819	47
14	8.96 005	138	8.96 186	139	1.03 813	9.99 818	46
15	8.96 143	138	8.96 325	139	1.03 674	9.99 817	45
16	8.96 280	137	8.96 464	138	1.03 536	9.99 816	44
17	8.96 417	137	8.96 602	138	1.03 398	9.99 815	43
18	8.96 553	136	8.96 739	137	1.03 260	9.99 814	42
19	8.96 689	136	8.96 876	137	1.03 123	9.99 813	41
20	8.96 825	135	8.97 013	137	1.02 986	9.99 811	40
21	8.96 960	135	8.97 149	136	1.02 850	9.99 810	39
22	8.97 094	134	8.97 285	136	1.02 714	9.99 809	38
23	8.97 229	134	8.97 421	135	1.02 579	9.99 808	37
24	8.97 363	134	8.97 556	135	1.02 444	9.99 807	36
25	8.97 496	133	8.97 690	134	1.02 309	9.99 805	35
26	8.97 629	133	8.97 825	134	1.02 175	9.99 804	34
27	8.97 762	132	8.97 958	133	1.02 041	9.99 803	33
28	8.97 894	132	8.98 092	133	1.01 908	9.99 802	32
29	8.98 026	132	8.98 225	133	1.01 775	9.99 801	31
30	8.98 157	131	8.98 357	132	1.01 642	9.99 799	30
31	8.98 288	131	8.98 490	132	1.01 510	9.99 798	29
32	8.98 419	130	8.98 621	131	1.01 378	9.99 797	28
33	8.98 549	130	8.98 753	131	1.01 247	9.99 796	27
34	8.98 679	130	8.98 884	131	1.01 116	9.99 794	26
35	8.98 808	129	8.99 015	131	1.00 985	9.99 793	25
36	8.98 937	129	8.99 145	130	1.00 855	9.99 792	24
37	8.99 066	128	8.99 275	130	1.00 725	9.99 791	23
38	8.99 194	128	8.99 404	129	1.00 596	9.99 789	22
39	8.99 322	127	8.99 533	129	1.00 467	9.99 788	21
40	8.99 449	127	8.99 662	129	1.00 337	9.99 787	20
41	8.99 577	127	8.99 791	128	1.00 209	9.99 786	19
42	8.99 703	126	8.99 919	128	1.00 081	9.99 784	18
43	8.99 830	126	9.00 046	127	0.99 953	9.99 783	17
44	8.99 956	126	9.00 174	127	0.99 826	9.99 782	16
45	9.00 081	125	9.00 300	126	0.99 699	9.99 781	15
46	9.00 207	125	9.00 427	126	0.99 573	9.99 779	14
47	9.00 332	125	9.00 553	126	0.99 446	9.99 778	13
48	9.00 456	124	9.00 679	125	0.99 321	9.99 777	12
49	9.00 580	124	9.00 804	125	0.99 195	9.99 776	11
50	9.00 704	124	9.00 930	125	0.99 070	9.99 774	10
51	9.00 828	123	9.01 054	124	0.98 945	9.99 773	9
52	9.00 951	123	9.01 179	124	0.98 821	9.99 772	8
53	9.01 073	122	9.01 303	124	0.98 697	9.99 770	7
54	9.01 196	122	9.01 427	123	0.98 573	9.99 769	6
55	9.01 318	122	9.01 550	123	0.98 450	9.99 768	5
56	9.01 440	121	9.01 673	123	0.98 327	9.99 766	4
57	9.01 561	121	9.01 796	122	0.98 204	9.99 765	3
58	9.01 682	120	9.01 918	122	0.98 081	9.99 764	2
59	9.01 803	120	9.02 040	122	0.97 959	9.99 763	1
60	9.01 923		9.02 162	121	0.97 838	9.99 761	0
	Log. Cos.	d.	Log. Cot.	c. d.	Log. Tan.	Log. Sin.	′

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	145	144	143	142	141
6	14.5	14.4	14.3	14.2	14.1
7	16.9	16.8	16.7	16.5	16.4
8	19.3	19.2	19.0	18.9	18.8
9	21.8	21.6	21.4	21.3	21.1
10	24.1	24.0	23.8	23.6	23.5
20	48.3	48.0	47.6	47.3	47.0
30	72.5	72.0	71.5	71.0	70.5
40	96.6	96.0	95.3	94.6	94.0
50	120.8	120.0	119.1	118.3	117.5

	140	139	138	137	136
6	14.0	13.9	13.8	13.7	13.6
7	16.3	16.2	16.1	16.0	15.8
8	18.6	18.5	18.4	18.2	18.1
9	21.0	20.8	20.7	20.5	20.4
10	23.3	23.1	23.0	22.8	22.6
20	46.6	46.3	46.0	45.6	45.3
30	70.0	69.5	69.0	68.5	68.0
40	93.3	92.6	92.0	91.3	90.6
50	116.6	115.8	115.0	114.1	113.3

	135	134	133	132
6	13.5	13.4	13.3	13.2
7	15.7	15.6	15.5	15.4
8	18.0	17.8	17.7	17.6
9	20.2	20.1	19.9	19.8
10	22.5	22.3	22.1	22.0
20	45.0	44.6	44.3	44.0
30	67.5	67.0	66.5	66.0
40	90.0	89.3	88.6	88.0
50	112.5	111.6	110.8	110.0

	131	130	129	128
6	13.1	13.0	12.9	12.8
7	15.3	15.1	15.0	14.9
8	17.4	17.3	17.2	17.0
9	19.6	19.5	19.3	19.2
10	21.8	21.6	21.5	21.3
20	43.6	43.3	43.0	42.6
30	65.5	65.0	64.5	64.0
40	87.3	86.6	86.0	85.3
50	109.1	108.3	107.5	106.6

	127	126	125	124	123
6	12.7	12.6	12.5	12.4	12.3
7	14.8	14.7	14.6	14.4	14.3
8	16.9	16.8	16.6	16.5	16.4
9	19.0	18.9	18.7	18.5	18.4
10	21.1	21.0	20.8	20.6	20.5
20	42.3	42.0	41.6	41.3	41.0
30	63.5	63.0	62.5	62.0	61.5
40	84.6	84.0	83.3	82.6	82.0
50	105.8	105.0	104.1	103.3	102.5

	122	121	120	2	1	0
6	12.2	12.1	12.0	0.1	0.1	0.0
7	14.2	14.1	14.0	0.2	0.1	0.0
8	16.2	16.1	16.0	0.2	0.1	0.0
9	18.3	18.1	18.0	0.2	0.1	0.1
10	20.3	20.1	20.0	0.2	0.1	0.1
20	40.6	40.3	40.0	0.5	0.3	0.1
30	61.0	60.5	60.0	0.7	0.5	0.2
40	81.3	80.6	80.0	1.0	0.6	0.3
50	101.6	100.8	100.0	1.2	0.8	0.4

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TABLE VII.—LOGARITHMIC SINES, COSINES, TANGENTS, AND COTANGENTS.
6°

′	Log. Sin.	d	Log. Tan.	c. d.	Log. Cot.	Log. Cos.	′
0	9.01 923̂	120	9.02 162	12î	0.97 838	9.99 76î	60
1	9.02 043̂	119̂	9.02 283̂	121	0.97 716̂	9.99 760	59
2	9.02 163	119̂	9.02 404̂	121	0.97 595̂	9.99 759	58
3	9.02 282̂	119	9.02 525	12ô	0.97 475̂	9.99 757̂	57
4	9.02 40î	119	9.02 645̂	12ô	0.97 354̂	9.99 756	56
5	9.02 520̂	119	9.02 765̂	120	0.97 234̂	9.99 754̂	55
6	9.02 638̂	118	9.02 885̂	11ĝ	0.97 115	9.99 753̂	54
7	9.02 756̂	118	9.03 004̂	11ĝ	0.96 995̂	9.99 752	53
8	9.02 874̂	118	9.03 123̂	119	0.96 876̂	9.99 750̂	52
9	9.02 992	117̂	9.03 242̂	119	0.96 757	9.99 749̂	51
10	9.03 109	117	9.03 361	118	0.96 639	9.99 748	50
11	9.03 225̂	117̂	9.03 479	118	0.96 521	9.99 746̂	49
12	9.03 342	116̂	9.03 597	118	0.96 403	9.99 745	48
13	9.03 458	116	9.03 714̂	117̂	0.96 285̂	9.99 744	47
14	9.03 574	116	9.03 83î	117	0.96 168	9.99 742̂	46
15	9.03 689	115̂	9.03 948	117	0.96 05î	9.99 741	45
16	9.03 805	115̂	9.04 065	116̂	0.95 935	9.99 739̂	44
17	9.03 91ô	114	9.04 18î	116̂	0.95 818̂	9.99 738̂	43
18	9.04 034	114̂	9.04 297	116	0.95 702̂	9.99 737	42
19	9.04 148	114̂	9.04 413	115̂	0.95 587	9.99 735̂	41
20	9.04 262̂	114	9.04 528̂	115	0.95 47î	9.99 734	40
21	9.04 376	113̂	9.04 643̂	115	0.95 356̂	9.99 732̂	39
22	9.04 489̂	113̂	9.04 758	114̂	0.95 242	9.99 73î	38
23	9.04 602̂	113	9.04 872̂	114	0.95 127	9.99 730	37
24	9.04 715̂	113	9.04 987	114̂	0.95 013	9.99 728̂	36
25	9.04 828	112̂	9.05 101	114	0.94 899	9.99 727	35
26	9.04 940	112	9.05 214̂	113̂	0.94 785̂	9.99 725̂	34
27	9.05 052	112	9.05 327̂	113	0.94 672̂	9.99 724	33
28	9.05 163̂	11î	9.05 440̂	113	0.94 55ô	9.99 723	32
29	9.05 275	11î	9.05 553̂	113	0.94 446̂	9.99 72î	31
30	9.05 386	111	9.05 666	112̂	0.94 334	9.99 720	30
31	9.05 496̂	11ô	9.05 778	112	0.94 222	9.99 718̂	29
32	9.05 607	11ô	9.05 890	112	0.94 110	9.99 717	28
33	9.05 717	110	9.06 00î	11î	0.93 998	9.99 715̂	27
34	9.05 827	110	9.06 113	11î	0.93 887	9.99 714	26
35	9.05 936̂	10ĝ	9.06 224	111	0.93 776	9.99 712̂	25
36	9.06 046	10ĝ	9.06 335	111	0.93 665	9.99 711	24
37	9.06 155	109	9.06 445̂	11ô	0.93 554̂	9.99 710	23
38	9.06 264	109	9.06 555̂	110	0.93 444̂	9.99 708̂	22
39	9.06 372̂	108̂	9.06 665̂	110	0.93 334̂	9.99 707	21
40	9.06 48ô	108	9.06 775	10ĝ	0.93 225	9.99 705̂	20
41	9.06 588̂	108	9.06 884̂	10ĝ	0.93 115̂	9.99 704	19
42	9.06 696̂	10î	9.06 994	10ĝ	0.93 006	9.99 702̂	18
43	9.06 803̂	10î	9.07 102̂	108	0.92 897̂	9.99 701	17
44	9.06 91ô	107	9.07 21î	109	0.92 788̂	9.99 699̂	16
45	9.07 017̂	107	9.07 319̂	108	0.92 68ô	9.99 698	15
46	9.07 124	10ô	9.07 428	10ô	0.92 572	9.99 696̂	14
47	9.07 23ô	10ô	9.07 535̂	10î	0.92 464̂	9.99 695	13
48	9.07 336̂	106	9.07 643	10î	0.92 357	9.99 693̂	12
49	9.07 44î	10ô	9.07 75ô	107	0.92 24ô	9.99 692	11
50	9.07 548	10ô	9.07 857̂	107	0.92 142̂	9.99 69ô	10
51	9.07 653̂	105	9.07 964̂	10ô	0.92 035̂	9.99 689	9
52	9.07 758	104̂	9.08 071	10ô	0.91 929	9.99 687̂	8
53	9.07 863	104̂	9.08 177̂	106	0.91 822̂	9.99 686	7
54	9.07 967	104̂	9.08 283̂	106	0.91 716̂	9.99 684̂	6
55	9.08 072	104	9.08 389	105	0.91 611	9.99 683	5
56	9.08 176	103̂	9.08 494̂	10ô	0.91 505̂	9.99 681	4
57	9.08 27ô	103̂	9.08 600	10ô	0.91 400	9.99 679̂	3
58	9.08 383	103̂	9.08 705	105	0.91 295	9.99 678	2
59	9.08 486̂	103	9.08 810	105	0.91 190	9.99 676̂	1
60	9.08 589̂		9.08 914̂	104̂	0.91 08ô	9.99 675	0
	Log. Cos.	d	Log. Cot.	c. d.	Log. Tan.	Log. Sin.	′

P. P.

	12î	121	120	119	118
6	12.î	12.1	12.0	11.9	11.8
7	14.2	14.1	14.0	13.9	13.7
8	16.2	16.î	16.0	15.8	15.7
9	18.2	18.î	18.0	17.8	17.7
10	20.2̂	20.î	20.0	19.8	19.6̂
20	40.5̂	40.3̂	40.0	39.8	39.3
30	60.7	60.5	60.0	59.5	59.0
40	81.0	80.6̂	80.0	79.3̂	78.6̂
50	101.2̂	100.8̂	100.0	99.î	98.3

	117	117	116	115
6	11.7	11.7	11.6	11.5
7	13.7	13.6̂	13.5̂	13.4
8	15.6	15.6	15.4	15.3̂
9	17.6	17.5̂	17.4	17.2
10	19.6	19.5	19.3̂	19.î
20	39.î	39.0	38.6̂	38.3̂
30	58.7	58.5	58.0	57.5
40	78.3̂	78.0	77.3̂	76.6̂
50	97.9	97.5	96.6̂	95.8

	114̂	114	113	112	11î
6	11.4̂	11.4	11.3	11.2	11.1
7	13.3̂	13.3	13.2	13.0̂	12.ĝ
8	15.2	15.2	15.0̂	14.ĝ	14.8
9	17.2̂	17.1	16.ĝ	16.8	16.6̂
10	19.1	19.0	18.8̂	18.6̂	18.5
20	38.î	38.0	37.6̂	37.3̂	37.0
30	57.2̂	57.0	56.5̂	56.0	55.5
40	76.3̂	76.0	75.3̂	74.6̂	74.0
50	95.4	95.0	94.î	93.3	92.5

	11ô	110	109	108
6	11.ô	11.0	10.9	10.8
7	12.9	12.8̂	12.7	12.6
8	14.7	14.6	14.5̂	14.4
9	16.6	16.5	16.3̂	16.2
10	18.4	18.3̂	18.î	18.0
20	36.8	36.6̂	36.3̂	36.0
30	55.2̂	55.0	54.5	54.0
40	73.6̂	73.3̂	72.6̂	72.0
50	92.1	91.6̂	90.8̂	90.0

	107	107	106	105	104
6	10.7̂	10.7	10.6	10.5	10.4
7	12.5̂	12.5	12.3̂	12.2	12.î
8	14.3̂	14.2	14.î	14.0	13.8
9	16.1	16.0̂	15.9	15.7̂	15.6
10	17.9	17.8̂	17.6̂	17.5	17.3̂
20	35.8	35.6̂	35.3̂	35.0	34.6̂
30	53.7	53.5	53.0	52.5	52.0
40	71.6̂	71.3̂	70.6̂	70.0	69.3̂
50	89.6	89.î	88.3̂	87.5	86.6̂

	103̂	103	2	î	1
6	10.3̂	10.3	0.2	0.î	0.1
7	12.1	12.0	0.2̂	0.2	0.î
8	13.8	13.7̂	0.2̂	0.2	0.î
9	15.5	15.4̂	0.3	0.2	0.î
10	17.2	17.î	0.3̂	0.2	0.î
20	34.5	34.3̂	0.6̂	0.5	0.3
30	51.7	51.5	1.0	0.7	0.5
40	69.0	68.6̂	1.3̂	1 0	0.6̂
50	86.2̂	85.8	1.6̂	1.2̂	0.8

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TABLE VII.—LOGARITHMIC SINES, COSINES, TANGENTS, AND COTANGENTS.
7°

′	Log. Sin.	d.	Log. Tan.	c. d.	Log. Cot.	Log. Cos.		P. P.
0	9.08 589		9.08 914		0.91 085	9.99 675	60	
1	9.08 692	102	9.09 018	104	0.90 981	9.99 673	59	
2	9.08 794	102	9.09 123	104	0.90 877	9.99 672	58	
3	9.08 897	102	9.09 226	103	0.90 773	9.99 670	57	
4	9.08 999	102	9.09 330	103	0.90 670	9.99 669	56	
5	9.09 101	102	9.09 433	103	0.90 566	9.99 667	55	
6	9.09 202	101	9.09 536	103	0.90 463	9.99 665	54	
7	9.09 303	101	9.09 639	103	0.90 360	9.99 664	53	
8	9.09 404	101	9.09 742	102	0.90 258	9.99 662	52	
9	9.09 505	101	9.09 844	102	0.90 155	9.99 661	51	
10	9.09 606	100	9.09 947	102	0.90 053	9.99 659	50	
11	9.09 706	100	9.10 048	101	0.89 951	9.99 658	49	
12	9.09 806	100	9.10 150	102	0.89 849	9.99 656	48	
13	9.09 906	100	9.10 252	101	0.89 748	9.99 655	47	
14	9.10 006	99	9.10 353	101	0.89 647	9.99 653	46	
15	9.10 105	99	9.10 454	101	0.89 546	9.99 651	45	
16	9.10 205	99	9.10 555	101	0.89 445	9.99 650	44	
17	9.10 303	98	9.10 655	100	0.89 344	9.99 648	43	
18	9.10 402	99	9.10 756	100	0.89 244	9.99 646	42	
19	9.10 501	98	9.10 856	100	0.89 144	9.99 645	41	
20	9.10 599	98	9.10 956	100	0.89 044	9.99 643	40	
21	9.10 697	98	9.11 055	99	0.88 944	9.99 641	39	
22	9.10 795	97	9.11 155	99	0.88 845	9.99 640	38	
23	9.10 892	97	9 11 254	99	0.88 745	9.99 638	37	
24	9.10 990	97	9.11 353	99	0.88 646	9.99 637	36	
25	9.11 087	97	9.11 452	98	0.88 548	9.99 635	35	
26	9.11 184	96	9.11 550	98	0.88 449	9.99 633	34	
27	9.11 281	97	9.11 649	98	0.88 351	9.99 632	33	
28	9.11 379	96	9.11 747	98	0.88 253	9.99 630	32	
29	9.11 473	96	9.11 845	98	0.88 155	9.99 628	31	
30	9.11 570	96	9.11 943	98	0.88 057	9.99 627	30	
31	9.11 663	95	9.12 040	97	0.87 959	9.99 625	29	
32	9.11 761	96	9.12 137	97	0.87 862	9.99 623	28	
33	9.11 856	95	9.12 235	97	0.87 765	9.99 622	27	
34	9.11 952	95	9.12 331	96	0.87 668	9.99 620	26	
35	9.12 047	95	9.12 428	97	0.87 571	9.99 618	25	
36	9.12 141	94	9.12 525	96	0.87 475	9.99 617	24	
37	9.12 236	94	9.12 621	96	0.87 379	9.99 615	23	
38	9.12 330	94	9.12 717	96	0.87 283	9.99 613	22	
39	9.12 425	94	9.12 813	96	0.87 187	9.99 611	21	
40	9.12 518	93	9.12 908	95	0.87 091	9.99 610	20	
41	9.12 612	94	9.13 004	95	0.86 996	9.99 608	19	
42	9.12 706	93	9.13 099	95	0.86 900	9.99 606	18	
43	9.12 799	93	9.13 194	95	0.86 805	9.99 605	17	
44	9.12 892	93	9.13 289	95	0.86 710	9.99 603	16	
45	9.12 985	93	9.13 384	94	0.86 616	9.99 601	15	
46	9.13 078	92	9.13 478	94	0.86 521	9.99 600	14	
47	9.13 170	92	9.13 572	94	0.86 427	9.99 598	13	
48	9.13 263	92	9.13 666	94	0.86 333	9.99 596	12	
49	9.13 355	92	9.13 760	94	0.86 239	9.99 594	11	
50	9.13 447	91	9.13 854	93	0.86 146	9.99 593	10	
51	9.13 538	92	9.13 947	93	0.86 052	9.99 591	9	
52	9.13 630	91	9.14 041	93	0.85 959	9.99 589	8	
53	9.13 721	91	9.14 134	93	0.85 866	9.99 587	7	
54	9.13 813	90	9.14 227	93	0.85 773	9.99 586	6	
55	9.13 903	91	9.14 319	92	0.85 680	9.99 584	5	
56	9.13 994	91	9.14 412	92	0.85 588	9.99 582	4	
57	9.14 085	90	9.14 504	92	0.85 495	9.99 580	3	
58	9.14 175	90	9.14 596	92	0.85 403	9.99 579	2	
59	9.14 265	90	9.14 688	92	0.85 311	9.99 577	1	
60	9.14 355		9.14 780		0.85 219	9.99 575	0	
	Log. Cos.	d.	Log. Cot.	c. d.	Log. Tan.	Log. Sin.	′	P. P.

P. P.

	104	103	102	101
6	10.4	10.3	10.2	10.1
7	12.1	12.0	11.9	11.8
8	13.8	13.7	13.6	13.4
9	15.6	15.4	15.3	15.1
10	17.3	17.1	17.0	16.8
20	34.6	34.3	34.0	33.6
30	52.0	51.5	51.0	50.5
40	69.3	68.6	68.0	67.3
50	86.6	85.8	85.0	84.1

	106	100	99	98
6	10.6	10.0	9.9	9.8
7	11.7	11.6	11.5	11.4
8	13.4	13.3	13.2	13.0
9	15.1	15.0	14.8	14.7
10	16.7	16.6	16.5	16.3
20	33.5	33.3	33.0	32.6
30	50.2	50.0	49.5	49.0
40	67.0	66.6	66.0	65.3
50	83.7	83.3	82.5	81.6

	97	97	96	95
6	9.7	9.7	9.6	9.5
7	11.4	11.3	11.2	11.1
8	13.0	12.9	12.8	12.6
9	14.6	14.5	14.4	14.2
10	16.2	16.1	16.0	15.8
20	32.5	32.3	32.0	31.6
30	48.7	48.5	48.0	47.5
40	65.0	64.6	64.0	63.3
50	81.2	80.8	80.0	79.1

	94	94	93	92
6	9.4	9.4	9.3	9.2
7	11.0	10.9	10.8	10.7
8	12.6	12.5	12.4	12.2
9	14.2	14.1	13.9	13.8
10	15.7	15.6	15.5	15.3
20	31.5	31.3	31.0	30.6
30	47.2	47.0	46.5	46.0
40	63.0	62.6	62.0	61.3
50	78.7	78.3	77.5	76.6

	91	91	90	2	1
6	9.1	9.1	9.0	0.2	0.1
7	10.7	10.6	10.5	0.2	0.2
8	12.2	12.1	12.0	0.2	0.2
9	13.7	13.6	13.5	0.3	0.2
10	15.2	15.1	15.0	0.3	0.2
20	30.5	30.3	30.0	0.6	0.5
30	45.7	45.5	45.0	1.0	0.7
40	61.0	60.6	60.0	1.3	1.0
50	76.2	75.8	75.0	1.6	1.2

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TABLE VII.—LOGARITHMIC SINES, COSINES, TANGENTS, AND COTANGENTS.

8°

′	Log. Sin	d.	Log. Tan.	c. d.	Log. Cot.	Log. Cos.	′
0	9.14355̂	90	9.14786̂	9î	0.85219̂	9.99575̂	60
1	9.14445̂	8ô	9.14872	91	0.85128	9.99573̂	59
2	9.14535̂	8ô	9.14963	9î	0.85037	9.99571̂	58
3	9.14624̂	89	9.15054̂	91	0.84945̂	9.99570	57
4	9.14713̂	89	9.15145̂	91	0.84854̂	9.99568	56
5	9.14802̂	89	9.15236	9î	0.84763̂	9.99566̂	55
6	9.14891̂	89	9.15327	9ô	0.84673	9.99564̂	54
7	9.14980	88	9.15417	9ô	0.84582̂	9.99563	53
8	9.15068̂	88	9.15507̂	90	0.84492̂	9.99561̂	52
9	9.15157	88	9.15598	9ô	0.84402	9.99559̂	51
10	9.15245	88	9.15687	8ô	0.84312̂	9.99557̂	50
11	9.15333	88	9.15777̂	90	0.84222̂	9.99555̂	49
12	9.15421	87	9.15867	8ô	0.84133	9.99553̂	48
13	9.15508̂	87	9.15956̂	8ô	0.84043̂	9.99552	47
14	9.15595̂	87	9.16045̂	89	0.83954̂	9.99550	46
15	9.15683	87	9.16134̂	89	0.83865̂	9.99548	45
16	9.15770	87	9.16223̂	89	0.83776	9.99546̂	44
17	9.15857	86	9.16312̂	89	0.83687̂	9.99544̂	43
18	9.15943̂	86	9.16401	88	0.83599	9.99542̂	42
19	9.16030	86	9.16489	88	0.83511	9.99541	41
20	9.16116̂	86	9.16577̂	88	0.83422̂	9.99539	40
21	9.16202̂	86	9.16665̂	88	0.83334̂	9.99537	39
22	9.16288̂	86	9.16753	87	0.83247	9.99535̂	38
23	9.16374̂	85	9.16841	88	0.83159	9.99533̂	37
24	9.16460	85	9.16928	87	0.83071̂	9.99531̂	36
25	9.16545̂	85	9.17015̂	87	0.82984̂	9.99529̂	35
26	9.16630̂	85	9.17103	87	0.82897	9.99528	34
27	9.16716	85	9.17190	87	0.82810	9.99526	33
28	9.16801	84	9.17276̂	86	0.82723̂	9.99524	32
29	9.16885̂	84	9.17363̂	87	0.82636̂	9.99522	31
30	9.16970	84	9.17450	86	0.82550	9.99520̂	30
31	9.17054̂	84	9.17536	86	0.82464	9.99518̂	29
32	9.17139	84	9.17622̂	86	0.82377̂	9.99516̂	28
33	9.17223	84	9.17708̂	86	0.82291̂	9.99514̂	27
34	9.17307	84	9.17794	85	0.82206	9.99512̂	26
35	9.17391	83	9.17880	86	0.82120	9.99511	25
36	9.17474̂	83	9.17965̂	85	0.82034̂	9.99509	24
37	9.17558	83	9.18051	85	0.81949	9.99507	23
38	9.17641	83	9.18136	85	0.81864	9.99505	22
39	9.17724	83	9.18221	85	0.81779	9.99503	21
40	9.17807	83	9.18306	85	0.81694	9.99501̂	20
41	9.17890	83	9.18390̂	84	0.81609̂	9.99499̂	19
42	9.17972̂	82	9.18475	84	0.81525	9.99497̂	18
43	9.18055	82	9.18559̂	84	0.81440̂	9.99495̂	17
44	9.18137̂	82	9.18644	84	0.81356	9.99493̂	16
45	9.18219̂	82	9.18728	84	0.81272	9.99491̂	15
46	9.18301̂	82	9.18812	84	0.81188	9.99489̂	14
47	9.18383̂	82	9.18896	84	0.81104	9.99487̂	13
48	9.18465	8î	9.18979̂	83	0.81020̂	9.99485̂	12
49	9.18546̂	8î	9.19063	83	0.80937	9.99484	11
50	9.18628	8î	9.19146	83	0.80854	9.99482	10
51	9.18709	81	9.19229̂	83	0.80770̂	9.99480	9
52	9.18790̂	8î	9.19312̂	83	0.80687̂	9.99478	8
53	9.18871	8ô	9.19395̂	83	0.80604̂	9.99476	7
54	9.18952	81	9.19478	8ẑ	0.80522	9.99474	6
55	9.19032̂	8ô	9.19560̂	8ẑ	0.80439̂	9.99472	5
56	9.19113	8ô	9.19643	8ẑ	0.80357	9.99470	4
57	9.19193̂	8ô	9.19725̂	8ẑ	0.80274̂	9.99468	3
58	9.19273̂	80	9.19807̂	82	0.80192̂	9.99466	2
59	9.19353̂	80	9.19889̂	82	0.80110̂	9.99464	1
60	9.19433	79̂	9.19971̂	82	0.80028̂	9.99462	0
	Log. Cos.	d.	Log. Cot.	c. d.	Log. Tan.	Log. Sin.	′

P. P.

	9î	91	90	89
6	9.î	9.1	9.0	8.9
7	10.7	10.6	10.5	10.4
8	12.2	12.î	12.0	11.8̂
9	13.7	13.6	13.5	13.3̂
10	15.2	15.î	15.0	14.8̂
20	30.5	30.3̂	30.0	29.6̂
30	45.7̂	45.5	45.0	44.5
40	61.0	60.6̂	60.0	59.3̂
50	76.2̂	75.8	75.0	74.î

	8ô	88	87	86
6	8.ô	8.8	8.7	8.6
7	10.3	10.2̂	10.î	10.ô
8	11.8	11.7̂	11.6	11.4̂
9	13.3	13.2	13.ô	12.9
10	14.7̂	14.6	14.5	14.3̂
20	29.5	29.3̂	29.0	28.6̂
30	44.2̂	44.0	43.5	43.0
40	59.0	58.6̂	58.0	57.3̂
50	73.7̂	73.3̂	72.5	71.6̂

	8ĝ	85	84	83
6	8.ĝ	8.5	8.4	8.3
7	10.0	9.9	9.8	9.7
8	11.4	11.3̂	11.2	11.ô
9	12.8	12.7̂	12.6	12.4̂
10	14.2̂	14.î	14.0	13.8̂
20	28.5	28.3̂	28.0	27.6̂
30	42.7̂	42.5	42.0	41.5
40	57.0	56.6̂	56.0	55.3̂
50	71.2̂	70.8	70.0	69.î

	8ẑ	82	81	80
6	8.ẑ	8.2	8.1	8.0
7	9.6	9.5̂	9.4̂	9.3̂
8	11.0	10.9̂	10.8	10.6̂
9	12.4	12.3	12.î	12.0
10	13.7̂	13.6̂	13.5	13.3̂
20	27.5	27.3̂	27.0	26.6̂
30	41.2̂	41.0	40.5	40.0
40	55.0	54.6̂	54.0	53.3̂
50	68.7̂	68.3̂	67.5	66.6̂

	7ĝ	2	î
6	7.ĝ	0.2	0.î
7	9.3	0.2̂	0.2
8	10.6	0.2̂	0.2
9	11.9	0.3	0.2
10	13.2̂	0.3̂	0.2
20	26.5	0.6̂	0.5
30	39.7̂	1.0	0.7̂
40	53.0	1.3̂	1.0
50	66.2̂	1.6̂	1.2

TABLE VII.—LOGARITHMIC SINES, COSINES, TANGENTS, AND COTANGENTS.

9°

'	Log. Sin.	d.	Log. Tan.	c. d.	Log. Cot.	Log. Cos.	'
0	9.19 433	80	9.19 97î	8î	0.80 028	9.99 462	60
1	9.19 513	7ô	9.20 053	8î	0.79 947	9.99 460	59
2	9.19 59ê	79	9.20 13ê	8î	0.79 86ê	9.99 458	58
3	9.19 672	79	9.20 216	81	0.79 784	9.99 456	57
4	9.19 751	79	9.20 297	81	0.79 703	9.99 454	56
5	9.19 830	79	9.20 378	81	0.79 622	9.99 452	55
6	9.19 909	79	9.20 459	81	0.79 541	9.99 450	54
7	9.19 988	7ô	9.20 540	8ô	0.79 460	9.99 448	53
8	9.20 06ô	7ô	9.20 62ô	81	0.79 37ô	9.99 446	52
9	9.20 145	7ê	9.20 70î	80	0.79 298	9.99 444	51
10	9.20 22ê	78	9.20 78î	8ô	0.79 21ê	9.99 442	50
11	9.20 30î	78	9.20 862	80	0.79 138	9.99 440	49
12	9.20 37ô	78	9.20 942	80	0.79 05ê	9.99ˑ437	48
13	9.20 457	78	9.21 022	80	0.78 978	9.99 43ê	47
14	9.20 53ê	78	9.21 102	80	0.78 898	9.99 43ê	46
15	9.20 613	77	9.21 18î	7ô	0.78 81ê	9.99 43î	45
16	9.20 69ô	7ô	9.21 261	7ô	0.78 739	9.99 42ô	44
17	9.20 768	77	9.21 34ô	79	0.78 65ô	9.99 42ô	43
18	9.20 845	77	9.21 420	79	0.78 580	9.99 42ê	42
19	9.20 922	77	9.21 499	79	0.78 501	9.99 42ê	41
20	9.20 999	77	9.21 578	79	0.78 422	9.99 421	40
21	9.21 076	77	9.21 657	79	0.78 343	9.99 419	39
22	9.21 15ê	76	9.21 73ê	7ô	0.78 26ê	9.99 417	38
23	9.21 229	76	9.21 814	78	0.78 186	9.99 415	37
24	9.21 30ê	76	9.21 89ê	78	0.78 10ô	9.99 413	36
25	9.21 382	76	9.21 971	7ê	0.78 029	9.99 411	35
26	9.21 458	76	9.22 049	78	0.77 95î	9.99 40ê	34
27	9.21 534	7ê	9.22 127	78	0.77 873	9.99 40ô	33
28	9.21 60ô	76	9.22 205	78	0.77 795	9.99 40â	32
29	9.21 68ê	75	9.22 283	78	0.77 717	9.99 40ê	31
30	9.21 761	75	9.22 36ô	7ô	0.77 63ô	9.99 40ô	30
31	9.21 83ô	75	9.22 438	7ô	0.77 562	9.99 398	29
32	9.21 91î	7ê	9.22 51ê	7ô	0.77 48ê	9.99 396	28
33	9.21 987	75	9.22 593	77	0.77 407	9.99 394	27
34	9.22 062	75	9.22 670	77	0.77 330	9.99 392	26
35	9.22 13ô	7â	9.22 747	77	0.77 253	9.99 38ô	25
36	9.22 21î	7ê	9.22 824	77	0.77 176	9.99 38î	24
37	9.22 286	7â	9.22 90ô	7ê	0.77 09ô	9.99 38ê	23
38	9.22 36ô	7â	9.22 97ô	77	0.77 02ê	9.99 383	22
39	9.22 435	7â	9.23 054	7ê	0.76 946	9.99 381	21
40	9.22 509	74	9.23 130	7ê	0.76 870	9.99 379	20
41	9.22 58ê	7â	9.23 20ô	7ê	0.76 79ê	9.99 377	19
42	9.22 65î	7â	9.23 282	7ê	0.76 71î	9.99 37â	18
43	9.22 731	73	9.23 35ô	7ê	0.76 64î	9.99 37ê	17
44	9 23 805	74	9 23 43â	7ê	0.76 56ê	9.99 37ô	16
45	9.22 878	7ê	9.23 51ô	7ê	0.76 48ô	9.99 368	15
46	9.22 952	7ê	9.23 586	7ê	0.76 414	9.99 366	14
47	9.23 025	7ê	9.23 66î	7ê	0.76 33ô	9.99 364	13
48	9.23 09ô	73	9.23 737	7ê	0.76 263	9.99 36î	12
49	9.23 17î	73	9.23 812	75	0.76 188	9.99 35ô	11
50	9.23 24â	73	9.23 887	75	0.76 113	9.99 357	10
51	9.23 317	7ê	9.23 962	75	0.76 038	9.99 355	9
52	9.23 390	73	9.24 037	75	0.75 963	9.99 353	8
53	9.23 46ê	7ê	9.24 112	75	0.75 888	9.99 35ô	7
54	9.23 535	7ê	9.24 18ô	7â	0.75 81ê	9.99 348	6
55	9.23 60ô	7ê	9.24 261	7â	0.75 739	9.99 346	5
56	9.23 67ô	72	9.24 33ô	7â	0.75 66â	9.99 344	4
57	9.23 75î	72	9.24 40ô	74	0.75 59ô	9.99 342	3
58	9.23 82ô	72	9.24 484	7â	0.75 516	9.99 33ô	2
59	9.23 89ê	72	9.24 558	74	0.75 442	9.99 33ô	1
60	9.23 967	7î	9.24 632	74	0.75 368	9.99 335	0
	Log. Cos.	d.	Log. Cot.	c. d.	Log. Tan.	Log. Sin.	'

P. P.

	8î	8ı	8o	79
6	8.î	8.1	8.0	7.9
7	9.5	9.4	9.3	9.2
8	10.ô	10.8	10.ô	10.ô
9	12.2	12.î	12.0	11.ô
10	13.6	13.5	13.ê	13.î
20	27.î	27.0	26.ô	26.ê
30	40.ô	40.5	40.0	39.5
40	54.ê	54.0	53.ê	52.ô
50	67.9	67.5	66.ô	65.8

	7ô	78	77
6	7.ô	7.8	7.7
7	9.î	9.1	9.0
8	10.â	10.4	10.ê
9	11.8	11.7	11.ô
10	13.1	13.0	12.ô
20	26.î	26.0	25.ô
30	39.ê	39.0	38.ê
40	52.ê	52.0	51.ê
50	65.4	65.0	64.î

	7ô	76	75	74
6	7.ô	7.6	7.5	7.4
7	8.9	8.ô	8.ô	8.6
8	10.2	10.î	10.0	9.ô
9	11.5	11.4	11.ê	11.î
10	12.ô	12.ô	12.5	12.ê
20	25.5	25.ê	25.0	24.ô
30	38.ê	38.0	37.5	37.0
40	51.0	50.ô	50.0	49.ê
50	63.ô	63.ê	62.5	61.ô

	7ê	73	72
6	7.ê	7.3	7.2
7	8.6	8.5	8.4
8	9.8	9.ô	9.6
9	11.0	10.ô	10.8
10	12.ê	12.î	12.0
20	24.5	24.ê	24.0
30	36.ô	36.5	36.0
40	49.0	48.ô	48.0
50	61.ê	60.ô	60.0

	7î	71	ê	2
6	7.î	7.1	0.ê	0.2
7	8.ê	8.3	0.ê	0.ê
8	9.ô	9.â	0.3	0.ê
9	10.7	10.ô	0.4	0.3
10	11.9	11.8	0.4	0.ê
20	23.ô	23.6	0.8	0.6
30	35.7	35.5	1.ê	1.0
40	47.ô	47.ê	1.ô	1.ê
50	59.6	59.î	2.1	1.6

P. P.

80°

TABLE VII.—LOGARITHMIC SINES, COSINES, TANGENTS, AND COTANGENTS.

10°

′	Log. Sin.	d.	Log. Tan.	c. d.	Log. Cot.	Log. Cos.		P. P.			
0	9.23 967	7î	9.24 632	7ŝ	0.75 368	9.99 335	60				
1	9.24 03ŝ	7î	9.24 70ŝ	74	0.75 294	9.99 333	59		74	7ŝ	73
2	9.24 110	7î	9.24 77ŝ	7ŝ	0.75 22ô	9.99 33ô	58	6	7.4	7.ŝ	7.3
3	9.24 18î	71	9.24 853	7ŝ	0.75 147	9.99 328	57	7	8.ô	8.6	8.5
4	9.24 25ŝ	71	9.24 926	7ŝ	0.75 07ŝ	9.99 326	56	8	9 ŝ	9.8	9.ŝ
5	9.24 32ŝ	71	9.25 000	7ŝ	0.75 000	9.99 324	55	9	11.1	11.0	10.ô
6	9.24 39ŝ	71	9.25 073	73	0.74 927	9.99 32î	54	10	12.ŝ	12.ŝ	12.1
7	9.24 46ŝ	71	9.25 146	73	0.74 854	9.99 319	53	20	24.ô	24.5	24.ŝ
8	9.24 536	71	9.25 219	73	0.74 781	9.99 317	52	30	37.0	36.ŝ	36.5
9	9.24 607	7ô	9.25 292	73	0.74 708	9.99 315	51	40	49.ŝ	49.0	48.ô
10	9.24 67ŝ	7ô	9.25 365	73	0.74 635	9.99 31ŝ	50	50	61.ô	61.ŝ	60.8
11	9.24 748	7ô	9.25 437	7ŝ	0.74 56ŝ	9.99 31ô	49		7ŝ	72	7î	71
12	9.24 818	70	9.25 510	7ŝ	0.74 490	9.99 308	48	6	7.ŝ	7.2	7.î	7.1
13	9.24 88ŝ	7ô	9.25 58ŝ	72	0.74 41ŝ	9.99 306	47	7	8.ŝ	8.4	8.ŝ	8.3
14	9.24 958	70	9.25 65ŝ	72	0.74 34ŝ	9.99 303	46	8	9.ŝ	9.6	9.ŝ	9.ŝ
15	9.25 028	6ô	9.25 727	72	0.74 273	9.99 30î	45	9	10.9	10.8	10.7	10.ô
16	9.25 098	70	9.25 799	72	0.74 201	9.99 299	44	10	12.1	12.0	11.9	11.ŝ
17	9.25 16ŝ	6ô	9.25 871	72	0.74 129	9.99 29ô	43	20	24.î	24.0	23.ŝ	23.6
18	9.25 23ŝ	70	9.25 943	72	0.74 057	9.99 29ŝ	42	30	36.ŝ	36.0	35.ŝ	35.5
19	9.25 306	69	9.26 01ŝ	7î	0.73 98ŝ	9.99 292	41	40	48.ŝ	48.0	47.6	47.ŝ
20	9.25 376	6ô	9.26 08ŝ	72	0.73 91ŝ	9.99 290	40	50	60.4	60.0	59.ŝ	59.î
21	9.25 44ŝ	6ô	9.26 158	7î	0.73 842	9.99 28ŝ	39		7ô	70	6ô	69
22	9.25 51ŝ	69	9.26 229	71	0.73 771	9.99 285	38	6	7.ô	7.0	6.ô	6.9
23	9.25 58ŝ	69	9.26 30ô	7î	0.73 69ô	9.99 283	37	7	8.2	8.î	8.1	8.ô
24	9.25 65ŝ	69	9.26 37î	7î	0.73 628	9.99 28ô	36	8	9.4	9.ŝ	9.ŝ	9.2
25	9.25 721	6ŝ	9.26 443	7î	0.73 557	9.99 27ŝ	35	9	10.6	10.5	10.4	10.ŝ
26	9.25 790	69	9.26 514	71	0.73 486	9.99 276	·34	10	11.ŝ	11.ô	11.ŝ	11.5
27	9.25 858	6ŝ	9.26 58ŝ	7ô	0.73 41ŝ	9.99 27ŝ	33	20	23.5	23.ŝ	23.î	23.0
28	9.25 927	6ŝ	9.26 65ŝ	71	0.73 34ŝ	9.99 27î	32	30	35.ŝ	35.0	34.ŝ	34.5
29	9.25 995	68	9.26 726	7ô	0.73 274	9.99 269	31	40	47.0	46.ô	46.ŝ	46.0
30	9.26 06ŝ	68	9.26 79ô	7ô	0.73 20ŝ	9.99 26ô	30	50	58.ŝ	58.ŝ	57.9	57.5
31	9.26 13î	68	9.26 867	7ô	0.73 133	9.99 26ŝ	29		6ŝ	68	6ŝ	67
32	9.26 19ô	68	9.26 93ŝ	70	0.73 06ŝ	9.99 262	28	6	6.8	6.8	6.ŝ	6.7
33	9.26 26ŝ	67	9.27 00ŝ	7ô	0.72 99ŝ	9.99 25ô	27	7	8.0	7.ô	7.9	7.8
34	9.26 335	67	9.27 078	7ô	0.72 922	9.99 257	26	8	9.î	9.ô	9.0	8.ô
35	9.26 40ŝ	68	9.27 148	70	0.72 852	9.99 255	25	9	10.3	10.2	10.1	10.ô
36	9.26 80ô	67	9.27 218	70	0.72 782	9.99 25ŝ	24	10	11.4	11.ŝ	11.ŝ	11.î
37	9.26 53ŝ	67	9.27 287	6ô	0.72 71ŝ	9.99 250	23	20	22.8	22.ô	22.5	22.ŝ
38	9.26 605	6ŝ	9.27 35ŝ	70	0.72 64ŝ	9.99 248	22	30	34.ŝ	34.0	33.ŝ	33.5
39	9.26 67ŝ	67	9.27 427	6ô	0.72 573	9.99 24ŝ	21	40	45.ô	45.ŝ	45.0	44.ô
40	9.26 73ô	67	9.27 49ô	6ô	0.72 50ŝ	9.99 243	20	50	57.1	56.ô	56.ŝ	55.8
41	9.26 80ŝ	67	9.27 566	6ô	0.72 434	9.99 24ô	19		6ŝ	66	6ŝ	65
42	9.26 87ŝ	67	9.27 635	6ô	0.72 365	9.99 238	18	6	6.ô	6.6	6.ŝ	6.5
43	9.26 940	6ô	9.27 70ŝ	69	0.72 29ŝ	9.99 236	17	7	7.ŝ	7.7	7.6	7.6
44	9.27 007	67	9.27 77ŝ	69	0.72 22ô	9.99 23ŝ	16	8	8.8	8.8	8.ŝ	8.ô
45	9.27 07ŝ	6ô	9.27 84ŝ	69	0.72 157	9.99 231	15	9	10.0	9.9	9.8	9.ŝ
46	9.27 140	6ô	9.27 91î	6ŝ	0.72 088	9.99 22ŝ	14	10	11.1	11.0	10.9	10.ŝ
47	9.27 20ô	66	9.27 980	69	0.72 020	9.99 226	13	20	22.î	22.0	21.ŝ	21.ô
48	9.27 27ŝ	66	9.28 049	6ŝ	0.71 951	9.99 224	12	30	33.ŝ	33.0	32.ŝ	32.5
49	9.27 339	6ô	9.28 11ŝ	6ŝ	0.71 88ŝ	9.99 22î	11	40	44.ŝ	44.0	43.6	43.ŝ
50	9.27 405	66	9.28 186	68	0.71 814	9.99 219	10	50	55.4	55.0	54.6	54.î
51	9.27 47î	6ŝ	9.28 254	6ŝ	0.71 746	9.99 21ô	9			ŝ	2		
52	9.27 53ô	66	9.28 32ŝ	68	0.71 67ŝ	9.99 214	8		6	0.ŝ	0.2		
53	9.27 60ŝ	66	9.28 39ô	68	0.71 60ô	9.99 212	7		7	0.3	0.ŝ		
54	9.27 668	6ŝ	9.28 459	68	0.71 541	9.99 20ô	6		8	0.ŝ	0.ŝ		
55	9.27 73ŝ	6ŝ	9.28 527	68	0.71 473	9.99 207	5		9	0.4	0.3		
56	9.27 799	6ŝ	9.28 59ŝ	6ŝ	0.71 40ŝ	9.99 20ŝ	4		10	0.4	0.ŝ		
57	9.27 86ŝ	6ŝ	9.28 66ŝ	68	0.71 33ŝ	9.99 202	3		20	0.8	0.6		
58	9.27 92ô	6ŝ	9.28 730	6ŝ	0.71 270	9.99 19ô	2		30	1.ŝ	1.0		
59	9.27 995	65	9.28 79ŝ	6ŝ	0.71 20ŝ	9.99 197	1		40	1.ô	1.ŝ		
60	9.28 060		9.28 865		0.71 135	9.99 19ŝ	0		50	2.1	1.6		
	Log. Cos.	d.	Log. Cot.	c. d.	Log. Tan.	Log. Sin.	′			P. P.		

TABLE VII.—LOGARITHMIC SINES, COSINES, TANGENTS, AND COTANGENTS.

11°

′	Log. Sin.	d.	Log. Tan.	c. d.	Log. Cot.	Log. Cos.	′
0	9.28 060	65	9.28 865	67	0.71 135	9.99 194	60
1	9.28 125	64	9.28 932	67	0.71 067	9.99 192	59
2	9.28 189	65	9.29 000	67	0.71 000	9.99 189	58
3	9.28 254	64	9.29 067	67	0.70 933	9.99 187	57
4	9.28 319	64	9.29 134	67	0.70 866	9.99 185	56
5	9.28 383	64	9.29 201	66	0.70 798	9.99 182	55
6	9.28 448	64	9.29 268	67	0.70 732	9.99 180	54
7	9.28 512	64	9.29 335	66	0.70 665	9.99 177	53
8	9.28 576	64	9.29 401	67	0.70 598	9.99 175	52
9	9.28 641	64	9.29 468	66	0.70 531	9.99 172	51
10	9.28 705	64	9.29 535	66	0.70 465	9.99 170	50
11	9.28 769	63	9.29 601	66	0.70 398	9.99 167	49
12	9.28 832	64	9.29 667	66	0.70 332	9.99 165	48
13	9.28 896	63	9.29 734	66	0.70 266	9.99 162	47
14	9.28 960	63	9.29 800	66	0.70 200	9.99 160	46
15	9.29 023	63	9.29 866	66	0.70 134	9.99 157	45
16	9.29 087	63	9.29 932	66	0.70 068	9.99 155	44
17	9.29 150	63	9.29 998	66	0.70 002	9.99 152	43
18	9.29 213	63	9.30 064	66	0.69 936	9.99 150	42
19	9.29 277	63	9.30 129	65	0.69 870	9.99 147	41
20	9.29 340	63	9.30 195	65	0.69 805	9.99 145	40
21	9.29 403	63	9.30 260	66	0.69 739	9.99 142	39
22	9.29 466	63	9.30 326	65	0.69 674	9.99 139	38
23	9.29 528	63	9.30 391	65	0.69 608	9.99 137	37
24	9.29 591	63	9.30 456	65	0.69 543	9.99 134	36
25	9.29 654	62	9.30 522	65	0.69 478	9.99 132	35
26	9.29 716	63	9.30 587	65	0.69 413	9.99 129	34
27	9.29 779	62	9.30 652	65	0.69 348	9.99 127	33
28	9.29 841	62	9.30 717	65	0.69 283	9.99 124	32
29	9.29 903	62	9.30 781	65	0.69 218	9.99 122	31
30	9.29 965	62	9.30 846	64	0.69 153	9.99 119	30
31	9.30 027	62	9.30 911	64	0.69 089	9.99 116	29
32	9.30 089	62	9.30 975	64	0.69 024	9.99 114	28
33	9.30 151	61	9.31 040	64	0.68 960	9.99 111	27
34	9.30 213	62	9.31 104	64	0.68 896	9.99 109	26
35	9.30 275	61	9.31 168	64	0.68 831	9.99 106	25
36	9.30 336	61	9.31 232	64	0.68 767	9.99 104	24
37	9.30 398	61	9.31 297	64	0.68 703	9.99 101	23
38	9.30 459	61	9.31 361	63	0.68 639	9.99 098	22
39	9.30 520	61	9.31 424	64	0.68 575	9.99 096	21
40	9.30 582	61	9.31 488	64	0.68 511	9.99 093	20
41	9.30 643	61	9.31 552	64	0.68 447	9.99 091	19
42	9.30 704	61	9.31 616	63	0.68 384	9.99 088	18
43	9.30 765	61	9.31 679	63	0.68 320	9.99 085	17
44	9.30 826	60	9.31 743	63	0.68 257	9.99 083	16
45	9.30 886	61	9.31 806	63	0.68 193	9.99 080	15
46	9.30 947	60	9.31 869	63	0.68 130	9.99 077	14
47	9.31 008	60	9.31 933	63	0.68 067	9.99 075	13
48	9.31 068	60	9.31 996	63	0.68 004	9.99 072	12
49	9.31 129	60	9.32 059	63	0.67 941	9.99 069	11
50	9.31 189	60	9.32 122	63	0.67 878	9.99 067	10
51	9.31 249	60	9.32 185	63	0.67 815	9.99 064	9
52	9.31 309	60	9.32 248	62	0.67 752	9.99 062	8
53	9.31 370	59	9.32 310	63	0.67 689	9.99 059	7
54	9.31 429	60	9.32 373	63	0.67 626	9.99 056	6
55	9.31 489	60	9.32 436	62	0.67 564	9.99 054	5
56	9.31 549	60	9.32 498	62	0.67 501	9.99 051	4
57	9.31 609	59	9.32 560	62	0.67 439	9.99 048	3
58	9.31 669	59	9.32 623	62	0.67 377	9.99 046	2
59	9.31 728	59	9.32 685	62	0.67 314	9.99 043	1
60	9.31 788	59	9.32 747	62	0.67 252	9.99 040	0
	Log. Cos.	d.	Log. Cot.	c. d.	Log. Tan.	Log. Sin.	′

P. P.

	67	67
6	6.7	6.7
7	7.9	7.8
8	9.0	8.9
9	10.1	10.0
10	11.2	11.1
20	22.5	22.3
30	33.7	33.5
40	45.0	44.6
50	56.2	55.8

	66	66	65	65
6	6.6	6.6	6.5	6.5
7	7.7	7.7	7.6	7.6
8	8.8	8.8	8.7	8.6
9	10.0	9.9	9.8	9.7
10	11.1	11.0	10.9	10.8
20	22.1	22.0	21.8	21.6
30	33.2	33.0	32.7	32.5
40	44.3	44.0	43.6	43.3
50	55.4	55.0	54.6	54.1

	64	64	63	63
6	6.4	6.4	6.3	6.3
7	7.5	7.4	7.4	7.3
8	8.6	8.5	8.4	8.4
9	9.7	9.6	9.5	9.4
10	10.7	10.6	10.6	10.5
20	21.5	21.3	21.1	21.0
30	32.2	32.0	31.7	31.5
40	43.0	42.6	42.3	42.0
50	53.7	53.3	52.9	52.5

	62	62	61	61
6	6.2	6.2	6.1	6.1
7	7.3	7.2	7.2	7.1
8	8.3	8.2	8.2	8.1
9	9.4	9.3	9.2	9.1
10	10.4	10.3	10.2	10.1
20	20.8	20.6	20.5	20.3
30	31.2	31.0	30.7	30.5
40	41.6	41.3	41.0	40.6
50	52.1	51.6	51.2	50.8

	60	60	59
6	6.0	6.0	5.9
7	7.0	7.0	6.9
8	8.0	8.0	7.9
9	9.1	9.0	8.9
10	10.1	10.0	9.9
20	20.1	20.0	19.8
30	30.2	30.0	29.7
40	40.3	40.0	39.6
50	50.4	50.0	49.6

	3	2	2
6	0.3	0.2	0.2
7	0.3	0.3	0.2
8	0.4	0.3	0.2
9	0.4	0.4	0.3
10	0.5	0.4	0.3
20	1.0	0.8	0.6
30	1.5	1.2	1.0
40	2.0	1.6	1.3
50	2.5	2.1	1.6

78°

TABLE VII.—LOGARITHMIC SINES, COSINES, TANGENTS, AND COTANGENTS.

12°

′	Log. Sin.	d.	Log. Tan.	c. d.	Log. Cot.	Log. Cos.	′
0	9.31 788	59	9.32 747	62	0.67 252	9.99 040	60
1	9.31 847	59	9.32 809	62	0.67 190	9.99 038	59
2	9.31 906	59	9.32 871	62	0.67 128	9.99 035	58
3	9.31 966	59	9.32 933	62	0.67 066	9.99 032	57
4	9.32 025	59	9.32 995	61	0.67 004	9.99 029	56
5	9.32 084	59	9.33 057	61	0.66 943	9.99 027	55
6	9.32 143	59	9.33 118	62	0.66 881	9.99 024	54
7	9.32 202	58	9.33 180	61	0.66 819	9.99 021	53
8	9.32 260	59	9.33 242	61	0.66 758	9.99 019	52
9	9.32 319	58	9.33 303	61	0.66 696	9.99 016	51
10	9.32 378	58	9.33 364	61	0.66 635	9.99 013	50
11	9.32 436	58	9.33 426	61	0.66 574	9.99 010	49
12	9.32 495	58	9.33 487	61	0.66 513	9.99 008	48
13	9.32 553	58	9.33 548	61	0.66 452	9.99 005	47
14	9.32 611	58	9.33 609	60	0.66 390	9.99 002	46
15	9.32 670	58	9.33 670	61	0.66 330	9.98 999	45
16	9.32 728	58	9.33 731	61	0.66 269	9.98 997	44
17	9.32 786	58	9.33 792	60	0.66 208	9.98 994	43
18	9.32 844	58	9.33 852	61	0.66 147	9.98 991	42
19	9.32 902	58	9.33 913	60	0.66 086	9.98 988	41
20	9.32 960	57	9.33 974	60	0.66 026	9.98 986	40
21	9.33 017	58	9.34 034	60	0.65 965	9.98 983	39
22	9.33 075	57	9.34 095	60	0.65 905	9.98 980	38
23	9.33 133	57	9.34 155	60	0.65 845	9.98 977	37
24	9.33 190	57	9.34 215	60	0.65 784	9.98 975	36
25	9.33 248	57	9.34 275	60	0.65 724	9.98 972	35
26	9.33 305	57	9.34 336	60	0.65 664	9.98 969	34
27	9.33 362	57	9.34 396	60	0.65 604	9.98 966	33
28	9.33 419	57	9.34 456	59	0.65 544	9.98 963	32
29	9.33 476	57	9.34 515	60	0.65 484	9.98 961	31
30	9.33 533	57	9.34 575	60	0.65 424	9.98 958	30
31	9.33 590	57	9.34 635	59	0.65 364	9.98 955	29
32	9.33 647	57	9.34 695	59	0.65 305	9.98 952	28
33	9.33 704	56	9.34 754	59	0.65 245	9.98 949	27
34	9.33 761	56	9.34 814	59	0.65 186	9.98 947	26
35	9.33 817	56	9.34 873	59	0.65 126	9.98 944	25
36	9.33 874	56	9.34 933	59	0.65 067	9.98 941	24
37	9.33 930	56	9.34 992	59	0.65 008	9.98 938	23
38	9.33 987	56	9.35 051	59	0.64 948	9.98 935	22
39	9.34 043	56	9.35 110	59	0.64 889	9.98 933	21
40	9.34 099	56	9.35 169	59	0.64 830	9.98 930	20
41	9.34 156	56	9.35 228	59	0.64 771	9.98 927	19
42	9.34 212	56	9.35 287	59	0.64 712	9.98 924	18
43	9.34 268	56	9.35 346	59	0.64 653	9.98 921	17
44	9.34 324	55	9.35 405	58	0.64 594	9.98 918	16
45	9.34 379	56	9.35 464	58	0.64 536	9.98 915	15
46	9.34 435	56	9.35 522	59	0.64 477	9.98 913	14
47	9.34 491	56	9.35 581	58	0.64 418	9.98 910	13
48	9.34 547	55	9.35 640	58	0.64 360	9.98 907	12
49	9.34 602	55	9.35 698	58	0.64 302	9.98 904	11
50	9.34 658	55	9.35 756	58	0.64 243	9.98 901	10
51	9.34 713	55	9.35 815	58	0.64 185	9.98 898	9
52	9.34 768	55	9.35 873	58	0.64 127	9.98 895	8
53	9.34 824	55	9.35 931	58	0.64 068	9.98 892	7
54	9.34 879	55	9.35 989	58	0.64 010	9.98 890	6
55	9.34 934	55	9.36 047	58	0.63 952	9.98 887	5
56	9.34 989	55	9.36 105	58	0.63 894	9.98 884	4
57	9.35 044	55	9.36 163	57	0.63 837	9.98 881	3
58	9.35 099	54	9.36 221	58	0.63 779	9.98 878	2
59	9.35 154	55	9.36 278	57	0.63 721	9.98 875	1
60	9.35 209	55	9.36 336	58	0.63 663	9.98 872	0
	Log. Cos.	d.	Log. Cot.	c. d.	Log. Tan.	Log. Sin.	′

P. P.

	62	61	61
6	6.2	6.1	6.1
7	7.2	7.2	7.1
8	8.2	8.2	8.1
9	9.3	9.2	9.1
10	10.3	10.2	10.1
20	20.6	20.5	20.3
30	31.0	30.7	30.5
40	41.3	41.0	40.6
50	51.6	51.2	50.8

	60	60	59	59
6	6.0	6.0	5.9	5.9
7	7.0	7.0	6.9	6.9
8	8.0	8.0	7.9	7.8
9	9.1	9.0	8.9	8.8
10	10.1	10.0	9.9	9.8
20	20.1	20.0	19.8	19.6
30	30.2	30.0	29.7	29.5
40	40.3	40.0	39.6	39.3
50	50.4	50.0	49.6	49.1

	58	58	57	57
6	5.8	5.8	5.7	5.7
7	6.8	6.7	6.7	6.6
8	7.8	7.7	7.6	7.6
9	8.8	8.7	8.6	8.5
10	9.7	9.6	9.6	9.5
20	19.5	19.3	19.1	19.0
30	29.2	29.0	28.7	28.5
40	39.0	38.6	38.3	38.0
50	48.7	48.3	47.9	47.5

	56	56	55	55
6	5.6	5.6	5.5	5.5
7	6.6	6.5	6.5	6.4
8	7.5	7.4	7.4	7.3
9	8.5	8.4	8.3	8.2
10	9.4	9.3	9.2	9.1
20	18.8	18.6	18.5	18.3
30	28.2	28.0	27.7	27.5
40	37.6	37.3	37.0	36.6
50	47.1	46.6	46.2	45.8

	54	3	2
6	5.4	0.3	0.2
7	6.3	0.3	0.3
8	7.2	0.4	0.3
9	8.2	0.4	0.4
10	9.1	0.5	0.4
20	18.1	1.0	0.8
30	27.2	1.5	1.2
40	36.3	2.0	1.6
50	45.4	2.5	2.1

TABLE VII.—LOGARITHMIC SINES, COSINES, TANGENTS, AND COTANGENTS.
13°

'	Log. Sin.	d.	Log. Tan.	c. d.	Log. Cot.	Log. Cos.	
0	9.35 209	54	9.36 336	57	0.63 663	9.98 872	60
1	9.35 263	54	9.36 394	57	0.63 606	9.98 869	59
2	9.35 318	54	9.36 451	57	0.63 548	9.98 866	58
3	9.35 372	54	9.36 509	57	0.63 491	9.98 863	57
4	9.35 427	54	9.36 566	57	0.63 433	9.98 860	56
5	9.35 481	54	9.36 623	57	0.63 376	9.98 858	55
6	9.35 536	54	9.36 681	57	0.63 319	9.98 855	54
7	9.35 590	54	9.36 738	57	0.63 262	9.98 852	53
8	9.35 644	54	9.36 795	57	0.63 204	9.98 849	52
9	9.35 698	54	9.36 852	57	0.63 147	9.98 846	51
10	9.35 752	54	9.36 909	57	0.63 090	9.98 843	50
11	9.35 806	54	9.36 966	57	0.63 033	9.98 840	49
12	9.35 860	54	9.37 023	56	0.62 977	9.98 837	48
13	9.35 914	53	9.37 080	57	0.62 920	9.98 834	47
14	9.35 968	54	9.37 136	56	0.62 863	9.98 831	46
15	9.36 021	53	9.37 193	57	0.62 806	9.98 828	45
16	9.36 075	53	9.37 250	56	0.62 750	9.98 825	44
17	9.36 128	53	9.37 306	56	0.62 693	9.98 822	43
18	9.36 182	53	9.37 363	56	0.62 637	9.98 819	42
19	9.36 235	53	9.37 419	56	0.62 580	9.98 816	41
20	9.36 289	53	9.37 475	56	0.62 524	9.98 813	40
21	9.36 342	53	9.37 532	56	0.62 468	9.98 810	39
22	9.36 395	53	9.37 588	56	0.62 412	9.98 807	38
23	9.36 448	53	9.37 644	56	0.62 356	9.98 804	37
24	9.36 501	53	9.37 700	56	0.62 299	9.98 801	36
25	9.36 554	53	9.37 756	56	0.62 243	9.98 798	35
26	9.36 607	53	9.37 812	56	0.62 188	9.98 795	34
27	9.36 660	53	9.37 868	56	0.62 132	9.98 792	33
28	9.36 713	52	9.37 924	56	0.62 076	9.98 789	32
29	9.36 766	53	9.37 979	56	0.62 020	9.98 786	31
30	9.36 818	52	9.38 035	56	0.61 964	9.98 783	30
31	9.36 871	52	9.38 091	55	0.61 909	9.98 780	29
32	9.36 923	52	9.38 146	55	0.61 853	9.98 777	28
33	9.36 976	52	9.38 202	55	0.61 798	9.98 774	27
34	9.37 028	52	9.38 257	55	0.61 742	9.98 771	26
35	9.37 081	52	9.38 313	55	0.61 687	9.98 768	25
36	9.37 133	52	9.38 368	55	0.61 632	9.98 765	24
37	9.37 185	52	9.38 423	55	0.61 576	9.98 762	23
38	9.37 237	52	9.38 478	55	0.61 521	9.98 759	22
39	9.37 289	52	9.38 533	55	0.61 466	9.98 755	21
40	9.37 341	52	9.38 589	55	0.61 411	9.98 752	20
41	9.37 393	52	9.38 644	54	0.61 356	9.98 749	19
42	9.37 445	52	9.38 698	55	0.61 301	9.98 746	18
43	9.37 497	51	9.38 753	55	0.61 246	9.98 743	17
44	9.37 548	52	9.38 808	55	0.61 191	9.98 740	16
45	9.37 600	52	9.38 863	55	0.61 137	9.98 737	15
46	9.37 652	51	9.38 918	54	0.61 082	9.98 734	14
47	9.37 703	52	9.38 972	55	0.61 027	9.98 731	13
48	9.37 755	51	9.39 027	54	0.60 973	9.98 728	12
49	9.37 806	51	9.39 081	55	0.60 918	9.98 725	11
50	9.37 857	52	9.39 136	54	0.60 864	9.98 721	10
51	9.37 909	51	9.39 190	54	0.60 809	9.98 718	9
52	9.37 960	51	9.39 244	54	0.60 755	9.98 715	8
53	9.38 011	51	9.39 299	54	0.60 701	9.98 712	7
54	9.38 062	51	9.39 353	54	0.60 647	9.98 709	6
55	9.38 113	51	9.39 407	54	0.60 592	9.98 706	5
56	9.38 164	51	9.39 461	54	0.60 538	9.98 703	4
57	9.38 215	50	9.39 515	54	0.60 484	9.98 700	3
58	9.38 266	51	9.39 569	54	0.60 430	9.98 696	2
59	9.38 317	51	9.39 623	54	0.60 376	9.98 693	1
60	9.38 367	50	9.39 677	53	0.60 323	9.98 690	0
	Log. Cos.	d.	Log. Cot.	c. d.	Log. Tan.	Log. Sin.	'

P. P.

	57	57	56	56
6	5.7	5.7	5.6	5.6
7	6.7	6.6	6.6	6.5
8	7.6	7.6	7.5	7.4
9	8.6	8.5	8.5	8.4
10	9.6	9.5	9.4	9.3
20	19.1	19.0	18.8	18.6
30	28.7	28.5	28.2	28.0
40	38.3	38.0	37.6	37.3
50	47.9	47.5	47.1	46.6

	55	55	54	54
6	5.5	5.5	5.4	5.4
7	6.5	6.4	6.3	6.3
8	7.4	7.3	7.2	7.2
9	8.3	8.2	8.2	8.1
10	9.2	9.1	9.1	9.0
20	18.5	18.3	18.1	18.0
30	27.7	27.5	27.2	27.0
40	37.0	36.6	36.3	36.0
50	46.2	45.8	45.4	45.0

	53	53	52	52
6	5.3	5.3	5.2	5.2
7	6.2	6.2	6.1	6.0
8	7.1	7.0	7.0	6.9
9	8.0	7.9	7.9	7.8
10	8.9	8.8	8.7	8.6
20	17.8	17.6	17.5	17.3
30	26.7	26.5	26.2	26.0
40	35.6	35.3	35.0	34.6
50	44.6	44.1	43.7	43.3

	51	51	50
6	5.1	5.1	5.0
7	6.0	5.9	5.9
8	6.8	6.8	6.7
9	7.7	7.6	7.6
10	8.6	8.5	8.4
20	17.1	17.0	16.8
30	25.7	25.5	25.2
40	34.3	34.0	33.6
50	42.9	42.5	42.1

	3	3	2
6	0.3	0.3	0.2
7	0.4	0.3	0.3
8	0.4	0.4	0.3
9	0.5	0.4	0.4
10	0.6	0.5	0.4
20	1.1	1.0	0.8
30	1.7	1.5	1.2
40	2.3	2.0	1.6
50	2.9	2.5	2.1

76°

TABLE VII.—LOGARITHMIC SINES, COSINES, TANGENTS, AND COTANGENTS.

14°

′	Log. Sin.	d.	Log. Tan.	c. d.	Log. Cot.	Log. Cos.	d.	′
0	9.38 367	56	9.39 677	54	0.60 323	9.98 696	3	60
1	9.38 418	56	9.39 731	54	0.60 269	9.98 687	3	59
2	9.38 468	56	9.39 784	53	0.60 215	9.98 684	3	58
3	9.38 519	56	9.39 838	54	0.60 161	9.98 681	3	57
4	9.38 569	56	9.39 892	53	0.60 108	9.98 678	3	56
5	9.38 620	56	9.39 945	53	0.60 054	9.98 674	3	55
6	9.38 670	56	9.39 999	53	0.60 001	9.98 671	3	54
7	9.38 720	50	9.40 052	53	0.59 947	9.98 668	3	53
8	9.38 771	50	9.40 106	53	0.59 894	9.98 665	3	52
9	9.38 821	50	9.40 159	53	0.59 841	9.98 662	3	51
10	9.38 871	50	9.40 212	53	0.59 787	9.98 658	3	50
11	9.38 921	50	9.40 265	53	0.59 734	9.98 655	3	49
12	9.38 971	50	9.40 318	53	0.59 681	9.98 652	3	48
13	9.39 021	50	9.40 372	53	0.59 628	9.98 649	3	47
14	9.39 071	50	9.40 425	53	0.59 575	9.98 646	3	46
15	9.39 120	49	9.40 478	53	0.59 522	9.98 642	3	45
16	9.39 170	50	9.40 531	52	0.59 469	9.98 639	3	44
17	9.39 220	49	9.40 583	53	0.59 416	9.98 636	3	43
18	9.39 269	49	9.40 636	53	0.59 363	9.98 633	3	42
19	9.39 319	49	9.40 689	53	0.59 311	9.98 630	3	41
20	9.39 368	49	9.40 742	52	0.59 258	9.98 626	3	40
21	9.39 418	49	9.40 794	53	0.59 205	9.98 623	3	39
22	9.39 467	49	9.40 847	52	0.59 153	9.98 620	3	38
23	9.39 516	49	9.40 899	53	0.59 100	9.98 617	3	37
24	9.39 566	49	9.40 952	52	0.59 048	9.98 613	3	36
25	9.39 615	49	9.41 004	52	0.58 995	9.98 610	3	35
26	9.39 664	49	9.41 057	52	0.58 943	9.98 607	3	34
27	9.39 713	49	9.41 109	52	0.58 891	9.98 604	3	33
28	9.39 762	49	9.41 161	52	0.58 838	9.98 600	3	32
29	9.39 811	49	9.41 213	52	0.58 786	9.98 597	3	31
30	9.39 860	49	9.41 266	52	0.58 734	9.98 594	3	30
31	9.39 909	49	9.41 318	52	0.58 682	9.98 591	3	29
32	9.39 957	48	9.41 370	52	0.58 630	9.98 587	3	28
33	9.40 006	48	9.41 422	52	0.58 578	9.98 584	3	27
34	9.40 055	49	9.41 474	52	0.58 526	9.98 581	3	26
35	9.40 103	48	9.41 525	52	0.58 474	9.98 578	3	25
36	9.40 152	48	9.41 577	52	0.58 422	9.98 574	3	24
37	9.40 200	48	9.41 629	52	0.58 370	9.98 571	3	23
38	9.40 249	48	9.41 681	51	0.58 319	9.98 568	3	22
39	9.40 297	48	9.41 732	51	0.58 267	9.98 564	3	21
40	9.40 345	48	9.41 784	52	0.58 216	9.98 561	3	20
41	9.40 394	48	9.41 836	51	0.58 164	9.98 558	3	19
42	9.40 442	48	9.41 887	51	0.58 112	9.98 554	3	18
43	9.40 490	48	9.41 938	51	0.58 061	9.98 551	3	17
44	9.40 538	48	9.41 990	51	0.58 010	9.98 548	3	16
45	9.40 586	48	9.42 041	51	0.57 958	9.98 544	3	15
46	9.40 634	48	9.42 092	51	0.57 907	9.98 541	3	14
47	9.40 682	48	9.42 144	51	0.57 856	9.98 538	3	13
48	9.40 730	48	9.42 195	51	0.57 805	9.98 534	3	12
49	9.40 777	47	9.42 246	51	0.57 753	9.98 531	3	11
50	9.40 825	48	9.42 297	51	0.57 702	9.98 528	3	10
51	9.40 873	47	9.42 348	51	0.57 651	9.98 524	3	9
52	9.40 920	47	9.42 399	51	0.57 600	9.98 521	3	8
53	9.40 968	47	9.42 450	51	0.57 549	9.98 518	3	7
54	9.41 015	47	9.42 501	50	0.57 499	9.98 514	3	6
55	9.41 063	47	9.42 552	51	0.57 448	9.98 511	3	5
56	9.41 110	47	9.42 602	50	0.57 397	9.98 508	3	4
57	9.41 158	47	9.42 653	51	0.57 346	9.98 504	3	3
58	9.41 205	47	9.42 704	50	0.57 296	9.98 501	3	2
59	9.41 252	47	9.42 754	50	0.57 245	9.98 498	3	1
60	9.41 299	47	9.42 805	50	0.57 195	9.98 494	3	0
	Log. Cos.	d.	Log. Cot.	c. d.	Log. Tan.	Log. Sin.	d.	′

P. P.

	54	53	53
6	5.4	5.3	5.3
7	6.3	6.2	6.2
8	7.2	7.1	7.0
9	8.1	8.0	7.9
10	9.0	8.9	8.8
20	18.0	17.8	17.6
30	27.0	26.7	26.5
40	36.0	35.6	35.3
50	45.0	44.6	44.1

	52	52	51	51
6	5.2	5.2	5.1	5.1
7	6.1	6.0	6.0	5.9
8	7.0	6.9	6.8	6.8
9	7.9	7.8	7.7	7.6
10	8.7	8.6	8.6	8.5
20	17.5	17.3	17.1	17.0
30	26.2	26.0	25.7	25.5
40	35.0	34.6	34.3	34.0
50	43.7	43.3	42.9	42.5

	50	50	49	49
6	5.0	5.0	4.9	4.9
7	5.9	5.8	5.8	5.7
8	6.7	6.6	6.6	6.5
9	7.6	7.5	7.4	7.3
10	8.4	8.3	8.2	8.1
20	16.8	16.6	16.5	16.3
30	25.2	25.0	24.7	24.5
40	33.6	33.3	33.0	32.6
50	42.1	41.6	41.2	40.8

	48	48	47	47
6	4.8	4.8	4.7	4.7
7	5.6	5.6	5.5	5.5
8	6.4	6.4	6.3	6.2
9	7.3	7.2	7.1	7.0
10	8.1	8.0	7.9	7.8
20	16.1	16.0	15.8	15.6
30	24.2	24.0	23.7	23.5
40	32.3	32.0	31.6	31.3
50	40.4	40.0	39.6	39.1

	3	3
6	0.3	0.3
7	0.4	0.3
8	0.4	0.4
9	0.5	0.4
10	0.6	0.5
20	1.1	1.0
30	1.7	1.5
40	2.3	2.0
50	2.9	2.5

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TABLE VII.—LOGARITHMIC SINES, COSINES, TANGENTS, AND COTANGENTS.
15°

′	Log. Sin.	d.	Log. Tan.	c. d.	Log. Cot.	Log. Cos.	d.	′
0	9.41 299̂		9.42 805̂		0.57 195	9.98 494̂		60
1	9.41 346̂	47	9.42 856̂	51	0.57 144	9.98 491	3	59
2	9.41 394	47̂	9.42 906̂	50	0.57 094	9.98 487̂	3	58
3	9.41 441	47	9.42 956̂	50̂	0.57 043̂	9.98 484	3	57
4	9.41 488	47	9.43 007	50̂	0.56 993	9.98 481	3	56
5	9.41 534̂	46̂	9.43 057̂	50̂	0.56 942̂	9.98 477̂	3	55
6	9.41 581̂	47	9.43 107	50	0.56 892̂	9.98 474	3	54
7	9.41 628̂	47	9.43 157	50	0.56 842̂	9.98 470̂	3	53
8	9.41 675	46̂	9.43 208	50̂	0.56 792	9.98 467	3	52
9	9.41 721̂	46	9.43 258	50	0.56 742	9.98 464	3	51
10	9.41 768	47	9.43 308	50	0.56 692	9.98 460̂	3	50
11	9.41 815	46̂	9.43 358	50	0.56 642	9.98 457	3	49
12	9.41 861̂	46	9.43 408	50	0.56 592	9.98 453̂	3	48
13	9.41 908	46̂	9.43 458	50	0.56 542	9.98 450	3	47
14	9.41 954̂	46	9.43 508	50	0.56 492	9.98 446̂	3	46
15	9.42 000̂	46	9.43 557̂	49	0.56 442̂	9.98 443	3	45
16	9.42 047	46̂	9.43 607	50	0.56 392̂	9.98 439̂	3	44
17	9.42 093̂	46̂	9.43 657	49̂	0.56 343	9.98 436̂	3	43
18	9.42 139̂	46	9.43 706̂	49	0.56 293̂	9.98 433	3	42
19	9.42 185̂	46	9.43 756̂	50	0.56 243̂	9.98 429̂	3	41
20	9.42 232	46̂	9.43 806	49̂	0.56 194	9.98 426	3	40
21	9.42 278	46	9.43 855̂	49̂	0.56 144̂	9.98 422̂	3	39
22	9.42 324	46	9.43 905	49̂	0.56 095	9.98 419	3	38
23	9.42 369̂	45̂	9.43 954̂	49	0.56 045̂	9.98 415̂	3	37
24	9.42 415̂	46	9.44 003̂	49	0.55 996	9.98 412	3	36
25	9.42 461̂	46	9.44 053	49̂	0.55 947	9.98 408̂	3	35
26	9.42 507	45̂	9.44 102	49	0.55 898	9.98 405	3	34
27	9.42 553	45	9.44 151̂	49̂	0.55 848	9.98 401̂	3	33
28	9.42 598	46	9.44 200̂	49	0.55 799̂	9.98 398	3	32
29	9.42 644̂	45̂	9.44 249̂	49̂	0.55 750̂	9.98 394̂	3	31
30	9.42 690	45̂	9.44 299	49	0.55 701	9.98 391	3	30
31	9.42 735̂	45̂	9.44 348	49	0.55 652	9.98 387̂	3	29
32	9.42 781	45	9.44 397	49	0.55 603	9.98 384	3	28
33	9.42 826̂	45	9.44 446	49	0.55 554	9.98 380̂	3	27
34	9.42 871̂	45	9.44 494̂	48	0.55 505̂	9.98 377	3	26
35	9.42 917	45̂	9.44 543̂	49	0.55 456̂	9.98 373̂	3	25
36	9.42 962̂	45	9.44 592̂	48	0.55 407̂	9.98 370	3	24
37	9.43 007̂	45	9.44 641	49	0.55 359	9.98 366̂	3	23
38	9.43 052̂	45̂	9.44 690	48	0.55 310	9.98 363	3	22
39	9.43 098	45	9.44 738̂	49	0.55 261̂	9.98 359̂	3	21
40	9.43 143	45	9.44 787	48	0.55 213	9.98 356	3	20
41	9.43 188	45	9.44 835̂	48	0.55 164̂	9.98 352̂	3	19
42	9.43 233	45	9.44 884	48	0.55 116	9.98 348̂	4	18
43	9.43 278	45	9.44 932̂	48	0.55 067	9.98 345	3	17
44	9.43 322̂	44̂	9.44 981	48	0.55 019	9.98 341̂	3	16
45	9.43 367̂	45	9.45 029̂	48	0.54 970̂	9.98 338	3	15
46	9.43 412	44̂	9.45 077̂	48	0.54 922̂	9.98 334̂	3	14
47	9.43 457	45̂	9.45 126	48	0.54 874	9.98 331	3	13
48	9.43 501̂	44̂	9.45 174̂	48	0.54 825̂	9.98 327̂	3	12
49	9.43 546	44̂	9.45 222̂	48	0.54 777̂	9.98 324	3	11
50	9.43 591	45	9.45 270̂	48	0.54 729̂	9.98 320	4	10
51	9.43 635̂	44̂	9.45 318̂	48	0.54 681̂	9.98 316̂	3	9
52	9.43 680	44̂	9.45 367	48	0.54 633	9.98 313	3	8
53	9.43 724	44̂	9.45 415	48	0.54 585	9.98 309̂	3	7
54	9.43 768̂	44̂	9.45 463	48	0.54 537	9.98 306	3	6
55	9.43 813	44̂	9.45 516̂	47̂	0.54 489̂	9.98 302̂	4	5
56	9.43 857	44	9.45 558̂	48	0.54 441̂	9.98 298̂	3	4
57	9.43 901̂	44̂	9.45 606̂	48	0.54 393̂	9.98 295	3	3
58	9.43 945̂	44	9.45 654	47̂	0.54 346	9.98 291̂	3	2
59	9.43 989̂	44	9.45 702	48	0.54 298	9.98 288	3	1
60	9.44 034	44̂	9.45 749̂	47̂	0.54 250̂	9.98 284	4	0
	Log. Cos.	d.	Log. Cot.	c. d.	Log. Tan.	Log. Sin.	d.	′

P. P.

	50̂	50
6	5.0̂	5.0
7	5.9	5.8
8	6.7	6.6
9	7.6	7.5
10	8.4	8.3
20	16.8	16.6
30	25.2	25.0
40	33.6	33.3
50	42.1	41.6

	49̂	49	48̂	48
6	4.9̂	4.9	4.8̂	4.8
7	5.8	5.7	5.6̂	5.6
8	6.6	6.5̂	6.4̂	6.4
9	7.4	7.3̂	7.3	7.2
10	8.2̂	8.1̂	8.1̂	8.0
20	16.5	16.3̂	16.1̂	16.0
30	24.7̂	24.5	24.2̂	24.0
40	33.0	32.6̂	32.3̂	32.0
50	41.2̂	40.8̂	40.4	40.0

	47̂	47	46̂	46
6	4.7̂	4.7	4.6̂	4.6
7	5.5̂	5.5	5.4̂	5.3̂
8	6.3̂	6.2	6.2	6.1̂
9	7.1̂	7.0̂	7.0	6.9
10	7.9̂	7.8	7.7̂	7.6̂
20	15.8̂	15.6̂	15.5	15.3̂
30	23.7̂	23.5	23.2̂	23.0
40	31.6̂	31.3̂	31.0	30.6̂
50	39.6̂	39.1̂	38.7̂	38.3̂

	45̂	45	44̂	44
6	4.5̂	4.5	4.4̂	4.4
7	5.3̂	5.2̂	5.2	5.1̂
8	6.0̂	6.0	5.9̂	5.8̂
9	6.8	6.7̂	6.7	6.6
10	7.6	7.5	7.4̂	7.3̂
20	15.1̂	15.0	14.8̂	14.6̂
30	22.7̂	22.5	22.2̂	22.0
40	30.3̂	30.0	29.6̂	29.3̂
50	37.9̂	37.5	37.1̂	36.6̂

	4	3̂	3
6	0.4	0.3̂	0.3
7	0.4̂	0.4	0.3̂
8	0.5̂	0.4̂	0.4
9	0.6	0.5	0.4̂
10	0.6̂	0.6	0.5
20	1.3̂	1.1̂	1.0
30	2.0	1.7̂	1.5
40	2.6̂	2.3̂	2.0
50	3.3̂	2.9	2.5

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TABLE VII.—LOGARITHMIC SINES, COSINES, TANGENTS, AND COTANGENTS.
16°

′	Log. Sin.	d.	Log. Tan.	c. d.	Log. Cot.	Log. Cos.	d.	′
0	9.44 034	44	9.45 749	48	0.54 250	9.98 284	3	60
1	9.44 078	44	9.45 797	47	0.54 202	9.98 280	3	59
2	9.44 122	44	9.45 845	47	0.54 155	9.98 277	3	58
3	9.44 166	44	9.45 892	47	0.54 107	9.98 273	3	57
4	9.44 209	43	9.45 940	47	0.54 060	9.98 269	4	56
5	9.44 253	44	9.45 987	47	0.54 012	9.98 266	3	55
6	9.44 297	44	9.46 035	47	0.53 965	9.98 262	3	54
7	9.44 341	43	9.46 082	47	0.53 917	9.98 258	4	53
8	9.44 384	43	9.46 129	47	0.53 870	9.98 255	3	52
9	9.44 428	44	9.46 177	47	0.53 823	9.98 251	3	51
10	9.44 472	43	9.46 224	47	0.53 776	9.98 247	4	50
11	9.44 515	43	9.46 271	47	0.53 728	9.98 244	3	49
12	9.44 559	43	9.46 318	47	0.53 681	9.98 240	3	48
13	9.44 602	43	9.46 366	47	0.53 634	9.98 236	4	47
14	9.44 646	43	9.46 413	47	0.53 587	9.98 233	3	46
15	9.44 689	43	9.46 460	47	0.53 540	9.98 229	3	45
16	9.44 732	43	9.46 507	47	0.53 493	9.98 225	4	44
17	9.44 776	43	9.46 554	47	0.53 446	9.98 222	3	43
18	9.44 819	43	9.46 601	47	0.53 399	9.98 218	4	42
19	9.44 862	43	9.46 647	46	0.53 352	9.98 214	3	41
20	9.44 905	43	9.46 694	47	0.53 305	9.98 211	4	40
21	9.44 948	43	9.46 741	46	0.53 258	9.98 207	3	39
22	9.44 991	43	9.46 788	46	0.53 212	9.98 203	3	38
23	9.45 034	43	9.46 834	46	0.53 165	9.98 200	4	37
24	9.45 077	43	9.46 881	47	0.53 118	9.98 196	3	36
25	9.45 120	43	9.46 928	46	0.53 072	9.98 192	4	35
26	9.45 163	42	9.46 974	46	0.53 025	9.98 188	3	34
27	9.45 206	43	9.47 021	46	0.52 979	9.98 185	4	33
28	9.45 249	42	9.47 067	46	0.52 932	9.98 181	3	32
29	9.45 291	43	9.47 114	46	0.52 886	9.98 177	4	31
30	9.45 334	43	9.47 160	46	0.52 839	9.98 173	3	30
31	9.45 377	42	9.47 207	46	0.52 793	9.98 170	4	29
32	9.45 419	43	9.47 253	46	0.52 747	9.98 166	3	28
33	9.45 462	42	9.47 299	46	0.52 700	9.98 162	4	27
34	9.45 504	43	9.47 345	46	0.52 654	9.98 158	4	26
35	9.45 547	42	9.47 392	46	0.52 608	9.98 155	3	25
36	9.45 589	42	9.47 438	46	0.52 562	9.98 151	4	24
37	9.45 631	43	9.47 484	46	0.52 516	9.98 147	3	23
38	9.45 674	42	9.47 530	46	0.52 469	9.98 143	4	22
39	9.45 716	42	9.47 576	46	0.52 423	9.98 140	4	21
40	9.45 758	42	9.47 622	46	0.52 377	9.98 136	3	20
41	9.45 800	42	9.47 668	45	0.52 331	9.98 132	4	19
42	9.45 842	42	9.47 714	46	0.52 286	9.98 128	4	18
43	9.45 885	42	9.47 760	46	0.52 240	9.98 124	3	17
44	9.45 927	42	9.47 806	46	0.52 194	9.98 121	4	16
45	9.45 969	42	9.47 851	45	0.52 148	9.98 117	3	15
46	9.46 011	42	9.47 897	46	0.52 102	9.98 113	4	14
47	9.46 052	41	9.47 943	46	0.52 057	9.98 109	4	13
48	9.46 094	42	9.47 989	45	0.52 011	9.98 105	4	12
49	9.46 136	42	9.48 034	45	0.51 965	9.98 102	3	11
50	9.46 178	41	9.48 080	45	0.51 920	9.98 098	4	10
51	9.46 220	42	9.48 125	45	0.51 874	9.98 094	4	9
52	9.46 261	41	9.48 171	45	0.51 829	9.98 090	3	8
53	9.46 303	41	9.48 216	45	0.51 783	9.98 086	4	7
54	9.46 345	42	9.48 262	45	0.51 738	9.98 082	4	6
55	9.46 386	41	9.48 307	45	0.51 692	9.98 079	3	5
56	9.46 428	41	9.48 353	45	0.51 647	9.98 075	4	4
57	9.46 469	41	9.48 398	45	0.51 602	9.98 071	4	3
58	9.46 511	41	9.48 443	45	0.51 556	9.98 067	3	2
59	9.46 552	41	9.48 488	45	0.51 511	9.98 063	4	1
60	9.46 593		9.48 534		0.51 466	9.98 059		0
	Log. Cos.	d.	Log. Cot.	c. d.	Log. Tan.	Log. Sin.	d.	′

P. P.

	48	47	47
6	4.8	4.7	4.7
7	5.6	5.5	5.5
8	6.4	6.3	6.2
9	7.2	7.1	7.0
10	8.0	7.9	7.8
20	16.0	15.8	15.6
30	24.0	23.7	23.5
40	32.0	31.6	31.3
50	40.0	39.6	39.1

	46	46	45	45
6	4.6	4.6	4.5	4.5
7	5.4	5.3	5.3	5.2
8	6.2	6.1	6.0	6.0
9	7.0	6.9	6.8	6.7
10	7.7	7.6	7.6	7.5
20	15.5	15.3	15.1	15.0
30	23.2	23.0	22.7	22.5
40	31.0	30.6	30.3	30.0
50	38.7	38.3	37.9	37.5

	44	43	43
6	4.4	4.3	4.3
7	5.1	5.1	5.0
8	5.8	5.8	5.7
9	6.6	6.5	6.4
10	7.3	7.2	7.1
20	14.6	14.5	14.3
30	22.0	21.7	21.5
40	29.3	29.0	28.6
50	36.6	36.2	35.8

	42	42	41	41
6	4.2	4.2	4.1	4.1
7	4.9	4.9	4.8	4.8
8	5.6	5.6	5.5	5.4
9	6.4	6.3	6.2	6.1
10	7.1	7.0	6.9	6.8
20	14.1	14.0	13.8	13.6
30	21.2	21.0	20.7	20.5
40	28.3	28.0	27.6	27.3
50	35.4	35.0	34.6	34.1

	4	3
6	0.4	0.3
7	0.4	0.4
8	0.5	0.4
9	0.6	0.5
10	0.6	0.6
20	1.3	1.1
30	2.0	1.7
40	2.6	2.3
50	3.3	2.9

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TABLE VII.—LOGARITHMIC SINES, COSINES, TANGENTS, AND COTANGENTS.

17°

′	Log. Sin.	d.	Log. Tan.	c. d.	Log. Cot.	Log. Cos.	d.		P. P.
0	9.46 593̂	4î	9.48 534	45	0.51 466	9.98 059̂	3̂	60	
1	9.46 635	4I	9.48 579	45	0.51 421	9.98 056	4	59	
2	9.46 676	4î	9.48 624	45	0.51 376	9.98 052	4	58	
3	9.46 717̂	4I	9.48 666̂	45̂	0.51 336̂	9.98 048	4	57	**45 45 4â 44**
4	9.46 758	4I	9.48 714̂	45	0.51 285̂	9.98 044	4	56	6 · 4.5 4.5 4.â 4.4
5	9.46 799̂	4I	9.48 759̂	45	0.51 240̂	9.98 040̂	3̂	55	7 · 5.3 5.â 5.2 5.î
6	9.46 840̂	4I	9.48 804̂	45	0.51 195̂	9.98 036̂	4	54	8 · 6.0 6.0 5.9 5.8
7	9.46 881̂	4I	9.48 849	44̂	0.51 151	9.98 032̂	4	53	9 · 6.8 6.7 6.7 6.6
8	9.46 922̂	4I	9.48 894	45	0.51 106	9.98 028̂	4	52	10 · 7.6 7.5 7.4 7.3̂
9	9.46 963̂	4I	9.48 939	45	0.51 061	9.98 024̂	4	51	20 · 15.î 15.0 14.8 14.6
10	9.47 004̂	4I	9.48 984	45̂	0.51 016	9.98 021	3̂	50	30 · 22.7̂ 22.5 22.2̂ 22.0
11	9.47 045̂	4I	9.49 028	44	0.50 97î	9.98 017	4	49	40 · 30.3̂ 30.0 29.6̂ 29.3̂
12	9.47 086̂	4I	9.49 073̂	45	0.50 926̂	9.98 013	4	48	50 · 37.9̂ 37.5 37.î 36.6̂
13	9.47 127	40̂	9.49 118	44̂	0.50 882	9.98 009	4	47	
14	9.47 168	4I	9.49 162̂	44	0.50 837̂	9.98 005	4	46	**4â 43**
15	9.47 208̂	40̂	9.49 207̂	45	0.50 792̂	9.98 001	3̂	45	6 · 4.3̂ 4.3
16	9.47 249	40̂	9.49 252	44̂	0.50 748	9.97 997̂	4	44	7 · 5.î 5.0
17	9.47 290	4I	9.49 296̂	44̂	0.50 703̂	9.97 993̂	4	43	8 · 5.8 5.7̂
18	9.47 330̂	40̂	9.49 341	44̂	0.50 659	9.97 989̂	4	42	9 · 6.5 6.4̂
19	9.47 371̂	40̂	9.49 385̂	44	0.50 614̂	9.97 985̂	4	41	10 · 7.2̂ 7.î
20	9.47 41î	40̂	9.49 430	44̂	0.50 570	9.97 98î	4	40	20 · 14.5 14.3̂
21	9.47 452	40̂	9.49 474̂	44̂	0.50 525̂	9.97 977	4	39	30 · 21.7̂ 21.5
22	9.47 492̂	40̂	9.49 518̂	44̂	0.50 48î	9.97 973̂	4	38	40 · 29.0 28.6̂
23	9.47 533̂	40̂	9.49 563	44̂	0.50 437	9.97 969̂	4̂	37	50 · 36.2̂ 35.8
24	9.47 573	40̂	9.49 607	44	0.50 392̂	9.97 966	3̂	36	
25	9.47 613̂	40	9.49 65î	44	0.50 348̂	9.97 962	4	35	**4î 41 4ô 40**
26	9.47 653̂	40	9.49 693̂	44̂	0.50 304̂	9.97 958	4	34	6 · 4.î 4.1 4.ô 4.0
27	9.47 694	40	9.49 740	44̂	0.50 260	9.97 954	4	33	7 · 4.8 4.8 4.7 4.6̂
28	9.47 734	40	9.49 784	44	0.50 216	9.97 950	4	32	8 · 5.5̂ 5.4̂ 5.4 5.3̂
29	9.47 774	40̂	9.49 828	44	0.50 172	9.97 946	4	31	9 · 6.2̂ 6.î 6.1 6.0
30	9.47 814	40	9.49 872	44	0.50 128	9.97 942	4	30	10 · 6.9 6.8 6.7̂ 6.6̂
31	9.47 854	40̂	9.49 916̂	4î	0.50 083̂	9.97 938	4	29	20 · 13.8̂ 13.6̂ 13.5 13.3̂
32	9.47 894	40	9.49 960̂	4Î	0.50 039̂	9.97 934	4	28	30 · 20.7̂ 20.5 20.2̂ 20.0
33	9.47 934	40	9.50 004	4â	0.49 996̂	9.97 930	4	27	40 · 27.6̂ 27.3̂ 27.0 26.6̂
34	9.47 974	40	9.50 048	44	0.49 952	9.97 926	4	26	50 · 34.6̂ 34.î 33.7̂ 33.3̂
35	9.48 014	40	9.50 092	44	0.49 908	9.97 922	4	25	
36	9.48 054	40	9.50 136	44	0.49 864	9.97 918	4	24	
37	9.48 093̂	39̂	9.50 179̂	43	0.49 820̂	9.97 914	4	23	**3â 39 3â**
38	9.48 133̂	39̂	9.50 223̂	44	0.49 776̂	9.97 910	4	22	6 · 3.â 3.9 3.8
39	9.48 173	40	9.50 267	43	0.49 733	9.97 906	4	21	7 · 4.6 4.5̂ 4.5
40	9.48 213	39̂	9.50 311	44	0.49 689	9.97 902	4	20	8 · 5.2̂ 5.2 5.î
41	9.48 252̂	39̂	9.50 354̂	43̂	0.49 645̂	9.97 898	4	19	9 · 5.9 5.8 5.8
42	9.48 292	39̂	9.50 398	43̂	0.49 602	9.97 894	4	18	10 · 6.6 6.5 6.4
43	9.48 33î	39̂	9.50 442	44̂	0.49 558	9.97 890	4	17	20 · 13.î 13.0 12.8̂
44	9.48 371	39̂	9.50 485̂	43̂	0.49 514̂	9.97 886	4	16	30 · 19.7̂ 19.5 19.2̂
45	9.48 410̂	39̂	9.50 529	43̂	0.49 471	9.97 88î	4̂	15	40 · 26.3̂ 26.0 25.6̂
46	9.48 450̂	39̂	9.50 572̂	43̂	0.49 427	9.97 877	4	14	50 · 32.9̂ 32.5 32.1
47	9.48 489̂	39̂	9.50 616	43̂	0.49 384	9.97 873̂	4	13	
48	9.48 529	39̂	9.50 659	43̂	0.49 340̂	9.97 869̂	4	12	
49	9.48 568	39	9.50 702̂	43	0.49 297̂	9.97 865̂	4	11	**4̂ 4 3̂**
50	9.48 607̂	39	9.50 746	43̂	0.49 254	9.97 86î	4	10	6 · 0.4̂ 0.4 0.3̂
51	9.48 646̂	39	9.50 789̂	43	0.49 210̂	9.97 857̂	4	9	7 · 0.5 0.4 0.4
52	9.48 686	39̂	9.50 832̂	43	0.49 167̂	9.97 853̂	4	8	8 · 0.6 0.5̂ 0.4̂
53	9.48 725	39	9.50 876	43	0.49 124	9.97 849̂	4	7	9 · 0.7 0.6 0.5
54	9.48 764	39̂	9.50 919	43	0.49 081	9.97 845	4̂	6	10 · 0.7̂ 0.6̂ 0.6
55	9.48 803̂	39	9.50 962	43̂	0.49 038̂	9.97 841	4	5	20 · 1.5 1.3̂ 1.î
56	9.48 842̂	39	9.51 005̂	43̂	0.48 994̂	9.97 837	4	4	30 · 2.2̂ 2.0 1.7̂
57	9.48 881̂	39	9.51 048	43	0.48 95î	9.97 833	4	3	40 · 3.0 2.6̂ 2.3̂
58	9.48 920̂	39	9.51 09î	43	0.48 908̂	9.97 829̂	4̂	2	50 · 3.7̂ 3.3̂ 2.9
59	9.48 959̂	39	9.51 134̂	43	0.48 865̂	9.97 824̂	4	1	
60	9.48 998	38	9.51 177̂	43	0.48 822̂	9.97 820̂	4	0	
	Log. Cos.	d.	Log. Cot.	c. d.	Log. Tan.	Log. Sin.	d.	′	P. P.

TABLE VII.—LOGARITHMIC SINES, COSINES, TANGENTS, AND COTANGENTS.
18°

′	Log. Sin.	d.	Log. Tan.	c. d.	Log. Cot.	Log. Cos.	d.	′
0	9.48 998		9.51 177		0.48 822	9.97 820		60
1	9.49 037	39	9.51 226	43	0.48 779	9.97 816	4	59
2	9.49 076	39	9.51 263	43	0.48 736	9.97 812	4	58
3	9.49 114	38	9.51 306	43	0.48 693	9.97 808	4	57
4	9.49 153	39	9.51 349	43	0.48 650	9.97 804	4	56
5	9.49 192	38	9.51 392	43	0.48 608	9.97 800	4	55
6	9.49 231	39	9.51 435	43	0.48 565	9.97 796	4	54
7	9.49 269	38	9.51 477	42	0.48 522	9.97 792	4	53
8	9.49 308	38	9.51 520	43	0.48 479	9.97 787	4	52
9	9.49 346	38	9.51 563	43	0.48 437	9.97 783	4	51
10	9.49 385	38	9.51 605	43	0.48 394	9.97 779	4	50
11	9.49 423	38	9.51 648	42	0.48 351	9.97 775	4	49
12	9.49 462	38	9.51 691	42	0.48 309	9.97 771	4	48
13	9.49 500	38	9.51 733	43	0.48 266	9.97 767	4	47
14	9.49 539	38	9.51 776	43	0.48 224	9.97 763	4	46
15	9.49 577	38	9.51 818	43	0.48 181	9.97 758	4	45
16	9.49 615	38	9.51 861	42	0.48 139	9.97 754	4	44
17	9.49 653	38	9.51 903	43	0.48 096	9.97 750	4	43
18	9.49 692	38	9.51 946	42	0.48 054	9.97 746	4	42
19	9.49 730	38	9.51 988	42	0.48 012	9.97 742	4	41
20	9.49 768	38	9.52 030	43	0.47 969	9.97 737	4	40
21	9.49 806	38	9.52 073	42	0.47 927	9.97 733	4	39
22	9.49 844	38	9.52 115	42	0.47 885	9.97 729	4	38
23	9.49 882	38	9.52 157	42	0.47 842	9.97 725	4	37
24	9.49 920	38	9.52 199	42	0.47 800	9.97 721	4	36
25	9.49 958	38	9.52 241	43	0.47 758	9.97 716	4	35
26	9.49 996	37	9.52 284	42	0.47 716	9.97 712	4	34
27	9.50 034	38	9.52 326	42	0.47 674	9.97 708	4	33
28	9.50 072	38	9.52 368	42	0.47 632	9.97 704	4	32
29	9.50 110	37	9.52 410	42	0.47 590	9.97 700	4	31
30	9.50 147	38	9.52 452	42	0.47 548	9.97 695	4	30
31	9.50 185	38	9.52 494	42	0.47 506	9.97 691	4	29
32	9.50 223	37	9.52 536	42	0.47 464	9.97 687	4	28
33	9.50 263	37	9.52 578	42	0.47 422	9.97 683	4	27
34	9.50 298	38	9.52 619	41	0.47 380	9.97 678	4	26
35	9.50 336	37	9.52 661	42	0.47 338	9.97 674	4	25
36	9.50 373	37	9.52 703	42	0.47 296	9.97 670	4	24
37	9.50 411	37	9.52 745	41	0.47 255	9.97 666	4	23
38	9.50 448	37	9.52 787	42	0.47 213	9.97 661	4	22
39	9.50 486	37	9.52 828	41	0.47 171	9.97 657	4	21
40	9.50 523	37	9.52 870	41	0.47 130	9.97 653	4	20
41	9.50 561	37	9.52 912	42	0.47 088	9.97 649	4	19
42	9.50 598	37	9.52 953	41	0.47 046	9.97 644	4	18
43	9.50 635	37	9.52 995	41	0.47 005	9.97 640	4	17
44	9.50 672	37	9.53 036	41	0.46 963	9.97 636	4	16
45	9.50 710	37	9.53 078	41	0.46 922	9.97 632	4	15
46	9.50 747	37	9.53 119	41	0.46 880	9.97 627	4	14
47	9.50 784	37	9.53 161	41	0.46 839	9.97 623	4	13
48	9.50 821	37	9.53 202	41	0.46 797	9.97 619	4	12
49	9.50 858	37	9.53 244	41	0.46 756	9.97 614	4	11
50	9.50 895	37	9.53 285	41	0.46 714	9.97 610	4	10
51	9.50 932	37	9.53 326	41	0.46 673	9.97 606	4	9
52	9.50 969	37	9.53 368	41	0.46 632	9.97 601	4	8
53	9.51 006	37	9.53 409	41	0.46 591	9.97 597	4	7
54	9.51 043	37	9.53 450	41	0.46 549	9.97 593	4	6
55	9.51 080	37	9.53 491	41	0.46 508	9.97 588	4	5
56	9.51 117	36	9.53 533	41	0.46 467	9.97 584	4	4
57	9.51 154	37	9.53 574	41	0.46 426	9.97 580	4	3
58	9.51 190	36	9.53 615	41	0.46 385	9.97 575	4	2
59	9.51 227	37	9.53 656	41	0.46 344	9.97 571	4	1
60	9.51 264	36	9.53 697		0.46 303	9.97 567		0
	Log. Cos.	d.	Log. Cot.	c. d.	Log. Tan.	Log. Sin.	d.	′

P. P.

	43	42	42
6	4.3	4.2	4.2
7	5.0	4.9	4.9
8	5.7	5.6	5.6
9	6.4	6.4	6.3
10	7.1	7.1	7.0
20	14.3	14.1	14.0
30	21.5	21.2	21.0
40	28.6	28.3	28.0
50	35.8	35.4	35.0

	41	41
6	4.1	4.1
7	4.8	4.8
8	5.5	5.4
9	6.2	6.1
10	6.9	6.8
20	13.8	13.6
30	20.7	20.5
40	27.6	27.3
50	34.6	34.1

	39	38	38
6	3.9	3.8	3.8
7	4.5	4.5	4.4
8	5.2	5.1	5.0
9	5.8	5.8	5.7
10	6.5	6.4	6.3
20	13.0	12.8	12.6
30	19.5	19.2	19.0
40	26.0	25.6	25.3
50	32.5	32.1	31.6

	37	37	36
6	3.7	3.7	3.6
7	4.4	4.3	4.2
8	5.0	4.9	4.8
9	5.6	5.5	5.5
10	6.2	6.1	6.1
20	12.5	12.3	12.1
30	18.7	18.5	18.2
40	25.0	24.6	24.3
50	31.2	30.8	30.4

	4	4
6	0.4	0 4
7	0.5	0.4
8	0.6	0.5
9	0.7	0.6
10	0.7	0.6
20	1.5	1.3
30	2.2	2.0
40	3.0	2.6
50	3.7	3.3

TABLE VII.—LOGARITHMIC SINES, COSINES, TANGENTS, AND COTANGENTS.
19°

′	Log. Sin.	d.	Log. Tan.	c. d.	Log. Cot.	Log. Cos.	d.		P. P.
0	9.51 264	37	9.53 697	41	0.46 303	9.97 567	4̂	60	
1	9.51 301	36	9.53 738	41	0.46 262	9.97 56̂2	4	59	
2	9.51 337̂	36	9.53 779	41	0.46 221	9.97 558	4	58	
3	9.51 374	36	9.53 820	41	0.46 180	9.97 554	4̂	57	41 4̂6 40
4	9.51 41̂6	36	9.53 861	41	0.46 139	9.97 54̂9	4̂	56	6 4.1 4.6 4.0
5	9.51 447	36	9.53 902	41	0.46 098	9.97 545	4	55	7 4.8 4.7 4.6̂
6	9.51 48̂3	36	9.53 943	4̂6	0.46 057	9.97 541	4̂	54	8 5.4̂ 5.4 5.3̂
7	9.51 520	36	9.53 98̂3	41	0.46 01̂6	9.97 536̂	4̂	53	9 6.1̂ 6.1 6.0
8	9.51 55̂6	36	9.54 024	41	0.45 97̂5	9.97 532	4̂	52	10 6.8̂ 6.7 6.6
9	9.51 593	36	9.54 06̂3	4̂6	0.45 934	9.97 52̂7	4	51	20 13.6̂ 13.5 13.3̂
10	9.51 62̂9	36	9.54 106	41	0.45 894	9.97 52̂3	4	50	30 20.5 20.2̂ 20.0
11	9.51 66̂5	36	9.54 147	4̂6	0.45 853	9.97 519	4̂	49	40 27.3̂ 27.0 26.6̂
12	9.51 702	36	9.54 18̂7	4̂6	0.45 812̂	9.97 51̂4	4̂	48	50 34.1̂ 33.7̂ 33.3̂
13	9.51 738	36	9.54 228	41	0.45 772	9.97 510	4̂	47	
14	9.51 77̂4	36	9.54 269	4̂6	0.45 731	9.97 50̂5	4	46	3̂9 39
15	9.51 81̂6	36	9.54 30̂9	4̂6	0.45 69̂0	9.97 50̂1	4̂	45	6 3.9̂ 3.9
16	9.51 847	36	9.54 350	4̂6	0.45 650	9.97 497	4̂	44	7 4.6 4.5
17	9.51 883	36	9.54 39̂0	4̂6	0.45 60̂9	9.97 49̂2	4̂	43	8 5.2̂ 5.2
18	9.51 919	36	9.54 431	4̂6	0.45 569	9.97 488	4̂	42	9 5.9 5.8
19	9.51 955	36	9.54 47̂1	4̂6	0.45 52̂8	9.97 48̂3	4	41	10 6.6 6.5
20	9.51 991	36	9.54 512	4̂6	0.45 488	9.97 479	4̂	40	20 13.1̂ 13.0
21	9.52 027	36	9.54 55̂2	4̂6	0.45 44̂7	9.97 475	4̂	39	30 19.7̂ 19.5
22	9.52 063	36	9.54 593	40	0.45 40̂7	9.97 47̂0	4̂	38	40 26.3̂ 26.0
23	9.52 099	36	9.54 633	4̂6	0.45 367	9.97 466	4̂	37	50 32.9 32.5
24	9.52 135	36	9.54 67̂3	4̂6	0.45 326	9.97 46̂1	4̂	36	
25	9.52 17̂0	35̂	9.54 714	40	0.45 286	9.97 457	4̂	35	37 3̂6 36
26	9.52 20̂6	36	9.54 754	4̂6	0.45 246	9.97 45̂2	4̂	34	6 3.7 3.6̂ 3.6
27	9.52 24̂2	36	9.54 79̂4	40	0.45 20̂5	9.97 448	4̂	33	7 4.3 4.2̂ 4.2
28	9.52 278	35̂	9.54 83̂4	40	0.45 16̂5	9.97 44̂3	4̂	32	8 4.9̂ 4.8̂ 4.8
29	9.52 314	36	9.54 87̂4	4̂6	0.45 12̂5	9.97 439	4̂	31	9 5.5̂ 5.5 5.4
30	9.52 34̂9	35̂	9.54 915	40	0.45 085	9.97 43̂4	4̂	30	10 6.1̂ 6.1 6.0
31	9.52 385	35̂	9.54 955	40	0.45 045	9.97 430	4̂	29	20 12.3̂ 12.1̂ 12.0
32	9.52 421	36	9.54 995	40	0.45 005	9.97 42̂5	4̂	28	30 18.5 18.2̂ 18.0
33	9.52 45̂6	35̂	9.55 035	40	0.44 965	9.97 421	4̂	27	40 24.6̂ 24.3̂ 24.0
34	9.52 492	35̂	9.55 075	4̂6	0.44 925	9.97 41̂6	4̂	26	50 30.8 30.4̂ 30.0
35	9.52 527	35̂	9.55 11̂5	3̂9	0.44 88̂4	9.97 412	4̂	25	
36	9.52 563	35̂	9.55 155	40	0.44 845	9.97 40̂7	4̂	24	
37	9.52 59̂8	35̂	9.55 195	40	0.44 805	9.97 403	4̂	23	35̂ 35 34̂
38	9.52 634	35̂	9.55 235	40	0.44 765	9.97 398	4̂	22	6 3.5̂ 3.5 3.4̂
39	9.52 66̂9	35	9.55 275	40	0.44 725	9.97 394	4̂	21	7 4.1̂ 4.1 4.0
40	9.52 70̂4	35	9.55 315	40	0.44 685	9.97 38̂9	4̂	20	8 4.7̂ 4.6̂ 4.6
41	9.52 740	35̂	9.55 355	3̂9	0.44 645	9.97 385	4̂	19	9 5.3 5.2̂ 5.2
42	9.52 77̂5	35	9.55 39̂4	40	0.44 60̂5	9.97 38̂0	4̂	18	10 5.9 5.8̂ 5.7̂
43	9.52 81̂6	35	9.55 43̂4	3̂9	0.44 56̂5	9.97 376	4̂	17	20 11.8̂ 11.6̂ 11.5
44	9.52 846	35	9.55 474	40	0.44 526	9.97 37̂1	4̂	16	30 17.7̂ 17.5 17.2̂
45	9.52 881	35	9.55 51̂4	3̂9	0.44 486	9.97 367	4̂	15	40 23.6̂ 23.3̂ 23.0
46	9.52 916	35̂	9.55 55̂3	40	0.44 44̂6	9.97 36̂2	4̂	14	50 29.6 29.1̂ 28.7̂
47	9.52 95̂1	35	9.55 59̂3	3̂9	0.44 40̂6	9.97 358	4̂	13	
48	9.52 98̂6	35	9.55 633	3̂9	0.44 367	9.97 35̂3	4̂	12	
49	9.53 02̂1	35	9.55 67̂2	39	0.44 327	9.97 349	4̂	11	5 4̂ 4
50	9.53 05̂6	35	9.55 712	3̂9	0.44 288	9.97 34̂4	4̂	10	6 0.5 0.4̂ 0.4
51	9.53 09̂1	35	9.55 75̂1	3̂9	0.44 24̂8	9.97 340	4	9	7 0.6 0.5̂ 0.4̂
52	9.53 12̂6	35	9.55 79̂1	40	0.44 20̂8	9.97 335	5	8	8 0.6̂ 0.6 0.5̂
53	9.53 16̂1	35	9.55 831	3̂9	0.44 169	9.97 33̂0	4̂	7	9 0.7̂ 0.7 0.6
54	9.53 19̂6	3̂4	9.55 87̂0	3̂9	0.44 12̂9	9.97 326	4̂	6	10 0.8̂ 0.7̂ 0.6̂
55	9.53 231	35	9.55 90̂9	39	0.44 09̂0	9.97 32̂1	4̂	5	20 1.6̂ 1.5 1.3̂
56	9.53 266	35	9.55 949	3̂9	0.44 051	9.97 317	4̂	4	30 2.5 2.2̂ 2.0
57	9.53 301	35	9.55 988	3̂9	0.44 01̂1	9.97 31̂2	4̂	3	40 3.3̂ 3.0 2.6̂
58	9.53 335	3̂4	9.56 028	3̂9	0.43 972	9.97 308	5	2	50 4.1̂ 3.7̂ 3.3̂
59	9.53 37̂0	35	9.56 067	39	0.43 93̂2	9.97 303	4̂	1	
60	9.53 405	3̂4	9.56 10̂6	39	0.43 89̂3	9.97 29̂8	4̂	0	
	Log. Cos.	d.	Log. Cot.	c. d.	Log. Tan.	Log. Sin.	d.	′	P. P.

TABLE VII.—LOGARITHMIC SINES, COSINES, TANGENTS, AND COTANGENTS.
20°

′	Log. Sin.	d.	Log. Tan.	c. d.	Log. Cot.	Log. Cos.	d.	′
0	9.53 405	35	9.56 106	39	0.43 893	9.97 298	4	60
1	9.53 440	34	9.56 146	39	0.43 854	9.97 294	4	59
2	9.53 474	34	9.56 185	39	0.43 815	9.97 289	4	58
3	9.53 509	35	9.56 224	39	0.43 775	9.97 285	4	57
4	9.53 544	34	9.56 263	39	0.43 736	9.97 280	5	56
5	9.53 578	34	9.56 303	39	0.43 697	9.97 275	4	55
6	9.53 613	34	9.56 342	39	0.43 658	9.97 271	4	54
7	9.53 647	34	9.56 381	39	0.43 619	9.97 266	4	53
8	9.53 682	34	9.56 420	39	0.43 580	9.97 261	5	52
9	9.53 716	34	9.56 459	39	0.43 540	9.97 257	4	51
10	9.53 750	34	9.56 498	39	0.43 501	9.97 252	4	50
11	9.53 785	34	9.56 537	39	0.43 462	9.97 248	4	49
12	9.53 819	34	9.56 576	39	0.43 423	9.97 243	5	48
13	9.53 854	34	9.56 615	39	0.43 384	9.97 238	4	47
14	9.53 888	34	9.56 654	39	0.43 346	9.97 234	5	46
15	9.53 922	34	9.56 693	39	0.43 307	9.97 229	4	45
16	9.53 956	34	9.56 732	39	0.43 268	9.97 224	4	44
17	9.53 990	34	9.56 771	39	0.43 229	9.97 220	5	43
18	9.54 025	34	9.56 810	38	0.43 190	9.97 215	4	42
19	9.54 059	34	9.56 848	39	0.43 151	9.97 210	5	41
20	9.54 093	34	9.56 887	39	0.43 112	9.97 206	4	40
21	9.54 127	34	9.56 926	39	0.43 074	9.97 201	5	39
22	9.54 161	34	9.56 965	38	0.43 035	9.97 196	4	38
23	9.54 195	34	9.57 003	39	0.42 996	9.97 191	5	37
24	9.54 229	34	9.57 042	39	0.42 958	9.97 187	4	36
25	9.54 263	34	9.57 081	38	0.42 919	9.97 182	5	35
26	9.54 297	34	9.57 119	39	0.42 880	9.97 177	4	34
27	9.54 331	34	9.57 158	38	0.42 842	9.97 173	5	33
28	9.54 365	33	9.57 196	39	0.42 803	9.97 168	5	32
29	9.54 398	34	9.57 235	39	0.42 765	9.97 163	4	31
30	9.54 432	34	9.57 274	38	0.42 726	9.97 159	5	30
31	9.54 466	34	9.57 312	38	0.42 687	9.97 154	5	29
32	9.54 500	34	9.57 350	39	0.42 649	9.97 149	5	28
33	9.54 534	33	9.57 389	38	0.42 611	9.97 144	4	27
34	9.54 567	34	9.57 427	39	0.42 572	9.97 140	5	26
35	9.54 601	33	9.57 466	38	0.42 534	9.97 135	5	25
36	9.54 634	34	9.57 504	38	0.42 495	9.97 130	5	24
37	9.54 668	34	9.57 542	39	0.42 457	9.97 125	4	23
38	9.54 702	33	9.57 581	38	0.42 419	9.97 121	5	22
39	9.54 735	34	9.57 619	38	0.42 380	9.97 116	5	21
40	9.54 769	33	9.57 657	39	0.42 342	9.97 111	5	20
41	9.54 802	34	9.57 696	38	0.42 304	9.97 106	4	19
42	9.54 836	33	9.57 734	38	0.42 266	9.97 102	5	18
43	9.54 869	33	9.57 772	38	0.42 227	9.97 097	5	17
44	9.54 902	34	9.57 810	38	0.42 189	9.97 092	5	16
45	9.54 936	33	9.57 848	38	0.42 151	9.97 087	5	15
46	9.54 969	33	9.57 886	39	0.42 113	9.97 082	4	14
47	9.55 002	34	9.57 925	38	0.42 075	9.97 078	5	13
48	9.55 036	33	9.57 963	38	0.42 037	9.97 073	5	12
49	9.55 069	33	9.58 001	38	0.41 999	9.97 068	5	11
50	9.55 102	33	9.58 039	38	0.41 961	9.97 063	5	10
51	9.55 135	33	9.58 077	38	0.41 923	9.97 058	4	9
52	9.55 168	34	9.58 115	38	0.41 885	9.97 054	5	8
53	9.55 202	33	9.58 153	37	0.41 847	9.97 049	5	7
54	9.55 235	33	9.58 190	38	0.41 809	9.97 044	5	6
55	9.55 268	33	9.58 228	38	0.41 771	9.97 039	5	5
56	9.55 301	33	9.58 266	38	0.41 733	9.97 034	5	4
57	9.55 334	33	9.58 304	38	0.41 695	9.97 029	4	3
58	9.55 367	33	9.58 342	38	0.41 658	9.97 025	5	2
59	9.55 400	33	9.58 380	37	0.41 620	9.97 020	5	1
60	9.55 433		9.58 417		0.41 582	9.97 015		0
	Log. Cos.	d.	Log. Cot.	c. d.	Log. Tan.	Log. Sin.	d.	′

P. P.

	39	39
6	3.9	3.9
7	4.6	4.5
8	5.2	5.2
9	5.9	5.8
10	6.6	6.5
20	13.1	13.0
30	19.7	19.5
40	26.3	26.0
50	32.9	32.5

	38	38	37
6	3.8	3.8	3.7
7	4.5	4.4	4.4
8	5.1	5.0	5.0
9	5.8	5.7	5.6
10	6.4	6.3	6.2
20	12.8	12.6	12.5
30	19.2	19.0	18.7
40	25.6	25.3	25.0
50	32.1	31.6	31.2

	35	34	34
6	3.5	3.4	3.4
7	4.1	4.0	3.9
8	4.6	4.6	4.5
9	5.2	5.2	5.1
10	5.8	5.7	5.6
20	11.6	11.5	11.3
30	17.5	17.2	17.0
40	23.3	23.0	22.6
50	29.1	28.7	28.3

	33	33
6	3.3	3.3
7	3.9	3.8
8	4.4	4.4
9	5.0	4.9
10	5.6	5.5
20	11.1	11.0
30	16.7	16.5
40	22.3	22.0
50	27.9	27.5

	5	4
6	0.5	0.4
7	0.6	0.5
8	0.6	0.6
9	0.7	0.7
10	0.8	0.7
20	1.6	1.5
30	2.5	2.2
40	3.3	3.0
50	4.1	3.7

TABLE VII.—LOGARITHMIC SINES, COSINES, TANGENTS, AND COTANGENTS.

21°

′	Log. Sin.	d.	Log. Tan.	c. d.	Log. Cot.	Log. Cos.	d.	′
0	9.55 433	33	9.58 417	38	0.41 58̂2	9.97 015	4̂	60
1	9.55 466	3̂2	9.58 455	37	0.41 54̂4	9.97 01̂0	5	59
2	9.55 49̂8	33	9.58 493	38	0.41 507	9.97 00̂5	5	58
3	9.55 53̂1	33	9.58 531	37	0.41 469	9.97 00̂0	5	57
4	9.55 56̂4	33	9.58 56̂8	37	0.41 43̂1	9.96 99̂5	5̂	56
		3̂2		37			4	
5	9.55 597	33	9.58 606	38	0.41 394	9.96 991	5	55
6	9.55 630	3̂2	9.58 644	37	0.41 356	9.96 986	5	54
7	9.55 66̂2	33	9.58 68̂1	37	0.41 31̂8	9.96 981	5̂	53
8	9.55 69̂5	33	9.58 719	37	0.41 281	9.96 976	5	52
9	9.55 728	3̂2	9.58 75̂6	37	0.41 24̂3	9.96 97̂1	5	51
10	9.55 76̂0	3̂2	9.58 794	37	0.41 206	9.96 96̂6	5	50
11	9.55 793	3̂2	9.58 83̂1	37	0.41 16̂8	9.96 96̂1	5̂	49
12	9.55 826	33	9.58 869	37	0.41 131	9.96 956	4	48
13	9.55 85̂8	3̂2	9.58 90̂6	37	0.41 09̂3	9.96 952	5	47
14	9.55 891	3̂2	9.58 944	37	0.41 056	9.96 947	5	46
15	9.55 92̂3	3̂2	9.58 98̂1	37	0.41 01̂8	9.96 942	5	45
16	9.55 956	3̂2	9.59 019	37	0.40 981	9.96 937	5	44
17	9.55 98̂8	3̂2	9.59 056	37	0.40 944	9.96 932	5	43
18	9.56 02̂0	3̂2	9.59 09̂3	37	0.40 90̂6	9.96 927	5̂	42
19	9.56 053	3̂2	9.59 131	37	0.40 869	9.96 92̂2	4	41
20	9.56 08̂5	3̂2	9.59 168	37	0.40 832	9.96 917	5	40
21	9.56 118	3̂2	9.59 20̂5	37	0.40 79̂4	9.96 91̂2	5	39
22	9.56 150	32	9.59 24̂2	37	0.40 75̂7	9.96 90̂7	5	38
23	9.56 18̂2	3̂2	9.59 280	37	0.40 720	9.96 90̂2	5	37
24	9.56 21̂4	32	9.59 317	37	0.40 683	9.96 897	5	36
25	9.56 247	3̂2	9.59 354	37	0.40 646	9.96 89̂2	5	35
26	9.56 279	32	9.59 39̂1	37	0.40 60̂8	9.96 88̂7	5	34
27	9.56 311	32	9.59 428	37	0.40 57̂1	9.96 88̂2	5	33
28	9.56 34̂3	3̂2	9.59 46̂5	37	0.40 53̂4	9.96 87̂7	5̂	32
29	9.56 37̂5	32	9.59 502	37	0.40 497	9.96 873	4	31
30	9.56 40̂7	32	9.59 540	37	0.40 460	9.96 868	5	30
31	9.56 43̂9	32	9.59 577	37	0.40 423	9.96 863	5	29
32	9.56 47̂1	32	9.59 614	37	0.40 386	9.96 858	5	28
33	9.56 50̂3	32	9.59 651	37	0.40 349	9.96 853	5	27
34	9.56 53̂5	32	9.59 688	37	0.40 312	9.96 848	5	26
35	9.56 56̂7	32	9.59 72̂4	36	0.40 27̂5	9.96 843	5	25
36	9.56 59̂9	32	9.59 76̂1	37	0.40 23̂8	9.96 838	5	24
37	9.56 63̂1	3̂1	9.59 798	37	0.40 20̂1	9.96 833	5	23
38	9.56 663	32	9.59 83̂5	37	0.40 16̂4	9.96 828	5	22
39	9.56 695	32	9.59 872	36	0.40 128	9.96 823	5	21
40	9.56 727	3̂1	9.59 909	37	0.40 091	9.96 818	5	20
41	9.56 75̂8	32	9.59 946	37	0.40 054	9.96 813	5	19
42	9.56 79̂0	3̂1	9.59 98̂2	36	0.40 01̂7	9.96 808	5̂	18
43	9.56 822	32	9.60 01̂9	37	0.39 98̂0	9.96 80̂2	5	17
44	9.56 85̂4	3̂1	9.60 056	36	0.39 944	9.96 797	5	16
45	9.56 88̂5	3̂1	9.60 093	37	0.39 907	9.96 79̂2	5	15
46	9.56 917	3̂1	9.60 12̂9	36	0.39 87̂0	9.96 78̂7	5	14
47	9.56 949	32	9.60 16̂5	37	0.39 83̂3	9.96 78̂2	5	13
48	9.56 98̂0	3̂1	9.60 203	36	0.39 797	9.96 77̂7	5	12
49	9.57 012	3̂1	9.60 23̂9	36	0.39 76̂0	9.96 77̂2	5	11
50	9.57 04̂3	3̂1	9.60 276	36	0.39 724	9.96 76̂7	5	10
51	9.57 075	3̂1	9.60 31̂2	36	0.39 68̂7	9.96 76̂2	5	9
52	9.57 10̂6	3̂1	9.60 34̂9	36	0.39 65̂0	9.96 75̂7	5	8
53	9.57 138	3̂1	9.60 386	36	0.39 614	9.96 752	5	7
54	9.57 16̂9	3̂1	9.60 42̂2	36	0.39 57̂7	9.96 747	5	6
55	9.57 201	3̂1	9.60 459	36	0.39 541	9.96 742	5	5
56	9.57 23̂2	3̂1	9.60 49̂5	36	0.39 50̂4	9.96 737	5	4
57	9.57 26̂3	3̂1	9.60 53̂1	36	0.39 46̂8	9.96 732	5	3
58	9.57 295	3̂1	9.60 568	36	0.39 432	9.96 72̂7	5	2
59	9.57 32̂6	3̂1	9.60 60̂4	36	0.39 39̂5	9.96 72̂1	5	1
60	9.57 35̂7	3̂1	9.60 641	36	0.39 359	9.96 71̂6	5	0
	Log. Cos.	d.	Log. Cot.	c. d.	Log. Tan.	Log. Sin.	d.	′

P. P.

	38	37	37
6	3.8	3.7	3.7
7	4.4	4.4	4.3
8	5.0	5.0	4.9
9	5.7	5.6	5.5
10	6.3	6.2	6.1
20	12.6	12.5	12.3
30	19.0	18.7	18.5
40	25.3	25.0	24.6
50	31.6	31.2	30.8

	36	36
6	3.6	3.6
7	4.2	4.2
8	4.8	4.8
9	5.5	5.4
10	6.1	6.0
20	12.1	12.0
30	18.2	18.0
40	24.3	24.0
50	30.4	30.0

	33	3̂2	32
6	3.3	3.2	3.2
7	3.8	3.8	3.7
8	4.4	4.3	4.2
9	4.9	4.9	4.8
10	5.5	5.4	5.3
20	11.0	10.8	10.6
30	16.5	16.2	16.0
40	22.0	21.6	21.3
50	27.5	27.1	26.6

	3̂1	31
6	3.1	3.1
7	3.7	3.6
8	4.2	4.1
9	4.7	4.6
10	5.2	5.1
20	10.5	10.3
30	15.7	15.5
40	21.0	20.6
50	26.2	25.8

	5̂	5	4̂
6	0.5	0.5	0.4
7	0.6	0.6	0.5
8	0.7	0.6	0.6
9	0.8	0.7	0.7
10	0.9	0.8	0.7
20	1.8	1.6	1.5
30	2.7	2.5	2.2
40	3.6	3.3	3.0
50	4.6	4.1	3.7

P. P.

TABLE VII.—LOGARITHMIC SINES, COSINES, TANGENTS, AND COTANGENTS.
22°

′	Log. Sin.	d.	Log. Tan.	c. d.	Log. Cot.	Log. Cos.	d.	′	P. P.
0	9.57 357̂	3î	9.60 641	36̂	0.39 359	9.96 71̂6	5	60	
1	9.57 389	31	9.60 677	36	0.39 32̂3	9.96 71̂1	5	59	
2	9.57 420	3I	9.60 71̂3	36̂	0.39 28̂6	9.96 70̂6	5	58	
3	9.57 451	3î	9.60 750	36	0.39 250	9.96 70̂1	5	57	36̂ 36
4	9.57 48̂2	31	9.60 78̂6	36	0.39 21̂3	9.96 696	5	56	6 3.6̂ 3.6
5	9.57 51̂3	31	9.60 82̂2	36	0.39 17̂7	9.96 691	5	55	7 4.2̂ 4.2
6	9.57 544	3î	9.60 859	36	0.39 141	9.96 686	5	54	8 4.8̂ 4.8
7	9.57 576	3I	9.60 895	36	0.39 105	9.96 681	5	53	9 5.5 5.4
8	9.57 607	31	9.60 931	36	0.39 069	9.96 67̂5	5	52	10 6.1 6.0
9	9.57 638	3I	9.60 967	36̂	0.39 03̂3	9.96 67̂0	5	51	20 12.î 12.0
10	9.57 669	31	9.61 00̂3	36	0.38 99̂6	9.96 66̂5	5	50	30 18.2̂ 18.0
11	9.57 700	31	9.61 03̂9	36̂	0.38 96̂0	9.96 660	5	49	40 24.3̂ 24.0
12	9.57 731	31	9.61 076	36	0.38 924	9.96 655	5	48	50 30.4 30.0
13	9.57 762	31	9.61 112	36	0.38 888	9.96 650	5	47	
14	9.57 79̂2	3ô	9.61 148	36	0.38 852	9.96 64̂4	5	46	
15	9.57 82̂3	31	9.61 184	36	0.38 816	9.96 63̂9	5	45	35̂ 35
16	9.57 85̂4	31	9.61 220	36	0.38 780	9.96 63̂4	5	44	6 3.5̂ 3.5
17	9.57 88̂5	3ô	9.61 256	36	0.38 744	9.96 629	5	43	7 4.î 4.1
18	9.57 916	31	9.61 292	36	0.38 708	9.96 624	5	42	8 4.7 4.6̂
19	9.57 947	3ô	9.61 328	36	0.38 672	9.96 619	5	41	9 5.3 5.2̂
20	9.57 97̂7	3I	9.61 364	36	0.38 636	9.96 61̂3	5	40	10 5.9 5.8̂
21	9.58 008	3ô	9.61 400	36	0.38 600	9.96 60̂8	5	39	20 11.8̂ 11.6
22	9.58 039	3I	9.61 436	36	0.38 564	9.96 60̂3	5	38	30 17.7̂ 17.5
23	9.58 070	3ô	9.61 472	36	0.38 528	9.96 598	5	37	40 23.6̂ 23.3̂
24	9.58 10ô	3ô	9.61 507	35̂	0.38 49̂3	9.96 593	5	36	50 29.6 29.î
25	9.58 131	3ô	9.61 54̂3	36	0.38 45̂6	9.96 58̂7	5	35	
26	9.58 162	31	9.61 57̂9	35̂	0.38 42̂0	9.96 58̂2	5	34	3î 31
27	9.58 19̂2	3ô	9.61 615	36	0.38 385	9.96 577	5	33	6 3.î 3.1
28	9.58 223	3ô	9.61 651	36	0.38 349	9.96 572	5	32	7 3.7 3.6
29	9.58 25̂3	3ô	9.61 68̂6	35̂	0.38 31̂3	9.96 567	5	31	8 4.2 4.î
30	9.58 284	3ô	9.61 72̂2	36	0.38 27̂7	9.96 56̂1	5	30	9 4.7 4.6̂
31	9.58 31̂4	3ô	9.61 758	35̂	0.38 242	9.96 55̂6	5	29	10 5.2̂ 5.î
32	9.58 345	3ô	9.61 794	36	0.38 206	9.96 551	5	28	20 10.5 10.3̂
33	9.58 37̂5	3ô	9.61 82̂9	35̂	0.38 17ô	9.96 546	5	27	30 15.7̂ 15.5
34	9.58 406	3ô	9.61 865	36	0.38 135	9.96 54ô	5	26	40 21.0̂ 20.6̂
35	9.58 436	30	9.61 901	35̂	0.38 099	9.96 53̂5	5	25	50 26.2̂ 25.8
36	9.58 46̂6	3ô	9.61 93̂6	36	0.38 06̂3	9.96 530	5	24	
37	9.58 497	30	9.61 972	35̂	0.38 028	9.96 525	5	23	3ô 30 2ĝ
38	9.58 527	3ô	9.62 007	35̂	0.37 99̂2	9.96 51̂9	5	22	6 3.ô 3.0 2.ĝ
39	9.58 55̂7	3ô	9.62 043	35̂	0.37 957	9.96 51̂4	5	21	7 3.5̂ 3.5 3.4̂
40	9.58 58̂7	30	9.62 07̂8	35̂	0.37 92̂1	9.96 509	5	20	8 4.ô 4.0 3.ĝ
41	9.58 618	3ô	9.62 114	35̂	0.37 886	9.96 50̂3	5	19	9 4.6 4.5 4.4
42	9.58 648	30	9.62 14̂9	35̂	0.37 85ô	9.96 498	5	18	10 5.î 5.0 4.9
43	9.58 67̂8	3ô	9.62 185	35	0.37 815	9.96 493	5	17	20 10.î 10.0 9.8̂
44	9.58 70̂8	30	9.62 22ô	35	0.37 77̂9	9.96 488	5	16	30 15.2̂ 15.0 14.7̂
45	9.58 738	30	9.62 256	35̂	0.37 744	9.96 48̂2	5	15	40 20.3̂ 20.0 19.6̂
46	9.58 769	3ô	9.62 29̂1	35̂	0.37 70̂8	9.96 47̂7	5	14	50 25.4 25.0 24.6
47	9.58 799	30	9.62 327	35	0.37 673	9.96 472	5	13	
48	9.58 829	30	9.62 36̂2	35̂	0.37 63̂7	9.96 46̂6	5	12	
49	9.58 859	30	9.62 39̂7	35	0.37 60̂2	9.96 46î	5	11	5̂ 5
50	9.58 889	30	9.62 433	35̂	0.37 567	9.96 456	5	10	6 0.5̂ 0.5
51	9.58 919	30	9.62 468	35̂	0.37 53̂1	9.96 45ô	5	9	7 0.6̂ 0.6
52	9.58 949	30	9.62 50̂3	35	0.37 49̂6	9.96 44̂5	5	8	8 0.7̂ 0.6̂
53	9.58 979	30	9.62 539	35̂	0.37 461	9.96 440	5	7	9 0.8 0.7̂
54	9.59 009	30	9.62 574	35	0.37 426	9.96 434	5	6	10 0.9 0.8̂
55	9.59 03̂8	2ĝ	9.62 60̂9	35̂	0.37 39ô	9.96 42̂9	5	5	20 1.8̂ 1.6
56	9.59 06̂8	30	9.62 64̂4	35	0.37 35̂5	9.96 424	5	4	30 2.7̂ 2.5
57	9.59 09̂8	30	9.62 67̂9	35̂	0.37 32ô	9.96 41̂8	5	3	40 3.6̂ 3.3̂
58	9.59 128	2ĝ	9.62 715	35̂	0.37 285	9.96 41̂3	5	2	50 4.6̂ 4.î
59	9.59 158	30	9.62 750	35	0.37 250	9.96 408	5	1	
60	9.59 188	30	9.62 785	35	0.37 215	9.96 40̂2	5	0	
	Log. Cos.	d.	Log. Cot.	c. d.	Log. Tan.	Log. Sin.	d.	′	P. P.

TABLE VII.—LOGARITHMIC SINES, COSINES, TANGENTS, AND COTANGENTS.

23°

′	Log. Sin.	d.	Log. Tan.	c. d.	Log. Cot.	Log. Cos.	d.	′
0	9.59 188	29	9.62 785	35	0.37 215	9.96 402	5	60
1	9.59 217	30	9.62 820	35	0.37 179	9.96 397	5	59
2	9.59 247	29	9.62 855	35	0.37 144	9.96 392	5	58
3	9.59 277	29	9.62 890	35	0.37 109	9.96 386	5	57
4	9.59 306	30	9.62 925	35	0.37 074	9.96 381	5	56
5	9.59 336	29	9.62 960	35	0.37 039	9.96 375	5	55
6	9.59 366	29	9.62 995	35	0.37 004	9.96 370	5	54
7	9.59 395	29	9.63 030	35	0.36 969	9.96 365	5	53
8	9.59 425	29	9.63 065	35	0.36 934	9.96 359	5	52
9	9.59 454	29	9.63 100	35	0.36 899	9.96 354	5	51
10	9.59 484	29	9.63 135	35	0.36 864	9.96 349	5	50
11	9.59 513	29	9.63 170	35	0.36 829	9.96 343	5	49
12	9.59 543	29	9.63 205	35	0.36 794	9.96 338	5	48
13	9.59 572	29	9.63 240	34	0.36 760	9.96 332	5	47
14	9.59 602	29	9.63 275	35	0.36 725	9.96 327	5	46
15	9.59 631	29	9.63 310	35	0.36 690	9.96 321	5	45
16	9.59 661	29	9.63 344	34	0.36 655	9.96 316	5	44
17	9.59 690	29	9.63 379	35	0.36 620	9.96 311	5	43
18	9.59 719	29	9.63 414	35	0.36 585	9.96 305	5	42
19	9.59 749	29	9.63 449	34	0.36 551	9.96 300	5	41
20	9.59 778	29	9.63 484	35	0.36 516	9.96 294	5	40
21	9.59 807	29	9.63 518	34	0.36 481	9.96 289	5	39
22	9.59 837	29	9.63 553	34	0.36 447	9.96 283	5	38
23	9.59 866	29	9.63 588	35	0.36 412	9.96 278	5	37
24	9.59 895	29	9.63 622	34	0.36 377	9.96 272	5	36
25	9.59 924	29	9.63 657	34	0.36 343	9.96 267	5	35
26	9.59 953	29	9.63 692	34	0.36 308	9.96 261	5	34
27	9.59 982	29	9.63 726	34	0.36 273	9.96 256	5	33
28	9.60 012	29	9.63 761	34	0.36 239	9.96 251	5	32
29	9.60 041	29	9.63 795	34	0.36 204	9.96 245	5	31
30	9.60 070	29	9.63 830	34	0.36 170	9.96 240	5	30
31	9.60 099	29	9.63 864	34	0.36 135	9.96 234	5	29
32	9.60 128	29	9.63 899	34	0.36 101	9.96 229	5	28
33	9.60 157	29	9.63 933	34	0.36 066	9.96 223	5	27
34	9.60 186	29	9.63 968	34	0.36 032	9.96 218	5	26
35	9.60 215	29	9.64 002	34	0.35 997	9.96 212	5	25
36	9.60 244	29	9.64 037	34	0.35 963	9.96 206	6	24
37	9.60 273	28	9.64 071	34	0.35 928	9.96 201	5	23
38	9.60 301	29	9.64 106	34	0.35 894	9.96 195	5	22
39	9.60 330	29	9.64 140	34	0.35 859	9.96 190	5	21
40	9.60 359	28	9.64 174	34	0.35 825	9.96 184	5	20
41	9.60 388	29	9.64 209	34	0.35 791	9.96 179	5	19
42	9.60 417	28	9.64 243	34	0.35 756	9.96 173	5	18
43	9.60 445	29	9.64 277	34	0.35 722	9.96 168	5	17
44	9.60 474	28	9 64 312	34	0.35 688	9.96 162	5	16
45	9.60 503	29	9.64 346	34	0.35 653	9.96 157	5	15
46	9.60 532	28	9.64 380	34	0.35 619	9.96 151	5	14
47	9.60 560	28	9.64 415	34	0.35 585	9.96 146	6	13
48	9.60 589	28	9.64 449	34	0.35 551	9.96 140	6	12
49	9.60 618	29	9.64 483	34	0.35 517	9.96 134	5	11
50	9.60 646	28	9.64 517	34	0.35 482	9.96 129	5	10
51	9.60 675	28	9.64 551	34	0.35 448	9.96 123	5	9
52	9.60 703	28	9.64 585	34	0.35 414	9.96 118	5	8
53	9.60 732	28	9.64 620	34	0.35 380	9.96 112	5	7
54	9.60 760	28	9.64 654	34	0.35 346	9.96 106	6	6
55	9.60 789	28	9.64 688	34	0.35 312	9.96 101	5	5
56	9.60 817	28	9.64 722	34	0.35 278	9.96 095	5	4
57	9.60 846	28	9.64 756	34	0.35 244	9.96 090	5	3
58	9.60 874	28	9.64 790	34	0.35 209	9.96 084	6	2
59	9.60 903	28	9.64 824	34	0.35 175	9.96 078	5	1
60	9.60 931		9.64 858		0.35 141	9.96 073		0
	Log. Cos.	d.	Log. Cot.	c. d.	Log. Tan.	Log. Sin.	d.	′

P. P.

```
      35⁸   35
6    3.⁵   3.5
7    4.¹   4.1
8    4.⁷   4.⁶
9    5.3   5.²
10   5.9   5.⁸
20  11.⁸  11.6
30  17.⁷  17.5
40  23.⁶  23.³
50  29.6  29.¹

      34⁴   34
6    3.⁴   3.4
7    4.0   3.⁹
8    4.6   4.⁵
9    5.2   5.1
10   5.⁷   5.6
20  11.5  11.³
30  17.²  17.0
40  23.0  22.6
50  28.⁷  28.³

       30
6     3.0
7     3.5
8     4.0
9     4.5
10    5.0
20   10.0
30   15.0
40   20.0
50   25.0

      29⁹   29   29⁸
6    2.⁹   2.9   2.⁸
7    3.⁴   3.4   3.3
8    3.⁹   3.8   3.8
9    4.4   4.³   4.3
10   4.9   4.8   4.⁷
20   9.8   9.6   9.5
30  14.⁷  14.5  14.²
40  19.⁶  19.³  19.0
50  24.6  24.¹  23.⁷

       6    5̂    5
6    0.6   0.⁵   0.5
7    0.7   0.⁶   0.6
8    0.8   0.⁷   0.6
9    0.9   0.8   0.⁷
10   1.0   0.9   0.⁸
20   2.0   1.⁸   1.6
30   3.0   2.⁷   2.5
40   4.0   3.⁶   3.³
50   5.0   4.6   4.¹
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TABLE VII.—LOGARITHMIC SINES, COSINES, TANGENTS, AND COTANGENTS.
24°

′	Log. Sin.	d.	Log. Tan.	c. d.	Log. Cot.	Log. Cos.	d.		P. P.
0	9.60 931	28	9.64 858	34	0.35 141	9.96 073	5	60	
1	9.60 959	28	9.64 892	34	0.35 107	9.96 067	5	59	
2	9.60 988	28	9.64 926	34	0.35 073	9.96 062	6	58	
3	9.61 016	28	9.64 960	33	0.35 040	9.96 056	5	57	
4	9.61 044	28	9.64 994	34	0.35 006	9.96 056	5	56	
5	9.61 073	28	9.65 028	34	0.34 972	9.96 045	6	55	34 · 33 · 33
6	9.61 101	28	9.65 062	34	0.34 938	9.96 039	5	54	6 · 3.4 · 3.3 · 3.3
7	9.61 129	28	9.65 096	34	0.34 904	9.96 033	5	53	7 · 3.9 · 3.9 · 3.8
8	9.61 157	28	9.65 129	34	0.34 870	9.96 028	6	52	8 · 4.5 · 4.4 · 4.4
9	9.61 186	28	9.65 163	34	0.34 836	9.96 022	5	51	9 · 5.1 · 5.0 · 4.9
10	9.61 214	28	9.65 197	34	0.34 802	9.96 016	5	50	10 · 5.6 · 5.6 · 5.5
11	9.61 242	28	9.65 231	33	0.34 769	9.96 011	6	49	20 · 11.3 · 11.1 · 11.0
12	9.61 270	28	9.65 265	34	0.34 735	9.96 005	5	48	30 · 17.0 · 16.7 · 16.5
13	9.61 298	28	9.65 299	33	0.34 701	9.95 999	6	47	40 · 22.6 · 22.3 · 22.0
14	9.61 326	28	9.65 332	34	0.34 667	9.95 994	5	46	50 · 28.3 · 27.9 · 27.5
15	9.61 354	28	9.65 366	33	0.34 633	9.95 988	6	45	
16	9.61 382	28	9.65 400	33	0.34 600	9.95 982	5	44	
17	9.61 410	28	9.65 433	34	0.34 566	9.95 977	6	43	
18	9.61 438	28	9.65 467	33	0.34 532	9.95 971	6	42	
19	9.61 466	28	9.65 501	34	0.34 499	9.95 965	5	41	28 · 28
20	9.61 494	28	9.65 535	33	0.34 465	9.95 959	6	40	6 · 2.8 · 2.8
21	9.61 522	27	9.65 568	33	0.34 431	9.95 954	6	39	7 · 3.3 · 3.2
22	9.61 550	28	9.65 602	33	0.34 398	9.95 948	6	38	8 · 3.8 · 3.7
23	9.61 578	28	9.65 635	33	0.34 364	9.95 942	5	37	9 · 4.3 · 4.2
24	9.61 606	28	9.65 669	34	0.34 331	9.95 937	6	36	10 · 4.7 · 4.6
25	9.61 634	27	9.65 703	33	0.34 297	9.95 931	6	35	20 · 9.5 · 9.3
26	9.61 661	28	9.65 736	33	0.34 263	9.95 925	5	34	30 · 14.2 · 14.0
27	9.61 689	28	9.65 770	33	0.34 230	9.95 919	6	33	40 · 19.0 · 18.6
28	9.61 717	28	9.65 803	33	0.34 196	9.95 914	6	32	50 · 23.7 · 23.3
29	9.61 745	27	9.65 837	33	0.34 163	9.95 908	6	31	
30	9.61 772	28	9.65 870	33	0.34 129	9.95 902	6	30	
31	9.61 800	28	9.65 904	33	0.34 096	9.95 896	5	29	
32	9.61 828	28	9.65 937	33	0.34 062	9.95 891	6	28	
33	9.61 856	27	9.65 971	33	0.34 029	9.95 885	6	27	27 · 27
34	9.61 883	27	9.66 004	33	0.33 996	9.95 879	6	26	6 · 2.7 · 2.7
35	9.61 911	27	9.66 037	33	0.33 962	9.95 873	6	25	7 · 3.2 · 3.1
36	9.61 938	27	9.66 071	33	0.33 929	9.95 867	5	24	8 · 3.6 · 3.6
37	9.61 966	28	9.66 104	33	0.33 895	9.95 862	6	23	9 · 4.1 · 4.0
38	9.61 994	27	9.66 137	33	0.33 862	9.95 856	6	22	10 · 4.6 · 4.5
39	9.62 021	27	9.66 171	33	0.33 829	9.95 850	6	21	20 · 9.1 · 9.0
40	9.62 049	27	9.66 204	33	0.33 795	9.95 844	6	20	30 · 13.7 · 13.5
41	9.62 076	27	9.66 237	33	0.33 762	9.95 838	5	19	40 · 18.3 · 18.0
42	9.62 104	27	9.66 271	33	0.33 729	9.95 833	6	18	50 · 22.9 · 22.5
43	9.62 131	27	9.66 304	33	0.33 696	9.95 827	6	17	
44	9.62 158	27	9.66 337	33	0.33 662	9.95 821	6	16	
45	9.62 186	27	9.66 370	33	0.33 629	9.95 815	6	15	
46	9.62 213	27	9.66 404	33	0.33 596	9.95 809	5	14	
47	9.62 241	27	9.66 437	33	0.33 563	9.95 804	6	13	6 · 5
48	9.62 268	27	9.66 470	33	0.33 529	9.95 798	6	12	6 · 0.6 · 0.5
49	9.62 295	27	9.66 503	33	0.33 496	9.95 792	6	11	7 · 0.7 · 0.6
50	9.62 323	27	9.66 536	33	0.33 463	9.95 786	6	10	8 · 0.8 · 0.7
51	9.62 350	27	9.66 570	33	0.33 430	9.95 786	6	9	9 · 0.9 · 0.8
52	9.62 377	27	9.66 603	33	0.33 397	9.95 774	6	8	10 · 1.0 · 0.9
53	9.62 404	27	9.66 636	33	0.33 364	9.95 768	5	7	20 · 2.0 · 1.8
54	9.62 432	27	9.66 669	33	0.33 331	9.95 763	6	6	30 · 3.0 · 2.7
55	9.62 459	27	9.66 702	33	0.33 298	9.95 757	6	5	40 · 4.0 · 3.6
56	9.62 486	27	9.66 735	33	0.33 265	9.95 751	6	4	50 · 5.0 · 4.6
57	9.62 513	27	9.66 768	33	0.33 232	9.95 745	6	3	
58	9.62 540	27	9.66 801	33	0.33 198	9.95 739	6	2	
59	9.62 567	27	9.66 834	33	0.33 165	9.95 733	6	1	
60	9.62 595		9.66 867	33	0.33 132	9.95 727		0	
	Log. Cos.	d.	Log. Cot.	c. d.	Log. Tan.	Log. Sin.	d.	′	P. P.

TABLE VII.—LOGARITHMIC SINES, COSINES, TANGENTS, AND COTANGENTS.
25°

′	Log. Sin.	d.	Log. Tan.	c. d.	Log. Cot.	Log. Cos.	d.	′
0	9.62 595	27	9.66 867	3̂3	0.33 13̂2	9.95 72̂7	6	60
1	9.62 622	27	9.66 900	33	0.33 100	9.95 72̂1	5	59
2	9.62 649	27	9.66 933	33	0.33 067	9.95 716	6	58
3	9.62 676	27	9.66 966	33	0.33 034	9.95 710	6	57
4	9.62 703	27	9.66 999	33	0.33 001	9.95 704	6	56
5	9.62 730	27	9.67 032	33	0.32 968	9.95 698	6	55
6	9.62 757	27	9.67 065	33	0.32 935	9.95 692	6	54
7	9.62 784	27	9.67 097	3̂3	0.32 90̂2	9.95 686	6	53
8	9.62 811	27	9.67 13̂0	33	0.32 86̂9	9.95 680	5	52
9	9.62 838	27	9.67 16̂3	33	0.32 836	9.95 67̂4	6	51
10	9.62 86̂4	26	9.67 196̂	33	0.32 80̂3	9.95 66̂8	6	50
11	9.62 89̂1	27	9.67 229	3̂3	0.32 771	9.95 66̂2	6	49
12	9.62 91̂8	27	9.67 262	33	0.32 738	9.95 656	6	48
13	9.62 94̂5	27	9.67 29̂4	3̂3	0.32 70̂5	9.95 65̂0	6	47
14	9.62 972	26	9.67 32̂7	3̂3	0.32 67̂2	9.95 64̂4	6	46
15	9.62 999	27	9.67 360	3̂3	0.32 640	9.95 63̂8	6	45
16	9.63 02̂5	26	9.67 393	33	0.32 60̂7	9.95 63̂2	5	44
17	9.63 05̂2	27	9.67 42̂5	3̂3	0.32 57̂4	9.95 627	6	43
18	9.63 079	26	9.67 45̂8	33	0.32 54̂1	9.95 621	6	42
19	9.63 106	27	9.67 491	3̂3	0.32 509	9.95 615	6	41
20	9.63 13̂2	26	9.67 52̂3	3̂3	0.32 47̂6	9.95 609	6	40
21	9.63 15̂9	27	9.67 55̂6	33	0.32 44̂3	9.95 603	6	39
22	9.63 186	26	9.67 589	3̂3	0.32 411	9.95 597	6	38
23	9.63 21̂2	26	9.67 62̂1	3̂3	0.32 37̂8	9.95 591	6	37
24	9.63 239	26	9.67 65̂4	33	0.32 34̂5	9.95 585	6	36
25	9.63 266	27	9.67 687	3̂3	0.32 31̂3	9.95 579	6	35
26	9.63 29̂2	26	9.67 71̂9	3̂3	0.32 28̂0	9.95 573	6	34
27	9.63 319	26	9.67 752	3̂3	0.32 248	9.95 567	6	33
28	9.63 34̂5	26	9.67 78̂4	3̂3	0.32 21̂5	9.95 561	6	32
29	9.63 372	26	9.67 817	3̂3	0.32 183	9.95 555	6	31
30	9.63 39̂8	26	9.67 84̂9	3̂3	0.32 15̂0	9.95 549	6	30
31	9.63 425	26	9.67 882	3̂3	0.32 118	9.95 543	6	29
32	9.63 45̂1	26	9.67 91̂4	3̂3	0.32 08̂5	9.95 537	6	28
33	9.63 478	26	9.67 947	3̂3	0.32 053	9.95 53̂0	6	27
34	9.63 504	26	9.67 97̂9	3̂3	0.32 02̂0	9.95 52̂4	6	26
35	9.63 53̂0	26	9.68 012	3̂3	0.31 98̂8	9.95 51̂8	6	25
36	9.63 557	26	9.68 04̂4	3̂3	0.31 95̂5	9.95 51̂2	6	24
37	9.63 58̂3	26	9.68 077	3̂3	0.31 923	9.95 50̂6	6	23
38	9.63 60̂9	26	9.68 109	32	0.31 891	9.95 50̂0	6	22
39	9.63 636	26	9.68 14̂1	3̂3	0.31 85̂8	9.95 49̂4	6	21
40	9.63 66̂2	26	9.68 174	3̂3	0.31 826	9.95 48̂8	6	20
41	9.63 68̂8	26	9.68 20̂6	3̂3	0.31 79̂3	9.95 48̂2	6	19
42	9.63 715	26	9.68 23̂8	32	0.31 76̂1	9.95 476	6	18
43	9.63 741	26	9.68 271	3̂2	0.31 729	9.95 470	6	17
44	9.63 76̂7	26	9.68 30̂3	3̂2	0.31 69̂6	9.95 464	6	16
45	9.63 79̂3	26	9.68 33̂5	32	0.31 66̂4	9.95 458	6	15
46	9.63 81̂9	26	9.68 368	3̂2	0.31 632	9.95 452	6	14
47	9.63 846	26	9.68 400	32	0.31 600	9.95 44̂5	6	13
48	9.63 872	26	9.68 43̂2	3̂2	0.31 56̂7	9.95 43̂9	6	12
49	9.63 898	26	9.68 46̂4	32	0.31 53̂5	9.95 43̂3	6	11
50	9.63 92̂4	26	9.68 497	3̂2	0.31 503	9.95 42̂7	6	10
51	9.63 95̂0	26	9.68 529	32	0.31 471	9.95 42̂1	6	9
52	9.63 97̂6	26	9.68 561	32	0.31 439	9.95 415	6	8
53	9.64 00̂2	26	9.68 59̂3	3̂2	0.31 40̂6	9.95 409	6	7
54	9.64 02̂8	26	9.68 62̂5	32	0.31 37̂4	9.95 403	6	6
55	9.64 05̂4	26	9.68 657	32	0.31 34̂2	9.95 397	6	5
56	9.64 08̂0	26	9.68 690	3̂2	0.31 310	9.95 39̂0	6	4
57	9.64 10̂6	26	9.68 722	32	0.31 278	9.95 38̂4	6	3
58	9.64 13̂2	26	9.68 754	32	0.31 246	9.95 37̂8	6	2
59	9.64 15̂8	26	9.68 786	32	0.31 214	9.95 372	6	1
60	9.64 184	2̂5	9.68 818	32	0.31 182	9.95 366	6	0
′	Log. Cos.	d.	Log. Cot.	c. d.	Log. Tan.	Log. Sin.	d.	′

P. P.

	33	3̂2	32
6	3.3	3.̂2	3.2
7	3.8	3.8	3.̂7
8	4.4	4.̂3	4.̂2
9	4.̂9	4.9	4.8
10	5.5	5.4	5.̂3
20	11.0	10.̂8	10.6
30	16.5	16.̂2	16.0
40	22.0	21.̂6	21.̂3
50	27.5	27.1	26.̂6

	27
6	2.7
7	3.̂1
8	3.6
9	4.̂0
10	4.5
20	9.0
30	13.5
40	18.0
50	22.5

	2̂6	26	2̂5
6	2.̂6	2.6	2.̂5
7	3.1	3.̂0	3.0
8	3.̂5	3.̂4	3.4
9	4.0	3.9	3.8
10	4.4	4.̂3	4.̂2
20	8.8	8.6	8.5
30	13.̂2	13.0	12.̂7
40	17.̂6	17.̂3	17.0
50	22.1	21.̂6	21.̂2

	6̂	6	5̂
6	0.̂6	0.6	0.̂5
7	0.̂7	0.7	0.6
8	0.8	0.8	0.̂7
9	1.0	0.9	0.8
10	1.1	1.0	0.9
20	2.̂1	2.0	1.̂8
30	3.̂2	3.0	2.̂7
40	4.̂3	4.0	3.̂6
50	5.4	5.0	4.6

64°

TABLE VII.—LOGARITHMIC SINES, COSINES, TANGENTS, AND COTANGENTS.
26°

′	Log. Sin.	d.	Log. Tan.	c. d.	Log. Cot.	Log. Cos.	d.	′	P. P.
0	9.64 184	26	9.68 818	32	0.31 182	9.95 366	6	60	
1	9.64 210	26	9.68 850	3̂2	0.31 150	9.95 360	6̂	59	
2	9.64 236	26	9.68 882	3̂2	0.31 117̂	9.95 353̂	6̂	58	
3	9.64 262	25̂	9.68 914̂	32	0.31 085̂	9.95 347̂	6	57	
4	9.64 287	26	9.68 946	32	0.31 053̂	9.95 341̂	6̂	56	
5	9.64 313̂	26	9.68 978	32	0.31 021̂	9.95 335	6	55	3̂2 32
6	9.64 339̂	26	9.69 010̂	32	0.30 989̂	9.95 329	6	54	6 | 3.2̂ | 3.2
7	9.64 365	25̂	9.69 042̂	3̂1	0.30 957̂	9.95 323	6	53	7 | 3.8 | 3.7̂
8	9.64 391	26	9.69 074	32	0.30 926	9.95 316̂	6̂	52	8 | 4.3̂ | 4.2̂
9	9.64 416̂	25̂	9.69 106	32	0.30 894	9.95 310̂	6	51	9 | 4.9 | 4.8
10	9.64 442̂	26	9.69 138	32	0.30 862	9.95 304	6̂	50	10 | 5.4 | 5.3̂
11	9.64 468	25̂	9.69 170	32	0.30 830	9.95 298	6	49	20 | 10.8 | 10.6̂
12	9.64 493̂	25̂	9.69 202	32	0.30 798	9.95 292	6̂	48	30 | 16.2̂ | 16.0
13	9.64 519̂	26	9.69 234	32	0.30 766	9.95 285̂	6	47	40 | 21.6̂ | 21.3̂
14	9.64 545	25̂	9.69 265̂	3̂1	0.30 734̂	9.95 279̂	6	46	50 | 27.1 | 26.6̂
15	9.64 570̂	25̂	9.69 297̂	32	0.30 702̂	9.95 273	6̂	45	
16	9.64 596	25̂	9.69 329̂	32	0.30 670̂	9.95 267	6̂	44	
17	9.64 622	26	9.69 361	3̂1	0.30 639	9.95 260̂	6̂	43	
18	9.64 647	25̂	9.69 393	32	0.30 607	9.95 254̂	6̂	42	
19	9.64 673	25̂	9.69 425	32	0.30 575	9.95 248	6̂	41	3̂1 31
20	9.64 698̂	25̂	9.69 456̂	3̂1	0.30 543̂	9.95 242	6	40	6 | 3.1̂ | 3.1
21	9.64 724	25̂	9.69 488̂	32	0.30 511̂	9.95 235̂	6̂	39	7 | 3.7 | 3.6
22	9.64 749̂	25̂	9.69 520	3̂1	0.30 480	9.95 229̂	6	38	8 | 4.2 | 4.1̂
23	9.64 775	25̂	9.69 552	32	0.30 448	9.95 223	6̂	37	9 | 4.7 | 4.6̂
24	9.64 800̂	25̂	9.69 583̂	3̂1	0.30 416̂	9.95 217	6̂	36	10 | 5.2̂ | 5.1̂
25	9.64 826	25̂	9.69 615̂	3̂1	0.30 384̂	9.95 210̂	6	35	20 | 10.5 | 10.3̂
26	9.64 851	25̂	9.69 647	3̂1	0.30 353	9.95 204̂	6̂	34	30 | 15.7̂ | 15.5
27	9.64 876	25̂	9.69 678̂	3̂1	0.30 321̂	9.95 198	6̂	33	40 | 21.0 | 20.6̂
28	9.64 902	25̂	9.69 710̂	32	0.30 289̂	9.95 191̂	6̂	32	50 | 26.2̂ | 25.8
29	9.64 927	25̂	9.69 742	3̂1	0.30 258	9.95 185̂	6̂	31	
30	9.64 952̂	25̂	9.69 773̂	3̂1	0.30 226̂	9.95 179	6̂	30	
31	9.64 978	25̂	9.69 805̂	32	0.30 194̂	9.95 173	6̂	29	
32	9.65 003̂	25̂	9.69 837	3̂1	0.30 163	9.95 166̂	6̂	28	
33	9.65 028	25̂	9.69 868̂	3̂1	0.30 131̂	9.95 160	6̂	27	26 2̂5 25
34	9.65 054	25	9.69 900	3̂1	0.30 100	9.95 154	6	26	6 | 2.6 | 2.5̂ | 2.5
35	9.65 079	25̂	9.69 931̂	3̂1	0.30 068̂	9.95 147̂	6̂	25	7 | 3.0̂ | 3.0 | 2.9
36	9.65 104	25̂	9.69 963	3̂1	0.30 037	9.95 141	6̂	24	8 | 3.4̂ | 3.4 | 3.3̂
37	9.65 129̂	25̂	9.69 994̂	3̂1	0.30 005̂	9.95 135	6̂	23	9 | 3.9 | 3.8 | 3.7̂
38	9.65 155	25	9.70 026	32	0.29 973̂	9.95 128̂	6̂	22	10 | 4.3̂ | 4.2̂ | 4.1̂
39	9.65 180	25̂	9.70 058	3̂1	0.29 942	9.95 122	6̂	21	20 | 8.6̂ | 8.5 | 8.3̂
40	9.65 205	25̂	9.70 089̂	3̂1	0.29 910̂	9.95 116̂	6	20	30 | 13.0 | 12.7̂ | 12.5
41	9.65 230̂	25̂	9.70 121	3̂1	0.29 879	9.95 109̂	6̂	19	40 | 17.3̂ | 17.0 | 16.6̂
42	9.65 255̂	25	9.70 152̂	3̂1	0.29 847̂	9.95 103	6̂	18	50 | 21.6̂ | 21.2̂ | 20.8
43	9.65 280̂	25	9.70 183̂	3̂1	0.29 816̂	9.95 097	6̂	17	
44	9.65 305̂	25	9.70 215	3̂1	0.29 785	9.95 090̂	6̂	16	
45	9.65 331	25̂	9.70 246̂	3̂1	0.29 753̂	9.95 084	6	15	
46	9.65 356	25	9.70 278	3̂1	0.29 722	9.95 078̂	6̂	14	
47	9.65 381	25	9.70 309̂	3̂1	0.29 690̂	9.95 071̂	6̂	13	2̂4 6̂ 6
48	9.65 406	25	9.70 341	3̂1	0.29 659	9.95 065	6̂	12	6 | 2.4̂ | 0.6̂ | 0.6
49	9.65 431	25	9.70 372	3̂1	0.29 628	9.95 058̂	6̂	11	7 | 2.8 | 0.7 | 0.7
50	9.65 456	25	9.70 403̂	3̂1	0.29 596̂	9.95 052	6̂	10	8 | 3.2̂ | 0.8 | 0.8
51	9.65 481	25	9.70 435	3̂1	0.29 565	9.95 046	6̂	9	9 | 3.7 | 1.0 | 0.9
52	9.65 506	25	9.70 466̂	3̂1	0.29 533̂	9.95 039̂	6̂	8	10 | 4.1 | 1.1 | 1.0
53	9.65 530̂	24	9.70 497̂	3̂1	0.29 502̂	9.95 033	6̂	7	20 | 8.1̂ | 2.1̂ | 2.0
54	9.65 555̂	25	9.70 529	3̂1	0.29 471	9.95 026̂	6̂	6	30 | 12.2̂ | 3.2̂ | 3.0
55	9.65 580̂	25	9.70 560̂	3̂1	0.29 439̂	9.95 020	6̂	5	40 | 16.3̂ | 4.3̂ | 4.0
56	9.65 605̂	25	9.70 591̂	3̂1	0.29 408	9.95 014̂	6̂	4	50 | 20.4 | 5.4 | 5.0
57	9.65 630	24	9.70 623	3̂1	0.29 377	9.95 007̂	6̂	3	
58	9.65 655	25	9.70 654	3̂1	0.29 346	9.95 001	6̂	2	
59	9.65 680̂	25	9.70 685̂	3̂1	0.29 314̂	9.94 994̂	6̂	1	
60	9.65 704̂	24	9.70 716̂	3̂1	0.29 283̂	9.94 988	6	0	
′	Log. Cos.	d.	Log. Cot.	c. d.	Log. Tan.	Log. Sin.	d.	′	P. P.

TABLE VII.—LOGARITHMIC SINES, COSINES, TANGENTS, AND COTANGENTS.

27°

′	Log. Sin.	d.	Log. Tan.	c. d.	Log. Cot.	Log. Cos.	u.		P. P.
0	9.65 704̂	25	9.70 716̂	3î	0.29 283̂	9.94 988	6̂	60	
1	9.65 729̂	24	9.70 748̂	3î	0.29 252	9.94 98î	6̂	59	
2	9.65 754	25	9.70 779	3î	0.29 221	9.94 975	6̂	58	
3	9.65 779	24̂	9.70 810	3î	0.29 190	9.94 969	6̂	57	
4	9.65 803̂	25	9.70 84î	3î	0.29 158	9.94 962̂	6̂	56	
5	9.65 828̂	24̂	9.70 872̂	3î	0.29 127̂	9.94 956	6̂	55	
6	9.65 853	25	9.70 903̂	3î	0.29 096̂	9.94 949̂	6̂	54	
7	9.65 878	24̂	9.70 935	3î	0.29 065̂	9.94 943	6̂	53	
8	9.65 902̂	24̂	9.70 966	3î	0.29 034	9.94 936̂	6̂	52	
-9	9.65 927	24̂	9.70 997	3î	0.29 003	9.94 930	6̂	51	
10	9.65 95î	25	9.71 028	3î	0.28 972	9.94 923̂	6̂	50	
11	9.65 976	24̂	9.71 059̂	3î	0.28 94î	9.94 917	6̂	49	
12	9.66 001	24̂	9.71 090̂	3î	0.28 90ô	9.94 91î	6̂	48	
13	9.66 025̂	24̂	9.71 12î	3î	0.28 878	9.94 904	6̂	47	
14	9.66 050	24̂	9.71 15â	3î	0.28 847̂	9.94 897	6̂	46	
15	9.66 074̂	24̂	9.71 183̂	3î	0.28 816̂	9.94 891	6̂	45	
16	9.66 099	24̂	9.71 214̂	3î	0.28 785̂	9.94 884̂	6̂	44	
17	9.66 123̂	24̂	9.71 245̂	3î	0.28 754̂	9.94 878	6̂	43	
18	9.66 148	24̂	9.71 276̂	3î	0.28 723̂	9.94 87î	6̂	42	
19	9.66 172̂	24̂	9.71 307	3î	0.28 692̂	9.94 865	6̂	41	
20	9.66 197	24̂	9.71 338̂	3î	0.28 66î	9.94 858	6̂	40	
21	9.66 22î	24̂	9.71 369̂	3î	0.28 630̂	9.94 852	6̂	39	
22	9.66 246	24̂	9.71 400̂	3î	0.28 599̂	9.94 845̂	6̂	38	
23	9.66 270̂	24	9.71 43î	3î	0.28 568̂	9.94 839	6̂	37	
24	9.66 294̂	24̂	9.71 462̂	3î	0.28 537̂	9.94 832̂	7	36	
25	9.66 319	24̂	9.71 493̂	30̂	0.28 506̂	9.94 825̂	6̂	35	
26	9.66 343̂	24	9.71 524	3î	0.28 476̂	9.94 819	6̂	34	
27	9.66 367̂	24̂	9.71 555	3î	0.28 445	9.94 812̂	6̂	33	
28	9.66 392	24̂	9.71 586	3î	0.28 414	9.94 806	6̂	32	
29	9.66 416̂	24	9.71 617	3î	0.28 383	9.94 799̂	6̂	31	
30	9.66 440̂	24̂	9.71 647̂	30̂	0.28 352̂	9.94 793	6̂	30	
31	9.66 465	24̂	9.71 678̂	3î	0.28 32î	9.94 786̂	7	29	
32	9.66 489	24̂	9.71 709̂	3î	0.28 290̂	9.94 779̂	6̂	28	
33	9.66 513̂	24̂	9.71 740	30̂	0.28 260	9.94 773	6̂	27	
34	9.66 537̂	24	9.71 771	3î	0.28 229	9.94 766̂	6̂	26	
35	9.66 56î	24	9.71 80î	30̂	0.28 198̂	9.94 760	6̂	25	
36	9.66 586	24̂	9.71 832̂	3î	0.28 167̂	9.94 753̂	6̂	24	
37	9.66 610	24	9.71 863̂	30̂	0.28 136̂	9.94 746̂	7	23	
38	9.66 634	24	9.71 894	3î	0.28 106̂	9.94 740	6̂	22	
39	9.66 658̂	24̂	9.71 925	3î	0.28 075	9.94 733̂	6̂	21	
40	9.66 682̂	24	9.71 955̂	30̂	0.28 044̂	9.94 727	6̂	20	
41	9.66 706̂	24	9.71 986	30̂	0.28 014	9.94 720̂	6̂	19	
42	9.66 730̂	24	9.72 017	3î	0.27 983	9.94 713̂	7	18	
43	9.66 754̂	24	9.72 047̂	30̂	0.27 952̂	9.94 707	6̂	17	
44	9.66 778̂	24	9.72 078	3î	0.27 92î	9.94 700̂	6̂	16	
45	9.66 802̂	24	9.72 109	30̂	0.27 891	9.94 693̂	7	15	
46	9.66 826̂	24	9.72 139̂	30̂	0.27 860̂	9.94 687	6̂	14	
47	9.66 850̂	24	9.72 170	30̂	0.27 830	9.94 680̂	6̂	13	
48	9.66 874̂	24	9.72 201	3î	0.27 799	9.94 674	7	12	
49	9.66 898̂	24	9.72 23î	30̂	0.27 768	9.94 667	7	11	
50	9.66 922̂	24	9.72 262	30̂	0.27 738	9.94 660̂	6̂	10	
51	9.66 946̂	24	9.72 292̂	30̂	0.27 707	9.94 654	7	9	
52	9.66 970̂	24	9.72 323	30̂	0.27 677	9.94 647	7	8	
53	9.66 994	23	9.72 354	3î	0.27 646	9.94 640̂	6̂	7	
54	9.67 018	24	9.72 384̂	30̂	0.27 615̂	9.94 633̂	7	6	
55	9.67 042	24	9.72 415	30̂	0.27 585	9.94 627	6̂	5	
56	9.67 066	24	9.72 445̂	30̂	0.27 554̂	9.94 620̂	6̂	4	
57	9.67 089̂	23	9.72 476	30̂	0.27 524	9.94 613̂	7	3	
58	9.67 11ŝ	24	9.72 506̂	30̂	0.27 493̂	9.94 607	7	2	
59	9.67 137	23̂	9.72 537	30̂	0.27 463	9.94 600	6̂	1	
60	9.67 161	24	9.72 567̂	30̂	0.27 432̂	9.94 593̂	6̂	0	
	Log. Cos.	d.	Log. Cot.	c. d.	Log. Tan.	Log. Sin.	d.	′	P. P.

P. P.

	3î	3I	30̂
6	3.î	3.1	3.0̂
7	3.7	3.6	3.5̂
8	4.2	4.î	4.0̂
9	4.7	4.6̂	4.6
10	5.2̂	5.î	5.1
20	10.5	10.3̂	10.î
30	15.7̂	15.5	15.2̂
40	21.0	20.6̂	20.3̂
50	26.2̂	25.8	25.4

	25
6	2.5
7	2.9
8	3.3
9	3.7̂
10	4.î
20	8.3̂
30	12.5
40	16.6̂
50	20.8̂

	24̂	24	23̂
6	2.4̂	2.4	2.3̂
7	2.8̂	2.8	2.7̂
8	3.2̂	3.2	3.î
9	3.7	3.6	3.5
10	4.î	4.0	3.9
20	8.î	8.0	7.8̂
30	12.2̂	12.0	11.7̂
40	16.3̂	16.0	15.6̂
50	20.4	20.0	19.6

	7	δ	6
6	0.7	0.6̂	0.6
7	0.8	0.7̂	0.7
8	0.9̂	0.8	0.8
9	1.0̂	1.0	0.9
10	1.î	1.1	1.0
20	2.3̂	2.î	2.0
30	3.5	3.2̂	3.0
40	4.6̂	4.3̂	4.0
50	5.8̂	5.4	5.0

TABLE VII.—LOGARITHMIC SINES, COSINES, TANGENTS, AND COTANGENTS.
28°

′	Log. Sin.	d.	Log. Tan.	c. d.	Log. Cot.	Log. Cos.	d.	′
0	9.67 161	23	9.72 567	30	0.27 432	9.94 593	6	60
1	9.67 184	24	9.72 598	30	0.27 402	9.94 587	7	59
2	9.67 208	23	9.72 628	30	0.27 371	9.94 580	6	58
3	9.67 232	24	9.72 659	30	0.27 341	9.94 573	7	57
4	9.67 256	23	9.72 689	30	0.27 311	9.94 566	6	56
5	9.67 279	23	9.72 719	30	0.27 280	9.94 560	7	55
6	9.67 303	24	9.72 750	30	0.27 250	9.94 553	6	54
7	9.67 327	23	9.72 780	30	0.27 219	9.94 546	7	53
8	9.67 350	23	9.72 811	30	0.27 189	9.94 539	6	52
9	9.67 374	23	9.72 841	30	0.27 159	9.94 533	7	51
10	9.67 397	23	9.72 871	30	0.27 128	9.94 526	6	50
11	9.67 421	24	9.72 902	30	0.27 098	9.94 519	7	49
12	9.67 445	23	9.72 932	30	0.27 067	9.94 512	6	48
13	9.67 468	23	9.72 962	30	0.27 037	9.94 506	7	47
14	9.67 492	23	9.72 993	30	0.27 007	9.94 499	7	46
15	9.67 515	23	9.73 023	30	0.26 976	9.94 492	7	45
16	9.67 539	23	9.73 053	30	0.26 946	9.94 485	6	44
17	9.67 562	23	9.73 084	30	0.26 916	9.94 478	6	43
18	9.67 586	23	9.73 114	30	0.26 886	9.94 472	7	42
19	9.67 609	23	9.73 144	30	0.26 855	9.94 465	7	41
20	9.67 633	23	9.73 174	30	0.26 825	9.94 458	7	40
21	9.67 656	23	9.73 205	30	0.26 795	9.94 451	6	39
22	9.67 679	23	9.73 235	30	0.26 765	9.94 444	7	38
23	9.67 703	23	9.73 265	30	0.26 734	9.94 437	7	37
24	9.67 726	23	9.73 295	30	0.26 704	9.94 431	6	36
25	9.67 750	23	9.73 325	30	0.26 674	9.94 424	7	35
26	9.67 773	23	9.73 356	30	0.26 644	9.94 417	7	34
27	9.67 796	23	9.73 386	30	0.26 614	9.94 410	6	33
28	9.67 819	23	9.73 416	30	0.26 584	9.94 403	7	32
29	9.67 843	23	9.73 446	30	0.26 553	9.94 396	7	31
30	9.67 866	23	9.73 476	30	0.26 523	9.94 390	6	30
31	9.67 889	23	9.73 506	30	0.26 493	9.94 383	7	29
32	9.67 913	23	9.73 536	30	0.26 463	9.94 376	7	28
33	9.67 936	23	9.73 567	30	0.26 433	9.94 369	6	27
34	9.67 959	23	9.73 597	30	0.26 403	9.94 362	7	26
35	9.67 982	23	9.73 627	30	0.26 373	9.94 355	7	25
36	9.68 005	23	9.73 657	30	0.26 343	9.94 348	7	24
37	9.68 029	23	9.73 687	30	0.26 313	9.94 341	6	23
38	9.68 052	23	9.73 717	30	0.26 283	9.94 335	7	22
39	9.68 075	23	9.73 747	30	0.26 253	9.94 328	7	21
40	9.68 098	23	9.73 777	30	0.26 223	9.94 321	7	20
41	9.68 121	23	9.73 807	30	0.26 193	9.94 314	7	19
42	9.68 144	23	9.73 837	30	0.26 163	9.94 307	7	18
43	9.68 167	23	9.73 867	30	0.26 133	9.94 300	6	17
44	9.68 190	23	9.73 897	30	0.26 103	9.94 293	7	16
45	9.68 213	23	9.73 927	30	0.26 073	9.94 286	7	15
46	9.68 236	23	9.73 957	30	0.26 043	9.94 279	7	14
47	9.68 259	23	9.73 987	30	0.26 013	9.94 272	7	13
48	9.68 282	23	9.74 017	30	0.25 983	9.94 265	7	12
49	9.68 305	23	9.74 047	30	0.25 953	9.94 258	7	11
50	9.68 328	23	9.74 076	29	0.25 923	9.94 251	6	10
51	9.68 351	23	9.74 106	30	0.25 893	9.94 245	7	9
52	9.68 374	22	9.74 136	30	0.25 863	9.94 238	7	8
53	9.68 397	23	9.74 166	30	0.25 833	9.94 231	7	7
54	9.68 420	23	9.74 196	29	0.25 804	9.94 224	7	6
55	9.68 443	23	9.74 226	30	0.25 774	9.94 217	7	5
56	9.68 466	22	9.74 256	30	0.25 744	9.94 210	7	4
57	9.68 488	23	9.74 286	30	0.25 714	9.94 203	7	3
58	9.68 511	23	9.74 315	29	0.25 684	9.94 196	7	2
59	9.68 534	23	9.74 345	30	0.25 654	9.94 189	7	1
60	9.68 557	22	9.74 375	29	0.25 625	9.94 182	7	0
	Log. Cos.	d.	Log. Cot.	c. d.	Log. Tan.	Log. Sin.	d.	′

P. P.

	36	30	29
6	3.6	3.0	2.9
7	3.5	3.5	3.4
8	4.0	4.0	3.9
9	4.6	4.5	4.4
10	5.1	5.0	4.9
20	10.1	10.0	9.8
30	15.2	15.0	14.7
40	20.3	20.0	19.6
50	25.4	25.0	24.6

	24
6	2.4
7	2.8
8	3.2
9	3.6
10	4.0
20	8.0
30	12.0
40	16.0
50	20.0

	23	23	22
6	2.3	2.3	2.2
7	2.7	2.7	2.6
8	3.1	3.0	3.0
9	3.5	3.4	3.4
10	3.9	3.8	3.7
20	7.8	7.6	7.5
30	11.7	11.5	11.2
40	15.6	15.3	15.0
50	19.6	19.1	18.7

	7	6
6	0.7	0.6
7	0.8	0.7
8	0.9	0.8
9	1.0	1.0
10	1.1	1.1
20	2.3	2.1
30	3.5	3.2
40	4.6	4.3
50	5.8	5.4

TABLE VII.—LOGARITHMIC SINES, COSINES, TANGENTS, AND COTANGENTS.

29°

'	Log. Sin.	d.	Log. Tan.	c. d.	Log. Cot.	Log. Cos.	d.	'	P. P.
0	9.68 557	23	9.74 375	30	0.25 625	9.94 182	7	60	
1	9.68 580	2̂3	9.74 405	30	0.25 595	9.94 175	7	59	
2	9.68 602	2̂3	9.74 435	2̂9	0.25 565	9.94 168	7	58	
3	9.68 625	2̂3	9.74 464	30	0.25 535	9.94 161	7	57	
4	9.68 648	23	9.74 494	2̂9	0.25 505	9.94 154	7	56	
5	9.68 671	2̂3	9.74 524	30	0.25 476	9.94 147	7	55	
6	9.68 693	23	9.74 554	2̂9	0.25 446	9.94 140	7	54	
7	9.68 716	2̂3	9.74 583	2̂9	0.25 416	9.94 133	7	53	
8	9.68 739	2̂3	9.74 613	30	0.25 387	9.94 126	7	52	
9	9.68 761	23	9.74 643	2̂9	0.25 357	9.94 118	7	51	
10	9.68 784	2̂3	9.74 672	30	0.25 327	9.94 111	7	50	
11	9.68 807	2̂2	9.74 702	2̂9	0.25 297	9.94 104	7	49	
12	9.68 829	2̂3	9.74 732	2̂9	0.25 268	9.94 097	7	48	
13	9.68 852	2̂2	9.74 761	2̂9	0.25 238	9.94 090	7	47	
14	9.68 874	2̂3	9.74 791	30	0.25 208	9.94 083	7	46	
15	9.68 897	23	9.74 821	2̂9	0.25 179	9.94 076	7	45	
16	9.68 920	2̂3	9.74 850	2̂9	0.25 149	9.94 069	7	44	
17	9.68 942	2̂3	9.74 880	2̂9	0.25 120	9.94 062	7	43	
18	9.68 965	2̂2	9.74 909	2̂9	0.25 090	9.94 055	7	42	
19	9.68 987	2̂3	9.74 939	30	0.25 060	9.94 048	7	41	
20	9.69 010	2̂2	9.74 969	2̂9	0.25 031	9.94 041	7	40	
21	9.69 032	2̂3	9.74 998	2̂9	0.25 001	9.94 034	7	39	
22	9.69 055	2̂2	9.75 028	2̂9	0.24 972	9.94 026	7	38	
23	9.69 077	2̂2	9.75 057	2̂9	0.24 942	9.94 019	7	37	
24	9.69 099	2̂3	9.75 087	2̂9	0.24 913	9.94 012	7	36	
25	9.69 122	2̂2	9.75 116	2̂9	0.24 883	9.94 005	7	35	
26	9.69 144	2̂3	9.75 146	2̂9	0.24 854	9.93 998	7	34	
27	9.69 167	2̂2	9.75 175	2̂9	0.24 824	9.93 991	7	33	
28	9.69 189	2̂2	9.75 205	2̂9	0.24 795	9.93 984	7	32	
29	9.69 211	2̂3	9.75 234	2̂9	0.24 765	9.93 977	7	31	
30	9.69 234	22	9.75 264	2̂9	0.24 736	9.93 969	7	30	
31	9.69 256	2̂2	9.75 293	2̂9	0.24 706	9.93 962	7	29	
32	9.69 278	2̂3	9.75 323	2̂9	0.24 677	9.93 955	7	28	
33	9.69 301	2̂2	9.75 352	2̂9	0.24 647	9.93 948	7	27	
34	9.69 323	2̂2	9.75 382	2̂9	0.24 618	9.93 941	7	26	
35	9.69 345	2̂2	9.75 411	2̂9	0.24 588	9.93 934	7	25	
36	9.69 367	2̂3	9.75 441	2̂9	0.24 559	9.93 926	7	24	
37	9.69 390	2̂2	9.75 470	2̂9	0.24 529	9.93 919	7	23	
38	9.69 412	22	9.75 499	2̂9	0.24 500	9.93 912	7	22	
39	9.69 434	2̂2	9.75 529	2̂9	0.24 471	9.93 905	7	21	
40	9.69 456	2̂2	9.75 558	2̂9	0.24 441	9.93 898	7	20	
41	9.69 478	2̂2	9.75 588	2̂9	0.24 412	9.93 891	7	19	
42	9.69 500	2̂3	9.75 617	2̂9	0.24 383	9.93 883	7	18	
43	9.69 523	22	9.75 646	2̂9	0.24 353	9.93 876	7	17	
44	9.69 545	22	9.75 676	2̂9	0.24 324	9.93 869	7	16	
45	9.69 567	22	9.75 705	2̂9	0.24 295	9.93 862	7	15	
46	9.69 589	22	9.75 734	2̂9	0.24 265	9.93 854	7	14	
47	9.69 611	2̂2	9.75 764	2̂9	0.24 236	9.93 847	7	13	
48	9.69 633	22	9.75 793	2̂9	0.24 207	9.93 840	7	12	
49	9.69 655	22	9.75 822	2̂9	0.24 177	9.93 833	7	11	
50	9.69 677	22	9.75 851	2̂9	0.24 148	9.93 826	7	10	
51	9.69 699	22	9.75 881	2̂9	0.24 119	9.93 818	7	9	
52	9.69 721	22	9.75 910	2̂9	0.24 090	9.93 811	7	8	
53	9.69 743	22	9.75 939	2̂9	0.24 060	9.93 804	7	7	
54	9.69 765	22	9.75 968	2̂9	0.24 031	9.93 796	7	6	
55	9.69 787	22	9.75 998	2̂9	0.24 002	9.93 789	7	5	
56	9.69 809	22	9.76 027	2̂9	0.23 973	9.93 782	7	4	
57	9.69 831	22	9.76 056	2̂9	0.23 943	9.93 775	7	3	
58	9.69 853	2̂1	9.76 085	2̂9	0.23 914	9.93 767	7	2	
59	9.69 875	22	9.76 115	2̂9	0.23 885	9.93 760	7	1	
60	9.69 897	22	9.76 144	29	0.23 856	9.93 753		0	
	Log. Cos.	d.	Log. Cot.	c. d.	Log. Tan.	Log. Sin.	d.	'	P. P.

P. P.

	30	2̂9	29
6	3.0	2.9	2.9
7	3.5	3.4	3.4
8	4.0	3.9	3.8
9	4.5	4 4	4.3
10	5.0	4.9	4.8
20	10.0	9.8	9.6
30	15.0	14.7	14.5
40	20.0	19.6	19.3
50	25.0	24.6	24.1

	23
6	2.3
7	2.7
8	3.0
9	3.4
10	3.8
20	7.6
30	11.5
40	15.3
50	19.1

	2̂2	22	2̂1
6	2.2	2.2	2.1
7	2.6	2.5	2.5
8	3.0	2.9	2.8
9	3.4	3.3	3.2
10	3.7	3.6	3.6
20	7.5	7.3	7.1
30	11.2	11.0	10.7
40	15.0	14.6	14.3
50	18.7	18.3	17.9

	7̂	7
6	0.7	0.7
7	0.9	0.8
8	1.0	0.9
9	1.1	1.0
10	1.2	1.1
20	2.5	2.3
30	3.7	3.5
40	5.0	4.6
50	6.2	5.8

60°

TABLE VII.—LOGARITHMIC SINES, COSINES, TANGENTS, AND COTANGENTS.
30°

,	Log. Sin.	d.	Log. Tan.	c. d.	Log. Cot.	Log. Cos.	d.		P. P.
0	9.69 897	22	9.76 144	29	0.23 856	9.93 753	7	60	
1	9.69 919	21	9.76 173	29	0.23 827	9.93 746	7	59	
2	9.69 94ô	22	9.76 20ô	29	0.23 79ô	9.93 738	7	58	
3	9.69 962	22	9.76 23î	29	0.23 768	9.93 731	7	57	
4	9.69 984	22	9.76 26ô	29	0.23 73ô	9.93 724	7	56	
5	9.70 006	21	9.76 28ô	29	0.23 71ô	9.93 71ô	7	55	
6	9.70 028	22	9.76 319	29	0.23 681	9.93 709	7	54	
7	9.70 050	22	9.76 348	29	0.23 652	9.93 702	7	53	
8	9.70 07î	21	9.76 377	29	0.23 623	9.93 694	7	52		29 29 28
9	9.70 093	22	9.76 406	29	0.23 594	9.93 687	7	51	6 2.ô 2.9 2.8
10	9.70 115	21	9.76 435	29	0.23 565	9.93 68ô	7	50	7 3.â 3.4 3.3
11	9.70 137	22	9.76 464	29	0.23 535	9.93 672	7	49	8 3.9 3.8 3.8
12	9.70 158	21	9.76 493	29	0.23 506	9.93 665	7	48	9 4.4 4.3 4.3
13	9.70 180	21	9.76 522	29	0.23 477	9.93 658	7	47	10 4.9 4.8 4.7
14	9.70 202	22	9.76 55î	29	0.23 448	9.93 65ô	7	46	20 9.8 9.6 9.5
15	9.70 223	21	9.76 58ô	29	0.23 41ô	9.93 643	7	45	30 14.7 14.5 14.2
16	9.70 245	21	9.76 60ô	29	0.23 39ô	9.93 635	7	44	40 19.ô 19.3 19.0
17	9.70 267	22	9.76 638	29	0.23 36î	9.93 628	7	43	50 24.6 24.î 23.7
18	9.70 288	21	9.76 667	29	0.23 332	9.93 621	7	42	
19	9.70 310	21	9.76 696	29	0.23 303	9.93 613	7	41	
20	9.70 33î	21	9.76 725	29	0.23 274	9.93 606	7	40	
21	9.70 353	22	9.76 754	29	0.23 245	9.93 599	7	39	
22	9.70 375	21	9.76 783	29	0.23 21ô	9.93 59î	7	38	
23	9.70 396	21	9.76 812	29	0.23 18ô	9.93 584	7	37	
24	9.70 418	21	9.76 84î	29	0.23 158	9.93 576	7	36	
25	9.70 439	21	9.76 87ô	29	0.23 12ô	9.93 569	7	35		22 21 21
26	9.70 461	21	9.76 899	28	0.23 101	9.93 562	7	34	6 2.2 2.î 2.1
27	9.70 482	21	9.76 928	29	0.23 072	9.93 554	7	33	7 2.5 2.5 2.4
28	9.70 504	21	9.76 957	29	0.23 043	9.93 547	7	32	8 2.9 2.8 2.8
29	9.70 525	21	9.76 986	29	0.23 014	9.93 53ô	7	31	9 3.3 3.2 3.1
30	9.70 547	21	9.77 015	28	0.22 985	9.93 532	7	30	10 3.ô 3.6 3.5
31	9.70 568	21	9.77 043	29	0.22 956	9.93 524	7	29	20 7.3 7.î 7.0
32	9.70 590	21	9.77 07î	29	0.22 92ô	9.93 517	7	28	30 11.0 10.7 10.5
33	9.70 611	21	9.77 10î	29	0.22 898	9.93 50ô	7	27	40 14.ô 14.3 14.0
34	9.70 632	21	9.77 13ô	29	0.22 86ô	9.93 502	7	26	50 18.3 17.9 17.5
35	9.70 654	21	9.77 159	28	0.22 841	9.93 495	7	25	
36	9.70 675	21	9.77 188	29	0.22 812	9.93 48ô	7	24	
37	9.70 696	21	9.77 217	28	0.22 783	9.93 480	7	23	
38	9.70 718	21	9.77 245	29	0.22 754	9.93 472	7	22	
39	9.70 739	21	9.77 274	29	0.22 725	9.93 465	7	21	
40	9.70 76ô	21	9.77 303	29	0.22 69ô	9.93 457	7	20	
41	9.70 782	21	9.77 332	28	0.22 668	9.93 450	7	19	
42	9.70 803	21	9.77 361	28	0.22 639	9.93 44â	7	18	
43	9.70 824	21	9.77 38ô	29	0.22 61ô	9.93 435	7	17		8 7 7
44	9.70 846	21	9.77 418	29	0.22 58î	9.93 427	7	16	6 0.8 0.7 0.7
45	9.70 867	21	9.77 447	28	0.22 553	9.93 420	7	15	7 0.ô 0.9 0.7
46	9.70 888	21	9.77 476	29	0.22 524	9.93 41â	7	14	8 1.ô 1.0 0.ô
47	9.70 90ô	21	9.77 504	28	0.22 49ô	9.93 405	7	13	9 1.2 1.1 1.ô
48	9.70 93ô	21	9.77 533	29	0.22 46ô	9.93 397	7	12	10 1.3 1.2 1.î
49	9.70 952	21	9.77 562	28	0.22 438	9.93 390	8	11	20 2.ô 2.5 2.3
50	9.70 973	21	9.77 591	28	0.22 409	9.93 382	7	10	30 4.0 3.7 3.5
51	9.70 994	21	9.77 61ô	29	0.22 38ô	9.93 374	7	9	40 5.3 5.0 4.6
52	9.71 01ô	21	9.77 648	28	0.22 352	9.93 367	7	8	50 6.ô 6.2 5.8
53	9.71 03ô	21	9.77 677	29	0.22 323	9.93 35ô	7	7	
54	9.71 05î	21	9.77 70ô	28	0.22 294	9.93 352	7	6	
55	9.71 078	21	9.77 734	28	0.22 266	9.93 34â	7	5	
56	9.71 09ô	21	9.77 763	29	0.22 237	9.93 337	7	4	
57	9.71 121	21	9.77 79î	28	0.22 20ô	9.93 32ô	8	3	
58	9.71 142	21	9.77 820	28	0.22 180	9.93 32î	7	2	
59	9.71 163	21	9.77 849	29	0.22 151	9.93 314	7	1	
60	9.71 184	21	9.77 87ô	28	0.22 12â	9.93 30ô		0	
	Log. Cos.	d.	Log. Cot.	c. d.	Log. Tan.	Log. Sin.	d.	,	P. P.

TABLE VII.—LOGARITHMIC SINES, COSINES, TANGENTS, AND COTANGENTS.
31°

′	Log. Sin.	d.	Log. Tan.	c. d.	Log. Cot.	Log. Cos.	d.	′	P. P.		
0	9.71 184		9.77 877		0.22 122	9.93 306		60			
1	9.71 205	21	9.77 906	28	0.22 094	9.93 299	7	59			
2	9.71 226	21	9.77 934	28	0.22 065	9.93 291	7	58			
3	9.71 247	21	9.77 963	28	0.22 037	9.93 284	8	57			
4	9.71 268	21	9.77 992	29	0.22 008	9.93 276	7	56			
5	9.71 289	21	9.78 020	28	0.21 979	9.93 268	7	55			
6	9.71 310	21	9.78 049	28	0.21 951	9.93 261	7	54			
7	9.71 331	21	9.78 077	28	0.21 922	9.93 253	8	53			
8	9.71 351	20	9.78 106	28	0.21 894	9.93 245	7	52		29	28	28
9	9.71 372	21	9.78 134	28	0.21 865	9.93 238	7	51	6	2.9	2.8	2.8
10	9.71 393	21	9.78 163	28	0.21 837	9.93 230	7	50	7	3.4	3.3	3.2
11	9.71 414	20	9.78 191	28	0.21 808	9.93 223	8	49	8	3.8	3.8	3.7
12	9.71 435	21	9.78 220	28	0.21 780	9.93 215	8	48	9	4.3	4.3	4.2
13	9.71 456	21	9.78 248	28	0.21 751	9.93 207	7	47	10	4.8	4.7	4.6
14	9.71 477	21	9.78 277	28	0.21 723	9.93 200	8	46	20	9.6	9.5	9.3
15	9.71 498	20	9.78 305	28	0.21 694	9.93 192	7	45	30	14.5	14.2	14.0
16	9.71 518	21	9.78 334	28	0.21 666	9.93 184	7	44	40	19.3	19.0	18.6
17	9.71 539	20	9.78 362	28	0.21 637	9.93 177	8	43	50	24.1	23.7	23.3
18	9.71 560	21	9.78 391	28	0.21 609	9.93 169	8	42			
19	9.71 581	20	9.78 419	28	0.21 580	9.93 161	8	41			
20	9.71 601	21	9.78 448	28	0.21 552	9.93 153	7	40			
21	9.71 622	20	9.78 476	28	0.21 523	9.93 146	8	39			
22	9.71 643	21	9.78 505	28	0.21 495	9.93 138	8	38			
23	9.71 664	20	9.78 533	28	0.21 467	9.93 130	7	37			
24	9.71 684	21	9.78 561	28	0.21 438	9.93 123	8	36			
25	9.71 705	20	9.78 590	28	0.21 410	9.93 115	8	35			
26	9.71 726	20	9.78 618	28	0.21 381	9.93 107	7	34		21	20	20
27	9.71 746	21	9.78 647	28	0.21 353	9.93 100	8	33	6	2.1	2.0	2.0
28	9.71 767	20	9.78 675	28	0.21 325	9.93 092	8	32	7	2.4	2.4	2.3
29	9.71 788	20	9.78 703	28	0.21 296	9.93 084	8	31	8	2.8	2.7	2.6
30	9.71 808	20	9.78 732	28	0.21 268	9.93 076	7	30	9	3.1	3.1	3.0
31	9.71 829	20	9.78 760	28	0.21 239	9.93 069	8	29	10	3.5	3.4	3.3
32	9.71 849	20	9.78 788	28	0.21 211	9.93 061	8	28	20	7.0	6.8	6.6
33	9.71 870	21	9.78 817	28	0.21 183	9.93 053	7	27	30	10.5	10.2	10.0
34	9.71 891	20	9.78 845	28	0.21 154	9.93 045	8	26	40	14.0	13.6	13.3
35	9.71 911	20	9.78 873	28	0.21 126	9.93 038	8	25	50	17.5	17.1	16.6
36	9.71 932	20	9.78 902	28	0.21 098	9.93 030	7	24			
37	9.71 952	20	9.78 930	28	0.21 070	9.93 022	8	23			
38	9.71 973	20	9.78 958	28	0.21 041	9.93 014	8	22			
39	9.71 993	20	9.78 987	28	0.21 013	9.93 006	8	21			
40	9.72 014	20	9.79 015	28	0.20 985	9.92 999	7	20			
41	9.72 034	20	9.79 043	28	0.20 956	9.92 991	8	19			
42	9.72 055	20	9.79 071	28	0.20 928	9.92 983	7	18			
43	9.72 075	20	9.79 100	28	0.20 900	9.92 975	8	17			
44	9.72 096	20	9.79 128	28	0.20 872	9.92 967	8	16		8	7
45	9.72 116	20	9.79 156	28	0.20 843	9.92 960	7	15	6	0.8	0.7
46	9.72 136	20	9.79 184	28	0.20 815	9.92 952	8	14	7	0.9	0.9
47	9.72 157	20	9.79 213	28	0.20 787	9.92 944	8	13	8	1.0	1.0
48	9.72 177	20	9.79 241	28	0.20 759	9.92 936	7	12	9	1.2	1.1
49	9.72 198	20	9.79 269	28	0.20 731	9.92 928	8	11	10	1.3	1.2
50	9.72 218	20	9.79 297	28	0.20 702	9.92 920	7	10	20	2.6	2.5
51	9.72 238	20	9.79 325	28	0.20 674	9.92 913	8	9	30	4.0	3.7
52	9.72 259	20	9.79 354	28	0.20 646	9.92 905	8	8	40	5.3	5.0
53	9.72 279	20	9.79 382	28	0.20 618	9.92 897	7	7	50	6.6	6.2
54	9.72 299	20	9.79 410	28	0.20 590	9.92 889	8	6			
55	9.72 319	20	9.79 438	28	0.20 561	9.92 881	8	5			
56	9.72 340	20	9.79 466	28	0.20 533	9.92 873	8	4			
57	9.72 360	20	9.79 494	28	0.20 505	9.92 865	7	3			
58	9.72 380	20	9.79 522	28	0.20 477	9.92 858	8	2			
59	9.72 400	20	9.79 551	28	0.20 449	9.92 850	8	1			
60	9.72 421		9.79 579		0.20 421	9.92 842		0			
	Log. Cos.	d.	Log. Cot.	c. d.	Log. Tan.	Log. Sin.	d.	′		P. P.	

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TABLE VII.—LOGARITHMIC SINES, COSINES, TANGENTS, AND COTANGENTS.
32°

'	Log. Sin.	d.	Log. Tan.	c. d.	Log. Cot.	Log. Cos.	d.	'
0	9.72 421	20	9.79 579	28	0.20 421	9.92 842	8	60
1	9.72 441	2ô	9.79 607	28	0.20 393	9.92 834	7̂	59
2	9.72 46î	20	9.79 635	28	0.20 365	9.92 82ô	8	58
3	9.72 48î	20	9.79 663	2ŝ	0.20 337	9.92 81ŝ	8	57
4	9.72 50î	2ô	9.79 69î	28	0.20 308	9.92 81ô	8	56
5	9.72 522	20	9.79 71ŷ	28	0.20 28ô	9.92 80ẑ	8	55
6	9.72 542	20	9.79 74ŷ	28	0.20 25ẑ	9.92 79â	8	54
7	9.72 562	2ô	9.79 77ŝ	28	0.20 22â	9.92 78ô	8	53
8	9.72 58ẑ	20	9.79 80ŝ	28	0.20 19ô	9.92 778	7̂	52
9	9.72 60ẑ	20	9.79 83î	28	0.20 16ŝ	9.92 771	8	51
10	9.72 62ẑ	20	9.79 85ŷ	28	0.20 14ô	9.92 763	8	50
11	9.72 64ẑ	20	9.79 887	28	0.20 11ẑ	9.92 755	8	49
12	9.72 66ẑ	20	9.79 91ŝ	28	0.20 08â	9.92 747	8	48
13	9.72 68ẑ	20	9.79 94ŝ	28	0.20 05ẑ	9.92 739	8	47
14	9.72 70ẑ	2ô	9.79 97î	28	0.20 02ŝ	9.92 731	8	46
15	9.72 723	20	9.79 99ŷ	28	0.20 00ô	9.92 723	8	45
16	9.72 743	20	9.80 02ŷ	28	0.19 97ẑ	9.92 715	8	44
17	9.72 763	20	9.80 05ŝ	28	0.19 94â	9.92 707	8	43
18	9.72 783	1ŷ	9.80 08ŝ	28	0.19 91ô	9.92 699	8	42
19	9.72 80ẑ	20	9.80 11î	28	0.19 888	9.92 691	8	41
20	9.72 82ẑ	20	9.80 13ŷ	28	0.19 86ô	9.92 683	8	40
21	9.72 84ẑ	20	9.80 16ŷ	28	0.19 83ẑ	9.92 675	8	39
22	9.72 86ẑ	20	9.80 19ŝ	28	0.19 80â	9.92 667	8	38
23	9.72 88ẑ	20	9.80 22ŝ	28	0.19 77ô	9.92 659	8	37
24	9.72 90ẑ	20	9.80 25î	2̂7	0.19 748	9.92 651	8	36
25	9.72 92ẑ	1ŷ	9.80 279	28	0.19 721	9 92 643	8	35
26	9.72 942	20	9.80 307	28	0.19 69ŝ	9.92 635	8	34
27	9.72 962	20	9.80 335	28	0.19 66ŝ	9.92 627	8	33
28	9.72 982	20	9.80 363	28	0.19 637	9.92 619	8	32
29	9.73 002	1ŷ	9.80 391	2̂7	0.19 609	9.92 611	8	31
30	9.73 02î	20	9.80 41ŝ	28	0.19 58î	9.92 603	8	30
31	9.73 04î	20	9.80 44ŝ	28	0.19 55ŝ	9.92 595	8	29
32	9.73 06î	1ŷ	9.80 47â	28	0.19 52ŝ	9.92 587	8	28
33	9.73 081	20	9.80 50ẑ	2̂7	0.19 49ŷ	9.92 579	8	27
34	9.73 101	1ŷ	9.80 530	28	0.19 470	9.92 57ô	8	26
35	9.73 12ô	1ŷ	9.80 558	28	0.19 442	9.92 56ẑ	8	25
36	9.73 14ô	1ŷ	9.80 586	2̂7	0.19 414	9.92 55â	8	24
37	9.73 160	20	9.80 61ŝ	28	0.19 38ŝ	9.92 546	8	23
38	9.73 180	1ŷ	9.80 64î	28	0.19 358	9.92 538	8	22
39	9.73 19ŷ	20	9.80 66ŷ	2̂7	0.19 33ô	9.92 53ô	8	21
40	9.73 21ŷ	1ŷ	9.80 697	28	0.19 303	9.92 522	8	20
41	9.73 239	1ŷ	9.80 725	2̂7	0.19 275	9.92 514	8	19
42	9:73 258	1ŷ	9.80 75ẑ	28	0.19 24̂7	9.92 506	8	18
43	9.73 278	20	9.80 78ô	28	0.19 21ŷ	9.92 498	8̂	17
44	9.73 298	1ŷ	9.80 80ŝ	2̂7	0.19 19î	9.92 48ŷ	8	16
45	9.73 31̂7	1ŷ	9.80 836	28	0.19 164	9.92 48î	8	15
46	9.73 33̂7	20	9.80 864	2̂7	0.19 136	9.92 47ĵ	8	14
47	9.73 357	1ŷ	9.80 89î	2̂7	0.19 108	9.92 46ŝ	8	13
48	9.73 37ô	1ŷ	9.80 91ŷ	28	0.19 08ô	9.92 457	8̂	12
49	9 73 396	1ŷ	9.80 947	2̂7	0.19 053	9.92 449	8	11
50	9.73 41ŝ	1ŷ	9.80 975	28	0.19 025	9.92 441	8	10
51	9.73 43ŝ	20	9.81 00ẑ	2̂7	0.18 99̂7	9.92 433	8	9
52	9.73 455	1ŷ	9.81 030	2̂7	0.18 97ô	9.92 424	8̂	8
53	9.73 474	1ŷ	9.81 058	28	0.18 942	9.92 41ô	8	7
54	9.73 494	1ŷ	9.81 08ŝ	2̂7	0.18 91â	9.92 408	8	6
55	9.73 51ŝ	1ŷ	9.81 11ŝ	28	0.18 88ô	9.92 400	8	5
56	9.73 533	1ŷ	9.81 141	2̂7	0.18 859	9.92 392	8̂	4
57	9.73 55ẑ	1ŷ	9.81 16ŝ	2̂7	0.18 83î	9.92 38ŝ	8	3
58	9.73 572	1ŷ	9.81 19ô	28	0.18 80ô	9.92 375	8̂	2
59	9.73 59î	1ŷ	9.81 224	2̂7	0.18 776	9.92 36̂7	8	1
60	9.73 611		9.81 25î	2̂7	0.18 74ŝ	9.92 359		0
	Log. Cos.	d.	Log. Cot.	c. d.	Log. Tan.	Log. Sin.	d.	'

P. P.

	2ŝ	28	2̂7
6	2.8	2.8	2.̂7
7	3.3	3.̂2	3.2
8	3.8	3.̂7	3.ô
9	4.3	4.2	4.1
10	4.̂7	4.ô	4.6
20	9.5	9.̂3	9.î
30	14.̂2	14.0	13.̂7
40	19.0	18.ô	18.̂3
50	23.̂7	23.̂3	22.9

	2ô	20	1ŷ
6	2.ô	2.0	1.ŷ
7	2.4	2.̂3	2.3
8	2.̂7	2.ô	2.6
9	3.1	3.0	2.ŷ
10	3.4	3.̂3	3.̂2
20	6.ŝ	6.ô	6.5
30	10.̂2	10.0	9.̂7
40	13.ô	13.̂3	13.0
50	17.1	16.ô	16.̂2

	ŝ	8	̂7
6	0.ŝ	0.8	0.̂7
7	1.0	0.ŷ	0.9
8	1.î	1.ô	1.0
9	1.3	1.2	1.1
10	1.4	1.̂3	1.̂2
20	2.ŝ	2.6	2.5
30	4.̂2	4.0	3.̂7
40	5.ô	5.̂3	5.0
50	7.1	6.ô	6.̂2

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TABLE VII.—LOGARITHMIC SINES, COSINES, TANGENTS, AND COTANGENTS.
33°

′	Log. Sin.	d.	Log. Tan.	c. d.	Log. Cot.	Log. Cos.	d.		P. P.
0	9.73 611	19	9.81 251	28	0.18 748	9.92 359	8	60	
1	9.73 630	19	9.81 279	27	0.18 720	9.92 351	8	59	
2	9.73 650	19	9.81 307	27	0.18 693	9.92 342	8	58	
3	9.73 669	19	9.81 334	28	0.18 665	9.92 334	8	57	
4	9.73 688	19	9.81 362	27	0.18 637	9.92 326	8	56	
5	9.73 708	19	9.81 390	27	0.18 610	9.92 318	8	55	
6	9.73 727	19	9.81 417	27	0.18 582	9.92 310	8	54	
7	9.73 746	19	9.81 445	28	0.18 555	9.92 301	8	53	
8	9 73 766	19	9.81 473	27	0.18 527	9.92 293	8	52	**28 27 27**
9	9.73 785	19	9.81 500	27	0.18 499	9.92 285	8	51	6 2.8 2.7 2.7
10	9.73 805	19	9.81 528	27	0.18 472	9.92 277	8	50	7 3.2 3.2 3.1
11	9.73 824	19	9.81 555	27	0.18 444	9.92 268	8	49	8 3.7 3.6 3.6
12	9.73 843	19	9.81 583	27	0.18 417	9.92 260	8	48	9 4.2 4.1 4.0
13	9.73 862	19	9.81 610	27	0.18 389	9.92 252	8	47	10 4.6 4.6 4.5
14	9.73 882	19	9.81 638	28	0.18 362	9.92 244	8	46	20 9.3 9.1 9.0
15	9.73 901	19	9.81 666	27	0.18 334	9.92 235	8	45	30 14.0 13.7 13.5
16	9.73 920	19	9.81 693	27	0.18 306	9.92 227	8	44	40 18.6 18.3 18.0
17	9.73 940	19	9.81 721	27	0.18 279	9.92 219	8	43	50 23.3 22.9 22.5
18	9.73 959	19	9.81 748	27	0.18 251	9.92 210	8	42	
19	9.73 978	19	9.81 776	27	0.18 224	9.92 202	8	41	
20	9.73 997	19	9.81 803	27	0.18 196	9.92 194	8	40	
21	9.74 016	19	9.81 831	27	0.18 169	9.92 185	8	39	
22	9.74 036	19	9.81 858	27	0.18 141	9.92 177	8	38	
23	9.74 055	19	9.81 886	27	0.18 114	9.92 169	8	37	
24	9.74 074	19	9.81 913	27	0.18 086	9.92 160	8	36	
25	9.74 093	19	9.81 941	27	0.18 059	9.92 152	8	35	**19 19 18**
26	9.74 112	19	9.81 968	27	0.18 031	9.92 144	8	34	6 1.9 1.9 1.8
27	9.74 131	19	9.81 996	27	0.18 004	9.92 135	8	33	7 2.3 2.2 2.1
28	9.74 151	19	9.82 023	27	0.17 976	9.92 127	8	32	8 2.6 2.5 2.4
29	9.74 170	19	9.82 051	27	0.17 949	9.92 119	8	31	9 2.9 2.8 2.8
30	9.74 189	19	9.82 078	27	0.17 921	9.92 110	8	30	10 3.2 3.1 3.1
31	9.74 208	19	9.82 105	27	0.17 894	9.92 102	8	29	20 6.5 6.3 6.1
32	9.74 227	19	9.82 133	27	0.17 867	9.92 094	8	28	30 9.7 9.5 9.2
33	9.74 246	19	9.82 160	27	0.17 839	9.92 085	8	27	40 13.0 12.6 12.3
34	9.74 265	19	9.82 188	27	0.17 812	9.92 077	8	26	50 16.2 15.8 15.4
35	9.74 284	19	9.82 215	27	0.17 784	9.92 069	8	25	
36	9.74 303	19	9.82 243	27	0.17 757	9.92 060	8	24	
37	9.74 322	19	9.82 270	27	0.17 729	9.92 052	8	23	
38	9.74 341	19	9.82 297	27	0.17 702	9.92 043	8	22	
39	9.74 360	18	9.82 325	27	0.17 675	9.92 035	8	21	
40	9.74 379	19	9.82 352	27	0.17 647	9.92 027	8	20	
41	9.74 398	19	9.82 380	27	0.17 620	9.92 018	8	19	
42	9.74 417	19	9.82 407	27	0.17 593	9.92 010	8	18	
43	9.74 436	19	9.82 434	27	0.17 565	9.92 001	8	17	**8 8**
44	9.74 455	19	9.82 462	27	0.17 538	9.91 993	8	16	6 0.8 0.8
45	9.74 474	19	9.82 489	27	0.17 510	9.91 984	8	15	7 1.0 0.9
46	9.74 493	18	9.82 516	27	0.17 483	9.91 976	8	14	8 1.1 1.0
47	9.74 511	19	9.82 544	27	0.17 456	9.91 967	8	13	9 1.3 1.2
48	9.74 530	19	9.82 571	27	0.17 428	9.91 959	8	12	10 1.4 1.3
49	9.74 549	19	9.82 598	27	0.17 401	9.91 951	8	11	20 2.8 2.6
50	9.74 568	19	9.82 626	27	0.17 374	9.91 942	8	10	30 4.2 4.0
51	9.74 587	18	9.82 653	27	0.17 347	9.91 934	8	9	40 5.6 5.3
52	9.74 606	19	9.82 680	27	0.17 319	9.91 925	8	8	50 7.1 6.6
53	9.74 625	19	9.82 708	27	0.17 292	9.91 917	8	7	
54	9.74 643	18	9.82 735	27	0.17 265	9.91 908	8	6	
55	9.74 662	19	9.82 762	27	0.17 237	9.91 900	8	5	
56	9.74 681	18	9.82 789	27	0.17 210	9.91 891	8	4	
57	9.74 700	18	9.82 817	27	0.17 183	9.91 883	8	3	
58	9.74 718	18	9.82 844	27	0.17 156	9.91 874	8	2	
59	9.74 737	19	9.82 871	27	0.17 128	9.91 866	8	1	
60	9 82 898		9 82 898	27	0.17 101	9.91 857	8	0	
	Log. Cos.	d.	Log. Cot.	c. d.	Log. Tan.	Log. Sin.	d.	′	P. P.

TABLE VII.—LOGARITHMIC SINES, COSINES, TANGENTS, AND COTANGENTS.
34°

′	Log. Sin.	d.	Log. Tan.	c. d.	Log. Cot.	Log. Cos.	d.		P. P.
0	9.74756		9.82898		0.17101	9.91857		60	
1	9.74775	19	9.82926	27	0.17074	9.91849	8	59	
2	9.74793	18	9.82953	27	0.17047	9.91840	8	58	
3	9.74812	19	9.82980	27	0.17019	9.91832	8	57	
4	9.74831	18	9.83007	27	0.16992	9.91823	8	56	
5	9.74849	18	9.83035	27	0.16965	9.91814	9	55	
6	9.74868	19	9.83062	27	0.16938	9.91806	8	54	
7	9.74887	18	9.83089	27	0.16910	9.91797	8	53	
8	9.74905	18	9.83116	27	0.16883	9.91789	8	52	27 27 26
9	9.74924	19	9.83143	27	0.16856	9.91780	8	51	6 | 2.7 | 2.7 | 2.6
10	9.74943	18	9.83171	27	0.16829	9.91772	8	50	7 | 3.2 | 3.1 | 3.1
11	9.74961	18	9.83198	27	0.16802	9.91763	8	49	8 | 3.6 | 3.6 | 3.5
12	9.74980	18	9.83225	27	0.16774	9.91755	8	48	9 | 4.1 | 4.0 | 4.0
13	9.74998	18	9.83252	27	0.16747	9.91746	9	47	10 | 4.6 | 4.5 | 4.4
14	9.75017	18	9.83279	27	0.16720	9.91737	8	46	20 | 9.1 | 9.0 | 8.8
15	9.75036	19	9.83307	27	0.16693	9.91729	8	45	30 | 13.7 | 13.5 | 13.2
16	9.75054	18	9.83334	27	0.16666	9.91720	8	44	40 | 18.3 | 18.0 | 17.6
17	9.75073	18	9.83361	27	0.16639	9.91712	8	43	50 | 22.9 | 22.5 | 22.1
18	9.75091	18	9.83388	27	0.16612	9.91703	9	42	
19	9.75110	18	9.83415	27	0.16584	9.91694	8	41	
20	9.75128	18	9.83442	27	0.16557	9.91686	8	40	
21	9.75147	18	9.83469	27	0.16530	9.91677	8	39	
22	9.75165	18	9.83496	27	0.16503	9.91668	9	38	
23	9.75184	18	9.83524	27	0.16476	9.91660	8	37	
24	9.75202	18	9.83551	27	0.16449	9.91651	8	36	
25	9.75221	18	9.83578	27	0.16422	9.91642	9	35	19 18 18
26	9.75239	18	9.83605	27	0.16395	9.91634	8	34	6 | 1.9 | 1.8 | 1.8
27	9.75257	18	9.83632	27	0.16368	9.91625	9	33	7 | 2.2 | 2.1 | 2.1
28	9.75276	18	9.83659	27	0.16340	9.91616	8	32	8 | 2.5 | 2.4 | 2.4
29	9.75294	18	9.83686	27	0.16313	9.91608	8	31	9 | 2.8 | 2.8 | 2.7
30	9.75313	18	9.83713	27	0.16286	9.91599	9	30	10 | 3.1 | 3.1 | 3.0
31	9.75331	18	9.83740	27	0.16259	9.91590	8	29	20 | 6.3 | 6.1 | 6.0
32	9.75349	18	9.83767	27	0.16232	9.91582	8	28	30 | 9.5 | 9.2 | 9.0
33	9.75368	18	9.83794	27	0.16205	9.91573	9	27	40 | 12.6 | 12.3 | 12.0
34	9.75386	18	9.83821	27	0.16178	9.91564	8	26	50 | 15.8 | 15.4 | 15.0
35	9.75404	18	9.83848	27	0.16151	9.91556	9	25	
36	9.75423	18	9.83875	27	0.16124	9.91547	9	24	
37	9.75441	18	9.83902	27	0.16097	9.91538	9	23	
38	9.75459	18	9.83929	27	0.16070	9.91529	8	22	
39	9.75478	18	9.83957	27	0.16043	9.91521	9	21	
40	9.75496	18	9.83984	27	0.16016	9.91512	8	20	
41	9.75514	18	9.84011	27	0.15989	9.91503	9	19	
42	9.75532	18	9.84038	27	0.15962	9.91495	8	18	
43	9.75551	18	9.84065	26	0.15935	9.91486	9	17	9 8
44	9.75569	18	9.84091	27	0.15908	9.91477	9	16	6 | 0.9 | 0.8
45	9.75587	18	9.84118	27	0.15881	9.91468	8	15	7 | 1.0 | 1.0
46	9.75605	18	9.84145	27	0.15854	9.91460	9	14	8 | 1.2 | 1.1
47	9.75623	18	9.84172	27	0.15827	9.91451	9	13	9 | 1.3 | 1.3
48	9.75642	18	9.84199	27	0.15800	9.91442	9	12	10 | 1.5 | 1.4
49	9.75660	18	9.84226	27	0.15773	9.91433	9	11	20 | 3.0 | 2.8
50	9.75678	18	9.84253	27	0.15746	9.91424	8	10	30 | 4.5 | 4.2
51	9.75696	18	9.84280	27	0.15719	9.91416	9	9	40 | 6.0 | 5.6
52	9.75714	18	9.84307	27	0.15692	9.91407	9	8	50 | 7.5 | 7.1
53	9.75732	18	9.84334	27	0.15665	9.91398	9	7	
54	9.75750	18	9.84361	26	0.15639	9.91389	8	6	
55	9.75769	18	9.84388	27	0.15612	9.91380	9	5	
56	9.75787	18	9.84415	27	0.15585	9.91372	8	4	
57	9.75805	18	9.84442	27	0.15558	9.91363	9	3	
58	9.75823	18	9.84469	27	0.15531	9.91354	9	2	
59	9.75841	18	9.84496	27	0.15504	9.91345	8	1	
60	9.75859	18	9.84522	26	0.15477	9.91336	9	0	
	Log. Cos.	d.	Log. Cot.	c. d.	Log. Tan.	Log. Sin.	d.	′	P. P.

TABLE VII.—LOGARITHMIC SINES, COSINES, TANGENTS, AND COTANGENTS.

35°

′	Log. Sin.	d.	Log. Tan.	c. d.	Log. Cot.	Log. Cos.	d.	′
0	9.75 859	18	9.84 522	27	0.15 477	9.91 336	9	60
1	9.75 877	18	9.84 549	27	0.15 450	9.91 327	9	59
2	9.75 895	18	9.84 576	27	0.15 423	9.91 318	8	58
3	9.75 913	18	9.84 603	27	0.15 396	9.91 310	9	57
4	9.75 931	18	9.84 630	26	0.15 370	9.91 301	9	56
5	9.75 949	18	9.84 657	27	0.15 343	9.91 292	9	55
6	9.75 967	18	9.84 684	27	0.15 316	9.91 283	8	54
7	9.75 985	18	9.84 711	27	0.15 289	9.91 274	9	53
8	9.76 003	18	9.84 737	26	0.15 262	9.91 265	9	52
9	9.76 021	18	9.84 764	27	0.15 235	9.91 256	9	51
10	9.76 039	18	9.84 791	27	0.15 208	9.91 247	9	50
11	9.76 057	18	9.84 818	26	0.15 182	9.91 239	8	49
12	9.76 075	18	9.84 845	27	0.15 155	9.91 230	9	48
13	9.76 092	17	9.84 871	26	0.15 128	9.91 221	9	47
14	9.76 110	18	9.84 898	27	0.15 101	9.91 212	9	46
15	9.76 128	18	9.84 925	27	0.15 074	9.91 203	9	45
16	9.76 146	18	9.84 952	26	0.15 048	9.91 194	9	44
17	9.76 164	17	9.84 979	26	0 15 021	9.91 185	8	43
18	9.76 182	18	9.85 005	27	0.14 994	9.91 176	9	42
19	9.76 200	18	9.85 032	27	0.14 967	9.91 167	9	41
20	9.76 217	17	9.85 059	27	0.14 940	9.91 158	9	40
21	9.76 235	18	9.85 086	26	0.14 914	9.91 149	9	39
22	9.76 253	18	9.85 113	27	0.14 887	9.91 140	9	38
23	9.76 271	18	9.85 139	26	0.14 860	9.91 131	9	37
24	9.76 289	18	9.85 166	27	0.14 833	9.91 122	9	36
25	9.76 306	17	9.85 193	26	0.14 807	9.91 113	9	35
26	9.76 324	18	9.85 220	27	0.14 780	9.91 104	9	34
27	9.76 342	17	9.85 246	26	0.14 753	9.91 095	9	33
28	9.76 360	18	9.85 273	27	0.14 726	9.91 086	9	32
29	9.76 377	17	9.85 300	26	0.14 700	9.91 077	9	31
30	9.76 395	17	9.85 327	27	0.14 673	9.91 068	9	30
31	9.76 413	18	9.85 353	26	0.14 646	9.91 059	9	29
32	9.76 431	17	9.85 380	26	0.14 620	9.91 050	9	28
33	9.76 448	17	9.85 407	27	0.14 593	9.91 041	9	27
34	9.76 466	18	9.85 433	26	0.14 566	9.91 032	9	26
35	9.76 484	17	9.85 460	27	0.14 539	9.91 023	9	25
36	9.76 501	17	9.85 487	26	0.14 513	9.91 014	9	24
37	9.76 519	17	9.85 513	26	0.14 486	9.91 005	9	23
38	9.76 536	18	9.85 540	27	0.14 459	9.90 996	9	22
39	9.76 554	17	9.85 567	26	0.14 433	9.90 987	9	21
40	9.76 572	17	9.85 594	27	0.14 406	9.90 978	9	20
41	9.76 589	17	9.85 620	26	0.14 379	9.90 969	9	19
42	9.76 607	17	9.85 647	26	0.14 353	9.90 960	9	18
43	9.76 624	18	9.85 673	26	0.14 326	9.90 951	9	17
44	9.76 642	17	9.85 700	27	0.14 299	9.90 942	9	16
45	9.76 660	17	9.85 727	26	0.14 273	9.90 933	9	15
46	9.76 677	17	9.85 753	26	0.14 246	9.90 923	9	14
47	9.76 695	17	9.85 780	27	0.14 219	9.90 914	9	13
48	9.76 712	17	9.85 807	26	0.14 193	9.90 905	9	12
49	9.76 730	17	9.85 833	26	0.14 166	9.90 896	9	11
50	9.76 747	17	9.85 860	26	0.14 140	9.90 887	9	10
51	9.76 765	17	9.85 887	27	0.14 113	9.90 878	9	9
52	9.76 782	17	9.85 913	26	0.14 086	9.90 869	9	8
53	9.76 800	17	9.85 940	26	0.14 060	9.90 860	9	7
54	9.76 817	17	9.85 966	26	0.14 033	9.90 850	9	6
55	9.76 835	17	9.85 993	27	0.14 007	9.90 841	9	5
56	9.76 852	17	9.86 020	26	0.13 980	9.90 832	9	4
57	9.76 869	17	9.86 046	27	0.13 953	9.90 823	9	3
58	9.76 887	17	9.86 073	26	0.13 927	9.90 814	9	2
59	9.76 904	17	9.86 099	26	0.13 900	9.90 805	9	1
60	9.76 922		9.86 126	26	0.13 874	9.90 796	9	0
	Log. Cos.	d.	Log. Cot.	c. d.	Log. Tan.	Log. Sin.	d.	′

P. P.

	27	26
6	2.7	2.6
7	3.1	3.1
8	3.6	3.5
9	4.0	4.0
10	4.5	4.4
20	9.0	8.8
30	13.5	13.2
40	18.0	17.6
50	22.5	22.1

	18	17	17
6	1.8	1.7	1.7
7	2.1	2.0	2.0
8	2.4	2.3	2.2
9	2.7	2.6	2.5
10	3.0	2.9	2.8
20	6.0	5.8	5.6
30	9.0	8.7	8.5
40	12.0	11.6	11.3
50	15.0	14.6	14.1

	9	9	8
6	0.9	0.9	0.8
7	1.1	1.0	1.0
8	1.2	1.2	1.1
9	1.4	1.3	1.3
10	1.6	1.5	1.4
20	3.1	3.0	2.8
30	4.7	4.5	4.2
40	6.3	6.0	5.6
50	7.9	7.5	7.1

TABLE VII.—LOGARITHMIC SINES, COSINES, TANGENTS, AND COTANGENTS.
36°

′	Log. Sin.	d.	Log. Tan.	c. d.	Log. Cot.	Log. Cos.	d.	′	P. P.		
0	9.76 922	17	9.86 126	26	0.13 874	9.90 796	9	60			
1	9.76 939	17	9.86 152	26	0.13 847	9.90 786	9	59			
2	9.76 956	17	9.86 179	27	0.13 821	9.90 777	9	58			
3	9.76 974	17	9.86 206	26	0.13 794	9.90 768	9	57			
4	9.76 991	17	9.86 232	26	0.13 767	9.90 759	9	56			
5	9.77 008	17	9.86 259	26	0.13 741	9.90 750	9	55			
6	9.77 026	17	9.86 285	26	0.13 714	9.90 740	9	54			
7	9.77 043	17	9.86 312	26	0.13 688	9.90 731	9	53		27	26	26
8	9.77 060	17	9.86 338	26	0.13 661	9.90 722	9	52	6	2.7	2.6	2.6
9	9.77 078	17	9.86 365	26	0.13 635	9.90 713	9	51	7	3.1	3.1	3.0
10	9.77 095	17	9.86 391	26	0.13 608	9.90 703	9	50	8	3.6	3.5	3.4
11	9.77 112	17	9.86 418	26	0.13 582	9.90 694	9	49	9	4.0	4.0	3.9
12	9.77 130	17	9.86 444	26	0.13 555	9.90 685	9	48	10	4.5	4.4	4.3
13	9.77 147	17	9.86 471	26	0.13 529	9.90 676	9	47	20	9.0	8.8	8.6
14	9.77 164	17	9.86 497	26	0.13 502	9.90 666	9	46	30	13.5	13.2	13.0
15	9.77 181	17	9.86 524	26	0.13 476	9.90 657	9	45	40	18.0	17.6	17.3
16	9.77 198	17	9.86 550	26	0.13 449	9.90 648	9	44	50	22.5	22.1	21.6
17	9.77 216	17	9.86 577	26	0.13 423	9.90 639	9	43				
18	9.77 233	17	9.86 603	26	0.13 396	9.90 629	9	42				
19	9.77 250	17	9.86 630	26	0.13 370	9.90 620	9	41				
20	9.77 267	17	9.86 656	26	0.13 343	9.90 611	9	40				
21	9.77 284	17	9.86 683	26	0.13 317	9.90 602	9	39				
22	9.77 302	17	9.86 709	26	0.13 290	9.90 592	9	38				
23	9.77 319	17	9.86 736	26	0.13 264	9.90 583	9	37				
24	9.77 336	17	9.86 762	26	0.13 237	9.90 574	9	36				
25	9.77 353	17	9.86 788	26	0.13 211	9.90 564	9	35		17	17	16
26	9.77 370	17	9.86 815	26	0.13 185	9.90 555	9	34	6	1.7	1.7	1.6
27	9.77 387	17	9.86 841	26	0.13 158	9.90 546	9	33	7	2.0	2.0	1.9
28	9.77 404	17	9.86 868	26	0.13 132	9.90 536	9	32	8	2.3	2.2	2.2
29	9.77 421	17	9.86 894	26	0.13 105	9.90 527	9	31	9	2.6	2.5	2.5
30	9.77 439	17	9.86 921	26	0.13 079	9.90 518	9	30	10	2.9	2.8	2.7
31	9.77 456	17	9.86 947	26	0.13 052	9.90 508	9	29	20	5.8	5.6	5.5
32	9.77 473	17	9.86 973	26	0.13 026	9.90 499	9	28	30	8.7	8.5	8.2
33	9.77 490	17	9.87 000	26	0.13 000	9.90 490	9	27	40	11.6	11.3	11.0
34	9.77 507	17	9.87 026	26	0.12 973	9.90 480	9	26	50	14.6	14.1	13.7
35	9.77 524	17	9.87 053	26	0.12 947	9.90 471	9	25				
36	9.77 541	17	9.87 079	26	0.12 920	9.90 461	9	24				
37	9.77 558	17	9.87 105	26	0.12 894	9.90 452	9	23				
38	9.77 575	17	9.87 132	26	0.12 868	9.90 443	9	22				
39	9.77 592	17	9.87 158	26	0.12 841	9.90 433	9	21				
40	9.77 609	17	9.87 185	26	0.12 815	9.90 424	9	20				
41	9.77 626	17	9.87 211	26	0.12 789	9.90 414	9	19				
42	9.77 643	17	9.87 237	26	0.12 762	9.90 405	9	18				
43	9.77 660	17	9.87 264	26	0.12 736	9.90 396	9	17				
44	9.77 677	16	9.87 290	26	0.12 709	9.90 386	9	16		9	9	
45	9.77 693	17	9.87 316	26	0.12 683	9.90 377	9	15	6	0.9	0.9	
46	9.77 710	17	9.87 343	26	0.12 657	9.90 367	9	14	7	1.1	1.0	
47	9.77 727	17	9.87 369	26	0.12 630	9.90 358	9	13	8	1.2	1.2	
48	9.77 744	17	9.87 395	26	0.12 604	9.90 348	9	12	9	1.4	1.3	
49	9.77 761	16	9.87 422	26	0.12 578	9.90 339	9	11	10	1.6	1.5	
50	9.77 778	17	9.87 448	26	0.12 551	9.90 330	9	10	20	3.1	3.0	
51	9.77 795	17	9.87 474	26	0.12 525	9.90 320	9	9	30	4.7	4.5	
52	9.77 812	16	9.87 501	26	0.12 499	9.90 311	9	8	40	6.3	6.0	
53	9.77 828	17	9.87 527	26	0.12 472	9.90 301	9	7	50	7.9	7.5	
54	9.77 845	17	9.87 553	26	0.12 446	9.90 292	9	6				
55	9.77 862	17	9.87 580	26	0.12 420	9.90 282	9	5				
56	9.77 879	16	9.87 606	26	0.12 393	9.90 273	9	4				
57	9.77 896	17	9.87 632	26	0.12 367	9.90 263	9	3				
58	9.77 913	17	9.87 659	26	0.12 341	9.90 254	9	2				
59	9.77 929	16	9.87 685	26	0.12 315	9.90 244	9	1				
60	9.77 946	17	9.87 711	26	0.12 288	9 90 235		0				
	Log. Cos.	d.	Log. Cot.	c. d.	Log. Tan.	Log. Sin.	d.	′		P. P.	

TABLE VII.—LOGARITHMIC SINES, COSINES, TANGENTS, AND COTANGENTS.

37°

′	Log. Sin.	d.	Log. Tan.	c. d.	Log. Cot.	Log. Cos.	d.		P. P.
0	9.77 946	16̂	9.87 71î	26	0.12 28ĝ	9.90 235	ĝ	60	
1	9.77 963	17	9.87 73ŷ	26	0.12 262	9.90 225	ĝ	59	
2	9.77 980	16	9.87 764	26	0.12 236	9.90 216	ĝ	58	
3	9.77 996	17	9.87 79ô	26	0.12 20ŷ	9.90 20ĝ	10	57	
4	9.78 013	16	9.87 81ĝ	26	0.12 18ĝ	9.90 19ĝ	ĝ	56	
5	9.78 030	16	9.87 843	26	0.12 157	9.90 187	ĝ	55	
6	9.78 046	17	9.87 869	26	0.12 131	9.90 17ŷ	ĝ	54	
7	9.78 06ĝ	16	9.87 89ŝ	26	0.12 10ŝ	9.90 168	ĝ	53	
8	9.78 080	17	9.87 92î	26	0.12 07ĝ	9.90 15ĝ	ĝ	52	
9	9.78 097	16	9.87 948	26	0.12 052	9.90 149	ĝ	51	
10	9.78 11ĝ	16	9.87 974	26	0.12 026	9.90 13ŷ	ĝ	50	2ĝ 26
11	9.78 130	17	9.88 00ô	26	0.11 99ĝ	9.90 130	10	49	6 2.ĝ 2.6
12	9.78 147	16	9.88 02ĝ	26	0.11 97ĝ	9.90 120	ĝ	48	7 3.1 3.ô
13	9.78 16ĝ	16	9.88 053	26	0.11 947	9.90 11ô	ĝ	47	8 3.ŝ 3.â
14	9.78 180	16	9.88 079	26	0.11 921	9.90 101	ĝ	46	9 4.0 3.9
15	9.78 19ĝ	16	9.88 105	26	0.11 895	9.90 09î	ĝ	45	10 4.4 4.ĝ
16	9.78 213	17	9.88 13î	26	0.11 86ĝ	9.90 082	10	44	20 8.ĝ 8.ĝ
17	9.78 230	16	9.88 15ŷ	26	0.11 84ĝ	9.90 072	ĝ	43	30 13.ĝ 13.0
18	9.78 24ĝ	16	9.88 184	26	0.11 816	9.90 06ĝ	ĝ	42	40 17.ĝ 17.ĝ
19	9.78 263	16	9.88 210	26	0.11 790	9.90 053	ĝ	41	50 22.1 21.ĝ
20	9.78 27ŷ	16	9.88 236	26	0.11 76ĝ	9.90 04ĝ	10	40	
21	9.78 296	16	9.88 262	26	0.11 73ŷ	9.90 03ĝ	ĝ	39	
22	9.78 31ĝ	16	9.88 28ĝ	26	0.11 71î	9.90 024	ĝ	38	
23	9.78 329	17	9.88 315	26	0.11 685	9.90 01â	10	37	
24	9.78 346	16	9.88 341	26	0.11 659	9.90 00â	ĝ	36	
25	9.78 36ĝ	16	9.88 367	26	0.11 633	9.89 995	ĝ	35	17 1ĝ 16
26	9.78 379	16	9.88 39ĝ	26	0.11 60ĝ	9.89 98ŝ	10	34	6 1.7 1.ĝ 1.6
27	9.78 39ŝ	16	9.88 41ŷ	26	0.11 58ô	9.89 97ŝ	ĝ	33	7 2.0 1.9 1.ĝ
28	9.78 412	16	9.88 44ŝ	26	0.11 55â	9.89 966	ĝ	32	8 2.ĝ 2.2 2.î
29	9.78 428	16	9.88 472	26	0.11 528	9.89 95ĝ	10	31	9 2.ŝ 2.5 2.4
30	9.78 44â	16	9.88 498	26	0.11 502	9.89 94ĝ	ĝ	30	10 2.ĝ 2.ŷ 2.ĝ
31	9.78 461	16	9.88 524	26	0.11 476	9.89 937	ĝ	29	20 5.ĝ 5.5 5.ĝ
32	9.78 477	16	9.88 55ô	26	0.11 44ŷ	9.89 927	10	28	30 8.5 8.ĝ 8.0
33	9.78 494	16	9.88 57ĝ	26	0.11 42ĝ	9.89 91ŷ	ĝ	27	40 11.ĝ 11.0 10.ĝ
34	9.78 51ô	16	9.88 60ĝ	26	0.11 397	9.89 908	10	26	50 14.î 13.ŷ 13.ĝ
35	9.78 527	16	9.88 629	26	0.11 371	9.89 898	ĝ	25	
36	9.78 54ĝ	16	9.88 655	26	0.11 345	9.89 88ĝ	10	24	
37	9.78 55ŷ	16	9.88 681	26	0.11 319	9.89 878	ĝ	23	
38	9.78 576	16	9.88 707	26	0.11 293	9.89 869	10	22	
39	9.78 59ĝ	16	9.88 73ĝ	26	0.11 26ĝ	9.89 859	ĝ	21	
40	9.78 609	16	9.88 75ŷ	26	0.11 24ô	9.89 84ŷ	10	20	
41	9.78 625	16	9.88 78ŝ	26	0.11 21â	9.89 83ŷ	ĝ	19	
42	9.78 64î	16	9.88 81î	26	0.11 18ĝ	9.89 830	10	18	
43	9.78 658	16	9.88 838	26	0.11 162	9.89 820	10	17	10 ĝ
44	9.78 674	16	9.88 864	26	0.11 136	9.89 81ô	ĝ	16	6 1.0 0.ŷ
45	9.78 69ô	16	9.88 890	26	0.11 110	9.89 80ô	ĝ	15	7 1.î 1.1
46	9.78 707	16	9.88 916	26	0.11 084	9.89 791	10	14	8 1.ĝ 1.ĝ
47	9.78 723	16	9.88 942	26	0.11 058	9.89 781	10	13	9 1.5 1.4
48	9.78 73ŷ	16	9.88 968	26	0.11 032	9.89 771	10	12	10 1.ĝ 1.6
49	9.78 755	16	9.88 99â	26	0.11 00ŝ	9.89 76î	ĝ	11	20 3.ĝ 3.î
50	9.78 772	16	9.89 02ô	26	0.10 979	9.89 75î	10	10	30 5.0 4.ŷ
51	9.78 78ĝ	16	9.89 04ĝ	26	0.10 95ĝ	9.89 742	ĝ	9	40 6.ĝ 6.ĝ
52	9.78 80â	16	9.89 07ĝ	26	0.10 927	9.89 732	10	8	50 8.ĝ 7.9
53	9.78 821	16	9.89 098	26	0.10 90î	9.89 722	10	7	
54	9.78 837	16	9.89 12â	26	0.10 87ŝ	9.89 71ĝ	ĝ	6	
55	9.78 853	16	9.89 15ô	26	0.10 84ŷ	9.89 70ĝ	10	5	
56	9.78 86ŷ	16	9.89 177	26	0.10 823	9.89 69ĝ	10	4	
57	9.78 88ŝ	16	9.89 203	26	0.10 797	9.89 683	ĝ	3	
58	9.78 902	16	9.89 229	26	0.10 771	9.89 673	10	2	
59	9.78 918	16	9.89 255	26	0.10 745	9.89 663	10	1	
60	9.78 934	16	9.89 281	26	0.10 719	9 89 653	10	0	
	Log. Cos.	d.	Log. Cot.	c. d.	Log. Tan.	Log. Sin.	d.	′	P. P.

TABLE VII.—LOGARITHMIC SINES, COSINES, TANGENTS, AND COTANGENTS.
38°

′	Log. Sin.	d.	Log. Tan.	c. d.	Log. Cot.	Log. Cos.	d.	′	P. P.
0	9.78 934	16	9.89 281	26	0.10 719	9.89 653	9	60	
1	9.78 950	16	9.89 307	26	0.10 693	9.89 643	10	59	
2	9.78 966	16	9.89 333	26	0.10 667	9.89 633	10	58	
3	9.78 982	16	9.89 359	26	0.10 641	9.89 623	10	57	
4	9.78 999	16	9.89 385	26	0.10 615	9.89 613	10	56	
5	9.79 015	16	9.89 411	26	0.10 589	9.89 604	9	55	
6	9.79 031	16	9.89 437	26	0.10 563	9.89 594	10	54	
7	9.79 047	16	9.89 463	26	0.10 537	9.89 584	10	53	
8	9.79 063	16	9.89 489	26	0.10 511	9.89 574	10	52	26 25
9	9.79 079	16	9.89 515	26	0.10 485	9.89 564	10	51	6 2.6 2.5
10	9.79 095	16	9.89 541	26	0.10 459	9.89 554	9	50	7 3.0 3.0
11	9.79 111	16	9.89 567	26	0.10 433	9.89 544	10	49	8 3.4 3.4
12	9.79 127	16	9.89 593	26	0.10 407	9.89 534	10	48	9 3.9 3.8
13	9.79 143	16	9.89 619	26	0.10 381	9.89 524	10	47	10 4.3 4.2
14	9.79 159	16	9.89 645	26	0.10 355	9.89 514	10	46	20 8.6 8.5
15	9.79 175	16	9.89 671	26	0.10 329	9.89 504	10	45	30 13.0 12.7
16	9.79 191	16	9.89 697	26	0.10 303	9.89 494	10	44	40 17.3 17.0
17	9.79 207	16	9.89 723	26	0.10 277	9.89 484	10	43	50 21.6 21.2
18	9.79 223	16	9.89 749	26	0.10 251	9.89 474	10	42	
19	9.79 239	16	9.89 775	26	0.10 225	9.89 464	10	41	
20	9.79 255	16	9.89 801	26	0.10 199	9.89 454	10	40	
21	9.79 271	16	9.89 827	26	0.10 173	9.89 444	10	39	
22	9.79 287	16	9.89 853	26	0.10 147	9.89 434	10	38	
23	9.79 303	16	9.89 879	26	0.10 121	9.89 424	10	37	
24	9.79 319	16	9.89 905	26	0.10 095	9.89 414	10	36	
25	9.79 335	16	9.89 931	26	0.10 069	9.89 404	10	35	18 16 15
26	9.79 351	16	9.89 957	25	0.10 043	9.89 394	10	34	6 1.6 1.6 1.5
27	9.79 367	15	9.89 982	26	0.10 017	9.89 384	10	33	7 1.9 1.8 1.8
28	9.79 383	16	9.90 008	26	0.09 991	9.89 374	10	32	8 2.2 2.1 2.0
29	9.79 399	16	9.90 034	26	0.09 965	9.89 364	10	31	9 2.5 2.4 2.3
30	9.79 415	16	9.90 060	26	0.09 939	9.89 354	10	30	10 2.7 2.6 2.6
31	9.79 431	15	9.90 086	26	0.09 913	9.89 344	10	29	20 5.5 5.3 5.1
32	9.79 446	16	9.90 112	26	0.09 887	9.89 334	10	28	30 8.2 8.0 7.7
33	9.79 462	16	9.90 138	25	0.09 861	9.89 324	10	27	40 11.0 10.6 10.3
34	9.79 478	15	9.90 164	26	0.09 836	9.89 314	16	26	50 13.7 13.3 12.9
35	9.79 494	16	9.90 190	26	0.09 810	9.89 304	10	25	
36	9.79 510	16	9.90 216	26	0.09 784	9.89 294	10	24	
37	9.79 526	15	9.90 242	26	0.09 758	9.89 284	10	23	
38	9.79 541	16	9.90 268	26	0.09 732	9.89 274	10	22	
39	9.79 557	16	9.90 294	25	0.09 706	9.89 264	10	21	
40	9.79 573	15	9.90 319	26	0.09 680	9.89 253	16	20	
41	9.79 589	16	9.90 345	26	0.09 654	9.89 243	10	19	
42	9.79 605	15	9.90 371	26	0.09 628	9.89 233	10	18	
43	9.79 620	16	9.90 397	25	0.09 602	9.89 223	10	17	16 10 9
44	9.79 636	15	9.90 423	26	0.09 577	9.89 213	16	16	6 1.6 1.0 0.9
45	9.79 652	16	9.90 449	26	0.09 551	9.89 203	10	15	7 1.2 1.1 1.1
46	9.79 668	15	9.90 475	26	0.09 525	9.89 193	10	14	8 1.4 1.3 1.2
47	9.79 683	16	9.90 501	25	0.09 499	9.89 182	16	13	9 1.6 1.5 1.4
48	9.79 699	15	9.90 526	26	0.09 473	9.89 172	10	12	10 1.7 1.6 1.6
49	9.79 715	15	9.90 552	26	0.09 447	9.89 162	10	11	20 3.5 3.3 3.1
50	9.79 730	16	9.90 578	26	0.09 421	9.89 152	10	10	30 5.2 5.0 4.7
51	9.79 746	15	9.90 604	25	0.09 395	9.89 142	16	9	40 7.0 6.6 6.3
52	9.79 762	15	9.90 630	26	0.09 370	9.89 132	10	8	50 8.7 8.3 7.9
53	9.79 777	15	9.90 656	26	0.09 344	9.89 121	16	7	
54	9.79 793	16	9.90 682	26	0.09 318	9.89 111	10	6	
55	9.79 809	15	9.90 707	25	0.09 292	9.89 101	10	5	
56	9.79 824	15	9.90 733	26	0.09 266	9.89 091	16	4	
57	9.79 840	16	9.90 759	26	0.09 240	9.89 081	10	3	
58	9.79 856	15	9.90 785	26	0.09 214	9.89 070	16	2	
59	9.79 871	15	9.90 811	25	0.09 189	9.89 060	10	1	
60	9.79 887	15	9.90 837	26	0.09 163	9.89 050	10	0	
	Log. Cos.	d.	Log. Cot.	c. d.	Log. Tan.	Log. Sin.	d.	′	P. P.

TABLE VII.—LOGARITHMIC SINES, COSINES, TANGENTS, AND COTANGENTS.

39°

′	Log. Sin.	d.	Log. Tan.	c. d.	Log. Cot.	Log. Cos.	d.		P. P.
0	9.79 887	16	9 90 837	26	0.09 163	9.89 05ô	1ô	60	
1	9.79 903	1ŝ	9.90 863	25	0.09 137	9.89 040	10	59	
2	9.79 918	1ŝ	9.90 88ŝ	26	0.09 11î	9.89 030	1ô	58	
3	9.79 934	1ŝ	9.90 91â	25	0.09 08ŝ	9.89 01ô	10	57	
4	9.79 94ô	1ŝ	9.90 940	26	0.09 060	9.89 00ô	1ô	56	
5	9.79 965	1ŝ	9.90 966	26	0.09 034	9.88 999	10	55	
6	9.79 98ô	1ŝ	9.90 992	25	0.09 008	9.88 989	1ô	54	
7	9.79 996	1ŝ	9 91 01î	26	0.08 98â	9.88 978	1ô	53	
8	9.80 01î	1ŝ	9.91 043	26	0.08 95ô	9.88 968	10	52	26 2ŝ
9	9.80 027	1ŝ	9.91 06ô	25	0.08 93ô	9.88 958	1ô	51	6 │ 2.6 │ 2.ŝ
10	9.80 04â	1ŝ	9.91 095	26	0.08 905	9.88 94î	10	50	7 │ 3.ô │ 3.0
11	9.80 058	1ŝ	9.91 121	25	0.08 879	9.88 937	1ô	49	8 │ 3.4 │ 3.4
12	9.80 07ŝ	1ŝ	9.91 14ô	26	0.08 85ŝ	9.88 927	10	48	9 │ 3.9 │ 3.8
13	9.80 089	1ŝ	9.91 17â	25	0.08 82ŝ	9.88 917	1ô	47	10 │ 4.ŝ │ 4.â
14	9.80 104	1ŝ	9.91 198	26	0.08 802	9.88 90ŝ	1ô	46	20 │ 8.ô │ 8.5
15	9.80 120	1ŝ	9.91 224	26	0.08 776	9.88 896	10	45	30 │ 13.0 │ 12.ŝ
16	9.80 13ŝ	1ŝ	9.91 250	25	0.08 750	9.88 886	1ô	44	40 │ 17.ŝ │ 17.0
17	9.80 151	1ŝ	9.91 27ŝ	26	0.08 72â	9.88 87ŝ	10	43	50 │ 21.ô │ 21.â
18	9.80 16ô	1ŝ	9.91 30î	25	0.08 69ô	9.88 865	1ô	42	
19	9.80 182	1ŝ	9.91 327	26	0.08 673	9.88 855	10	41	
20	9.80 19ô	1ŝ	9.91 353	25	0.08 647	9.88 84â	1ô	40	
21	9.80 213	1ŝ	9.91 378	26	0.08 62î	9.88 834	1ô	39	
22	9.80 228	1ŝ	9.91 40â	25	0.08 59ŝ	9.88 82ŝ	10	38	
23	9.80 24ŝ	1ŝ	9.91 430	26	0.08 570	9.88 81ŝ	1ô	37	
24	9.80 259	1ŝ	9.91 456	25	0.08 544	9.88 803	10	36	
25	9.80 27â	15	9.91 48î	26	0.08 51ô	9.88 79â	1ô	35	16 1ŝ 15
26	9.80 28ô	15	9.91 507	25	0.08 49â	9.88 782	10	34	6 │ 1.6 │ 1.ŝ │ 1.5
27	9.80 305	1ŝ	9.91 533	26	0.08 467	9.88 772	1ô	33	7 │ 1.ô │ 1.8 │ 1.ŝ
28	9.80 32ô	15	9.91 559	25	0.08 441	9.88 76î	1ô	32	8 │ 2.î │ 2.ô │ 2.0
29	9.80 33ŝ	1ŝ	9.91 58â	26	0.08 41ŝ	9.88 751	1ô	31	9 │ 2.4 │ 2.3 │ 2.â
30	9.80 351	1ŝ	9.91 61ô	25	0.08 38ô	9.88 74ô	1ô	30	10 │ 2.ô │ 2.6 │ 2.5
31	9.80 36ô	1ŝ	9.91 636	26	0.08 364	9.88 730	1ô	29	20 │ 5.ŝ │ 5.î │ 5.0
32	9.80 38î	1ŝ	9.91 662	25	0.08 338	9.88 720	10	28	30 │ 8.0 │ 7.ŝ │ 7.5
33	9.80 397	1ŝ	9.91 687	26	0.08 31â	9.88 70ô	10	27	40 │ 10.ô │ 10.ŝ │ 10.0
34	9.80 41â	15	9.91 71ŝ	25	0.08 28ô	9.88 699	1ô	26	50 │ 13.ŝ │ 12.9 │ 12.5
35	9.80 427	15	9.91 739	26	0.08 261	9.88 68ô	1ô	25	
36	9.80 443	15	9.91 765	25	0.08 235	9.88 678	1ô	24	
37	9.80 458	15	9.91 79ô	26	0.08 20ô	9.88 667	1ô	23	
38	9.80 47ŝ	15	9.91 816	25	0.08 18ŝ	9.88 657	1ô	22	
39	9 80 48ŝ	1ŝ	9.91 842	25	0.08 158	9.88 646	1ô	21	
40	9.80 504	1ŝ	9.91 86ô	26	0.08 13â	9.88 636	1ô	20	
41	9.80 519	1ŝ	9.91 89ŝ	25	0.08 10ô	9.88 62ŝ	1ô	19	
42	9.80 53â	15	9.91 919	26	0.08 081	9.88 615	1ô	18	
43	9.80 54ô	15	9.91 94ŝ	25	0.08 055	9.88 60â	1ô	17	11 1ô 10
44	9.80 56â	15	9.91 97ô	26	0.08 02ô	9.88 594	1ô	16	6 │ 1.1 │ 1.ô │ 1.0
45	9.80 580	15	9.91 996	25	0.08 004	9.88 58ŝ	15	15	7 │ 1.3 │ 1.2 │ 1.î
46	9.80 595	15	9.92 022	26	0.07 978	9.88 573	1ô	14	8 │ 1.4 │ 1.4 │ 1.ŝ
47	9.80 61ô	1ŝ	9.92 047	2ŝ	0.07 95â	9.88 56â	1ô	13	9 │ 1.6 │ 1.6 │ 1.5
48	9.80 62ŝ	15	9.92 07ŝ	26	0.07 92ô	9.88 552	1ô	12	10 │ 1.8 │ 1.ŝ │ 1.ô
49	9.80 64ô	15	9.92 099	2ŝ	0.07 901	9.88 54î	1ô	11	20 │ 3.6 │ 3.5 │ 3.ŝ
50	9.80 65ŝ	15	9.92 12â	2ŝ	0.07 87ŝ	9.88 531	1ô	10	30 │ 5.5 │ 5.â │ 5.0
51	9.80 671	1ŝ	9.92 15ô	26	0.07 84ô	9.88 52ô	1ô	9	40 │ 7.ŝ │ 7.0 │ 6.ô
52	9.80 686	15	9.92 176	2ŝ	0.07 824	9.88 510	1ô	8	50 │ 9.î │ 8.ŝ │ 8.3
53	9.80 701	15	9.92 20î	2ŝ	0.07 79ô	9.88 499	1ô	7	
54	9.80 716	15	9.92 227	26	0.07 77â	9.88 489	1ô	6	
55	9.80 73î	15	9.92 253	2ŝ	0.07 747	9.88 47ô	11	5	
56	9.80 74ô	15	9.92 27ô	2ŝ	0.07 72î	9.88 46î	1ô	4	
57	9.80 76î	15	9.92 304	26	0.07 69ŝ	9.88 457	1ô	3	
58	9.80 77ô	15	9.92 330	2ŝ	0.07 670	9.88 44ô	1ô	2	
59	9.80 79î	15	9.92 35ŝ	2ŝ	0.07 64â	9.88 436	1ô	1	
60	9.80 80ô	15	9.92 38î	26	0.07 61ô	9.88 42ŝ	1ô	0	
	Log. Cos.	d.	Log. Cot.	c. d.	Log. Tan.	Log. Sin.	d.	′	P. P.

TABLE VII.—LOGARITHMIC SINES, COSINES, TANGENTS, AND COTANGENTS.

40°

′	Log. Sin.	d.	Log. Tan.	c. d.	Log. Cot.	Log. Cos.	d.	′
0	9.80 806	15	9.92 381	26	0.07 618	9.88 425	10	60
1	9.80 822	15	9.92 407	25	0.07 593	9.88 415	11	59
2	9.80 837	15	9.92 432	26	0.07 567	9.88 404	11	58
3	9.80 852	15	9.92 458	26	0.07 541	9.88 393	10	57
4	9.80 867	15	9.92 484	25	0.07 516	9.88 383	11	56
5	9.80 882	15	9.92 509	26	0.07 490	9.88 372	11	55
6	9.80 897	15	9.92 535	26	0.07 465	9.88 361	10	54
7	9.80 912	15	9.92 561	25	0.07 439	9.88 351	11	53
8	9.80 927	15	9.92 586	26	0.07 413	9.88 340	11	52
9	9.80 942	15	9.92 612	26	0.07 388	9.88 329	10	51
10	9.80 957	15	9.92 638	25	0.07 362	9.88 319	11	50
11	9.80 972	15	9.92 663	26	0.07 336	9.88 308	11	49
12	9.80 987	14	9.92 689	25	0.07 311	9.88 297	10	48
13	9.81 001	15	9.92 714	26	0.07 285	9.88 287	11	47
14	9.81 016	15	9.92 740	26	0.07 259	9.88 276	11	46
15	9.81 031	15	9.92 766	25	0.07 234	9.88 265	10	45
16	9.81 046	15	9.92 791	26	0.07 208	9.88 255	11	44
17	9.81 061	15	9.92 817	25	0.07 183	9.88 244	11	43
18	9.81 076	15	9.92 842	26	0.07 157	9.88 233	10	42
19	9.81 091	15	9.92 868	26	0.07 131	9.88 223	11	41
20	9.81 106	15	9.92 894	25	0.07 106	9.88 212	11	40
21	9.81 121	15	9.92 919	26	0.07 080	9.88 201	11	39
22	9.81 136	14	9.92 945	26	0.07 055	9.88 190	10	38
23	9.81 150	15	9.92 971	25	0.07 029	9.88 180	11	37
24	9.81 165	15	9.92 996	26	0.07 003	9.88 169	11	36
25	9.81 180	15	9.93 022	25	0.06 978	9.88 158	11	35
26	9.81 195	15	9.93 047	26	0.06 952	9.88 147	10	34
27	9.81 210	15	9.93 073	25	0.06 927	9.88 137	11	33
28	9.81 225	15	9.93 098	26	0.06 901	9.88 126	11	32
29	9.81 239	14	9.93 124	26	0.06 875	9.88 115	11	31
30	9.81 254	15	9.93 150	25	0.06 850	9.88 104	10	30
31	9.81 269	15	9.93 175	26	0.06 824	9.88 094	11	29
32	9.81 284	15	9.93 201	25	0.06 799	9.88 083	11	28
33	9.81 299	15	9.93 226	26	0.06 773	9.88 072	11	27
34	9.81 313	15	9.93 252	26	0.06 748	9.88 061	11	26
35	9.81 328	15	9.93 278	25	0.06 722	9.88 050	11	25
36	9.81 343	15	9.93 303	26	0.06 696	9.88 039	10	24
37	9.81 358	14	9.93 329	25	0.06 671	9.88 029	11	23
38	9.81 372	15	9.93 354	26	0.06 645	9.88 018	11	22
39	9.81 387	15	9.93 380	25	0.06 620	9.88 007	11	21
40	9.81 402	14	9.93 405	26	0.06 594	9.87 996	11	20
41	9.81 416	15	9.93 431	25	0.06 569	9.87 985	11	19
42	9.81 431	15	9.93 456	26	0.06 543	9.87 974	11	18
43	9.81 446	14	9.93 482	26	0.06 518	9.87 963	10	17
44	9.81 460	15	9.93 508	25	0.06 492	9.87 953	11	16
45	9.81 475	15	9.93 533	26	0.06 466	9.87 942	11	15
46	9.81 490	14	9.93 559	25	0.06 441	9.87 931	11	14
47	9.81 504	15	9.93 584	26	0.06 415	9.87 920	11	13
48	9.81 519	15	9.93 610	25	0.06 390	9.87 909	11	12
49	9.81 534	14	9.93 635	26	0.06 364	9.87 898	11	11
50	9.81 548	15	9.93 661	25	0.06 339	9.87 887	11	10
51	9.81 563	15	9.93 686	26	0.06 313	9.87 876	11	9
52	9.81 578	14	9.93 712	25	0.06 288	9.87 865	11	8
53	9.81 592	15	9.93 737	26	0.06 262	9.87 854	10	7
54	9.81 607	14	9.93 763	25	0.06 237	9.87 844	11	6
55	9.81 621	15	9.93 788	26	0.06 211	9.87 833	11	5
56	9.81 636	14	9.93 814	26	0.06 186	9.87 822	11	4
57	9.81 650	15	9.93 840	25	0.06 160	9.87 811	11	3
58	9.81 665	15	9.93 865	26	0.06 134	9.87 800	11	2
59	9.81 680	14	9.93 891	25	0.06 109	9.87 789	11	1
60	9.81 694		9.93 916		0.06 083	9.87 778		0
	Log. Cos.	d.	Log. Cot.	c. d.	Log. Tan.	Log. Sin.	d.	′

P. P.

	26	25
6	2.6	2.5
7	3.0	3.0
8	3.4	3.4
9	3.9	3.8
10	4.3	4.2
20	8.6	8.5
30	13.0	12.7
40	17.3	17.0
50	21.6	21.2

	15	15	14
6	1.5	1.5	1.4
7	1.8	1.7	1.7
8	2.0	2.0	1.9
9	2.3	2.2	2.2
10	2.6	2.5	2.4
20	5.1	5.0	4.8
30	7.7	7.5	7.2
40	10.3	10.0	9.6
50	12.9	12.5	12.1

	11	10
6	1.1	1.0
7	1.3	1.2
8	1.4	1.4
9	1.6	1.6
10	1.8	1.7
20	3.6	3.5
30	5.5	5.2
40	7.3	7.0
50	9.1	8.7

49°

TABLE VII.—LOGARITHMIC SINES, COSINES, TANGENTS, AND COTANGENTS.
41°

′	Log. Sin.	d.	Log. Tan.	c. d.	Log. Cot.	Log. Cos.	d.	′
0	9.81 694	14	9.93 916	25	0.06 083	9.87 778	11	60
1	9.81 709	14	9.93 942	25	0.06 058	9.87 767	11	59
2	9.81 723	14	9.93 967	25	0.06 032	9.87 756	11	58
3	9.81 738	14	9.93 993	25	0.06 007	9.87 745	11	57
4	9.81 752	14	9.94 018	25	0.05 981	9.87 734	11	56
5	9.81 767	14	9.94 044	25	0.05 956	9.87 723	11	55
6	9.81 781	14	9.94 069	25	0.05 930	9.87 712	11	54
7	9.81 796	14	9.94 095	25	0.05 905	9.87 701	11	53
8	9.81 810	14	9.94 120	25	0.05 879	9.87 690	11	52
9	9.81 824	14	9.94 146	25	0.05 854	9.87 679	11	51
10	9.81 839	14	9.94 171	25	0.05 828	9.87 668	11	50
11	9.81 853	14	9.94 197	25	0.05 803	9.87 657	11	49
12	9.81 868	14	9.94 222	25	0.05 777	9.87 645	11	48
13	9.81 882	14	9.94 248	25	0.05 752	9.87 634	11	47
14	9.81 897	14	9.94 273	25	0.05 726	9.87 623	11	46
15	9.81 911	14	9.94 299	25	0.05 701	9.87 612	11	45
16	9.81 925	14	9.94 324	25	0.05 675	9.87 601	11	44
17	9.81 940	14	9.94 350	25	0.05 650	9.87 590	11	43
18	9.81 954	14	9.94 375	25	0.05 625	9.87 579	11	42
19	9.81 969	14	9.94 400	25	0.05 599	9.87 568	11	41
20	9.81 983	14	9.94 426	25	0.05 574	9.87 557	11	40
21	9.81 997	14	9.94 451	25	0.05 548	9.87 546	11	39
22	9.82 012	14	9.94 477	25	0.05 523	9.87 535	11	38
23	9.82 026	14	9.94 502	25	0.05 497	9.87 523	11	37
24	9.82 040	14	9.94 528	25	0.05 472	9.87 512	11	36
25	9.82 055	14	9.94 553	25	0.05 446	9.87 501	11	35
26	9.82 069	14	9.94 579	25	0.05 421	9.87 490	11	34
27	9.82 083	14	9.94 604	25	0.05 395	9.87 479	11	33
28	9.82 098	14	9.94 630	25	0.05 370	9.87 468	11	32
29	9.82 112	14	9.94 655	25	0.05 344	9.87 457	11	31
30	9.82 126	14	9.94 681	25	0.05 319	9.87 445	11	30
31	9.82 140	14	9.94 706	25	0.05 293	9.87 434	11	29
32	9.82 155	14	9.94 732	25	0.05 268	9.87 423	11	28
33	9.82 169	14	9.94 757	25	0.05 243	9.87 412	11	27
34	9.82 183	14	9.94 782	25	0.05 217	9.87 401	11	26
35	9.82 197	14	9.94 808	25	0.05 192	9.87 389	11	25
36	9.82 212	14	9.94 833	25	0.05 166	9.87 378	11	24
37	9.82 226	14	9.94 859	25	0.05 141	9.87 367	11	23
38	9.82 240	14	9.94 884	25	0.05 115	9.87 356	11	22
39	9.82 254	14	9.94 910	25	0.05 090	9.87 345	11	21
40	9.82 269	14	9.94 935	25	0.05 064	9.87 333	11	20
41	9.82 283	14	9.94 961	25	0.05 039	9.87 322	11	19
42	9.82 297	14	9.94 986	25	0.05 014	9.87 311	11	18
43	9.82 311	14	9.95 011	25	0.04 988	9.87 300	11	17
44	9.82 325	14	9.95 037	25	0.04 963	9.87 288	11	16
45	9.82 339	14	9.95 062	25	0.04 937	9.87 277	11	15
46	9.82 354	14	9.95 088	25	0.04 912	9.87 266	11	14
47	9.82 368	14	9.95 113	25	0.04 886	9.87 254	11	13
48	9.82 382	14	9.95 139	25	0.04 861	9.87 243	11	12
49	9.82 396	14	9.95 164	25	0.04 836	9.87 232	11	11
50	9.82 410	14	9.95 189	25	0.04 810	9.87 221	11	10
51	9.82 424	14	9.95 215	25	0.04 785	9.87 209	11	9
52	9.82 438	14	9.95 240	25	0.04 759	9.87 198	11	8
53	9.82 452	14	9.95 266	25	0.04 734	9.87 187	11	7
54	9.82 467	14	9.95 291	25	0.04 708	9.87 175	11	6
55	9.82 481	14	9.95 316	25	0.04 683	9.87 164	11	5
56	9.82 495	14	9.95 342	25	0.04 658	9.87 153	11	4
57	9.82 509	14	9.95 367	25	0.04 632	9.87 141	11	3
58	9.82 523	14	9.95 393	25	0.04 607	9.87 130	11	2
59	9.82 537	14	9.95 418	25	0.04 581	9.87 118	11	1
60	9.82 551	14	9.95 443	25	0.04 556	9.87 107	11	0
	Log. Cos.	d.	Log. Cot.	c. d.	Log. Tan.	Log. Sin.	d.	′

P. P.

	25	25
6	2.5	2.5
7	3.0	2.9
8	3.4	3.3
9	3.8	3.7
10	4.2	4.1
20	8.5	8.3
30	12.7	12.5
40	17.0	16.6
50	21.2	20.8

	14	14
6	1.4	1.4
7	1.7	1.6
8	1.9	1.8
9	2.2	2.1
10	2.4	2.3
20	4.8	4.6
30	7.2	7.0
40	9.6	9.3
50	12.1	11.6

	11	11
6	1.1	1.1
7	1.3	1.3
8	1.5	1.4
9	1.7	1.6
10	1.9	1.8
20	3.8	3.6
30	5.7	5.5
40	7.6	7.3
50	9.6	9.1

48°

TABLE VII.—LOGARITHMIC SINES, COSINES, TANGENTS, AND COTANGENTS.
42°

′	Log. Sin.	d	Log. Tan.	c. d.	Log. Cot.	Log. Cos.	d.			P. P.
0	9.82 551	14	9.95 443̂	25̂	0.04 556̂	9.87 107	1 î	60		
1	9.82 565	14	9.95 469	25̂	0.04 531	9.87 096	1 î	59		
2	9.82 579	14	9.95 494̂	25̂	0.04 505̂	9.87 084̂	1 î	58		
3	9.82 593	14	9.95 520	25̂	0.04 480	9.87 073	1 î	57		
4	9.82 607	14	9.95 545̂	25̂	0.04 454̂	9.87 062	1 1	56		
5	9.82 621	14	9.95 571	25	0.04 429	9.87 050̂	1 î	55		
6	9.82 635	14	9.95 596	25	0.04 404	9.87 039	1 î	54		
7	9.82 649	14	9.95 621̂	25̂	0.04 378	9.87 027̂	1 î	53		
8	9.82 663	14	9.95 647	25̂	0.04 353	9.87 016	1 î	52			25̂	25
9	9.82 677	14	9.95 672̂	25̂	0.04 327̂	9.87 004̂	1 î	51	6	2.5̂	2.5
10	9.82 691	14	9.95 697̂	25̂	0.04 302̂	9.86 993̂	1 1	50	7	3.0	2.9
11	9.82 705	14	·9.95 723̂	25̂	0.04 277	9.86 982	1 î	49	8	3.4	3.3̂
12	9.82 719	14	9.95 748̂	25	0.04 251̂	9.86 970̂	1 î	48	9	3.8	3.7̂
13	9.82 733	13̂	9.95 774	25	0.04 226	9.86 959	1 î	47	10	4.2̂	4.1̂
14	9.82 746̂	14	9.95 799̂	25	0.04 200̂	9.86 947̂	1 î	46	20	8.5	8.3̂
15	9.82 760̂	14	9.95 824̂	25̂	0.04 175̂	9.86 936	1 î	45	30	12.7̂	12.5
16	9.82 774̂	14	9.95 850	25̂	0.04 150	9.86 924̂	1 î	44	40	17.0	16.6̂
17	9.82 788̂	14	9.95 875̂	25̂	0.04 124̂	9.86 913	1 î	43	50	21.2̂	20.8
18	9.82 802̂	13̂	9.95 901	25	0.04 099	9.86 901̂	1 î	42		
19	9.82 816	14	9.95 926	25̂	0.04 074	9.86 890	1 î	41		
20	9.82 830	14	9.95 951̂	25̂	0.04 048̂	9.86 878̂	1 î	40		
21	9.82 844	14	9.95 977	25̂	0.04 023	9.86 867	1 î	39		
22	9.82 858	14	9.96 002̂	25̂	0.03 997̂	9.86 855̂	1 î	38		
23	9.82 871̂	14	9.96 027	25̂	0.03 972̂	9.86 844	1 î	37		
24	9.82 885̂	14	9.96 053	25̂	0.03 947	9.86 832̂	1 î	36		
25	9.82 899̂	13̂	9.96 078̂	25̂	0.03 921̂	9.86 821	1 î	35			14	13̂
26	9.82 913	14	9.96 104	25̂	0.03 896	9.86 809̂	1 î	34	6	1.4	1.3̂
27	9.82 927	13̂	9.96 129	25̂	0.03 871	9.86 798	1 2	33	7	1.6̂	1.6
28	9.82 940̂	14	9.96 154̂	25	0.03 845̂	9.86 786	1 î	32	8	1.8	1.8
29	9.82 954̂	14	9.96 180	25	0.03 820	9.86 774̂	1 î	31	9	2.1	2.0
30	9.82 968̂	13̂	9.96 205	25̂	0.03 795	9.86 763	1 î	30	10	2.3̂	2.2̂
31	9.82 982	14	9.96 230̂	25̂	0.03 769̂	9.86 751̂	1 î	29	20	4.6	4.5
32	9.82 996	13̂	9.96 256	25̂	0.03 744	9.86 740	1 î	28	30	7.0	6.7̂
33	9.83 009̂	14	9.96 281̂	25̂	0.03 718̂	9.86 728̂	1 î	27	40	9.3̂	9.0
34	9.83 023̂	13̂	9.96 306̂	25	0.03 693̂	9.86 716	1 2	26	50	11.6̂	11.2̂
35	9.83 037	14	9.96 332	25̂	0.03 668	9.86 705	1 î	25		
36	9.83 051	13̂	9.96 357	25̂	0.03 642̂	9.86 693̂	1 î	24		
37	9.83 064̂	14	9.96 383	25	0.03 617	9.86 682	1 î	23		
38	9.83 078̂	13̂	9.96 408̂	25̂	0.03 592	9.86 670̂	1 2	22		
39	9.83 092	14	9.96 433̂	25̂	0.03 566̂	9.86 658̂	1 î	21		
40	9.83 106	13̂	9.96 459	25̂	0.03 541	9.86 647	1 î	20		
41	9.83 119̂	13̂	9.96 484	25	0.03 516	9.86 635̂	1 î	19		
42	9.83 133	14	9.96 509̂	25̂	0.03 490̂	9.86 623̂	1 2	18		
43	9.83 147	13̂	9.96 535	25̂	0.03 465	9.86 612	1 î	17			12	1î	11
44	9.83 160̂	13̂	9.96 560	25	0.03 440	9.86 600̂	1 î	16	6	1.2	1.î	1.1
45	9.83 174	14	9.96 585̂	25̂	0.03 414̂	9.86 588̂	1 2	15	7	1.4	1.3̂	1.3
46	9.83 188	13̂	9.96 611	25̂	0.03 389	9.86 577	1 î	14	8	1.6	1.5̂	1.4̂
47	9.83 201̂	13̂	9.96 636	25	0.03 364	9.86 565̂	1 î	13	9	1.8	1.7	1.6
48	9.83 215	13̂	9.96 661̂	25̂	0.03 338̂	9.86 553̂	1 2	12	10	2.0	1.9	1.8̂
49	9.83 229	14	9.96 687	25̂	0.03 313	9.86 542	1 î	11	20	4.0	3.8̂	3.6̂
50	9.83 242̂	13̂	9.96 712̂	25̂	0.03 287̂	9.86 530̂	1 2	10	30	6.0	5.7̂	5.5
51	9.83 256	13̂	9.96 737̂	25̂	0.03 262̂	9.86 518̂	1 î	9	40	8.0	7.6̂	7.3̂
52	9.83 269̂	13̂	9.96 763	25̂	0.03 237	9.86 507	1 î	8	50	10.0	9.6	9.î
53	9.83 283̂	14	9.96 788̂	25̂	0.03 211̂	9.86 495	1 2	7		
54	9.83 297	13̂	9.96 813̂	25	0.03 186̂	9.86 483̂	1 î	6		
55	9.83 310̂	13̂	9.96 839	25̂	0.03 161	9.86 471̂	1 2	5		
56	9.83 324	13̂	9.96 864̂	25̂	0.03 135̂	9.86 460	1 î	4		
57	9.83 337̂	13̂	9.96 889̂	25̂	0.03 110̂	9.86 448	1 2	3		
58	9.83 351	13̂	9.96 915	25̂	0.03 085	9.86 436̂	1 î	2		
59	9.83 365	14	9.96 940̂	25̂	0.03 059̂	9.86 424̂	1 2	1		
60	9.83 378̂	13̂	9.96 965̂	25	0.03 034̂	9.86 412̂	1 2	0		
	Log. Cos.	d.	Log. Cot.	c. d.	Log. Tan.	Log. Sin.	d.	′		P. P.

TABLE VII.—LOGARITHMIC SINES, COSINES, TANGENTS, AND COTANGENTS.
43°

′	Log. Sin.	d.	Log. Tan.	c. d.	Log. Cot.	Log. Cos.	d.	′		P. P.
0	9.83 37ŝ	1ŝ	9.96 96ŝ	2ŝ	0.03 03â	9.86 41ẑ	1î	**60**		
1	9.83 392	1ŝ	9.96 991	25	0.03 009	9.86 401	12	59		
2	9.83 40ŝ	1ŝ	9.97 016	2ŝ	0.02 984	9.86 389	1î	58		
3	9.83 419	1ŝ	9.97 04î	2ŝ	0.02 958	9.86 37ŝ	1î	57		
4	9.83 43ẑ	1ŝ	9.97 067	2ŝ	0.02 933	9.86 36ŝ	1î	56		
5	9.83 446	13	9.97 092	25	0.02 908	9.86 354	12	55		
6	9.83 45ŝ	1ŝ	9.97 11ŝ	2ŝ	0.02 88ẑ	9.86 342	12	54		
7	9.83 473	1ŝ	9.97 143	2ŝ	0.02 857	9.86 330	1î	53		
8	9.83 48ŝ	1ŝ	9.97 168	2ŝ	0.02 832	9.86 31ŝ	12	52		2ŝ 25
9	9.83 500	1ŝ	9.97 19ŝ	2ŝ	0.02 80ŝ	9.86 30ŝ	12	51	6	2.ŝ 2.5
10	9.83 51ŝ	1ŝ	9.97 219	25	0.02 781	9.86 29â	12	**50**	7	3.0 2.9
11	9.83 527	1ŝ	9.97 244	25	0.02 756	9.86 28ẑ	1î	49	8	3.4 3.ŝ
12	9.83 54ŝ	1ŝ	9.97 26ŝ	2ŝ	0.02 73ŝ	9.86 271	12	48	9	3.8 3.ŝ
13	9.83 554	13	9.97 295	25	0.02 705	9.86 259	12	47	10	4.ẑ 4.î
14	9.83 567	13	9.97 320	25	0.02 680	9.86 247	1î	46	20	8.5 8.ŝ
15	9.83 58ŝ	13	9.97 345	2ŝ	0.02 65â	9.86 23ŝ	12	45	30	12.ŝ 12.5
16	9.83 594	1ŝ	9.97 37ŝ	2ŝ	0.02 62ŝ	9.86 22ŝ	12	44	40	17.0 16.6
17	9.83 607	1ŝ	9.97 396	2ŝ	0.02 604	9.86 21î	12	43	50	21.ẑ 20.ŝ
18	9.83 621	1ŝ	9.97 42î	25	0.02 57ŝ	9.86 19ŝ	12	42		
19	9.83 63â	1ŝ	9.97 446	25	0.02 55ŝ	9.86 18ŝ	1î	41		
20	9.83 647	13	9.97 472	2ŝ	0.02 528	9.86 176	12	**40**		
21	9.83 661	1ŝ	9.97 49ŝ	2ŝ	0.02 50ẑ	9.86 164	12	39		
22	9.83 67â	1ŝ	9.97 52ẑ	2ŝ	0.02 47ŝ	9.86 152	12	38		
23	9.83 688	1ŝ	9.97 548	25	0.02 452	9.86 140	12	37		
24	9.83 701	13	9.97 573	25	0.02 427	9.86 128	12	36		
25	9.83 71â	1ŝ	9.97 59ŝ	2ŝ	0.02 40î	9.86 116	12	35		1ŝ 13
26	9.83 728	1ŝ	9.97 624	25	0.02 376	9.86 104	12	34	6	1.ŝ 1.3
27	9.83 74î	1ŝ	9.97 649	2ŝ	0.02 351	9.86 092	12	33	7	1.6 1.5
28	9.83 75â	1ŝ	9.97 67â	2ŝ	0.02 32ŝ	9.86 080	12	32	8	1.8 1.ŝ
29	9.83 768	1ŝ	9.97 69ŝ	2ŝ	0.02 30ŝ	9.86 068	12	31	9	2.0 1.ŝ
30	9.83 781	13	9.97 725	2ŝ	0.02 275	9.86 056	12	**30**	10	2.ẑ 2.î
31	9.83 79â	1ŝ	9.97 75ŝ	25	0.02 24ŝ	9.86 044	12	29	20	4.5 4.ŝ
32	9.83 808	1ŝ	9.97 77ŝ	2ŝ	0.02 22â	9.86 032	12	28	30	6.ŝ 6.5
33	9.83 821	1ŝ	9.97 801	2ŝ	0.02 199	9.86 020	12	27	40	9.0 8.ŝ
34	9 83 83â	13	9.97 826	25	0.02 174	9.86 008	12	26	50	11.ẑ 10.ŝ
35	9.83 847	13	9.97 85î	2ŝ	0.02 14ŝ	9.85 996	12	25		
36	9.83 861	1ŝ	9.97 877	25	0.02 123	9.85 984	12	24		
37	9.83 874	1ŝ	9.97 902	25	0.02 098	9.85 972	12	23		
38	9.83 88ŝ	13	9.97 927	25	0.02 07â	9.85 960	12	22		
39	9.83 90ŝ	1ŝ	9.97 95ẑ	2ŝ	0.02 04ŝ	9.85 948	12	21		
40	9.83 914	13	9.97 978	2ŝ	0.02 022	9.85 936	12	**20**		
41	9.83 927	13	9.98 00ŝ	25	0.01 99ŝ	9.85 924	12	19		
42	9.83 94ŝ	13	9.98 028	25	0.01 97î	9.85 912	12	18		
43	9.83 95ŝ	1ŝ	9.98 054	25	0.01 946	9.85 900	1ŝ	17		1ẑ 12 1î
44	9.83 967	13	9.98 079	25	0.01 921	9.85 887	12	16	6	1.ẑ 1.2 1.î
45	9.83 980	13	9.98 10â	2ŝ	0.01 89ŝ	9.85 87ŝ	12	15	7	1.â 1.4 1.ŝ
46	9.83 993	1ŝ	9.98 12ŝ	2ŝ	0.01 87ŝ	9.85 86ŝ	12	14	8	1.ŝ 1.6 1.ŝ
47	9.84 00ŝ	1ŝ	9.98 155	2ŝ	0.01 845	9.85 85î	12	13	9	1.9 1.8 1.7
48	9.84 01ŝ	1ŝ	9.98 18ŝ	25	0.01 81ŝ	9.85 83ŝ	12	12	10	2.1 2.0 1.9
49	9.84 033	1ŝ	9.98 20ŝ	25	0.01 79â	9.85 827	1î	11	20	4.î 4.0 3.8
50	9.84 046	13	9.98 231	2ŝ	0.01 769	9.85 815	12	**10**	30	6.ẑ 6.0 5.7
51	9.84 059	13	9.98 256	25	0.01 744	9.85 803	12	9	40	8.ŝ 8.0 7.ŝ
52	9.84 072	1ŝ	9.98 28î	2ŝ	0.01 71ŝ	9.85 791	12	8	50	10.4 10.0 9.6
53	9.84 08ŝ	1ŝ	9.98 30ŝ	25	0.01 693	9.85 77ŝ	1ẑ	7		
54	9.84 09ŝ	13	9.98 332	2ŝ	0.01 668	9.85 766	12	6		
55	9.84 11î	13	9.98 357	2ŝ	0.01 64â	9.85 75â	12	5		
56	9.84 12â	1ŝ	9.98 38ẑ	2ŝ	0.01 617	9.85 742	1ẑ	4		
57	9.84 138	1ŝ	9.98 408	2ŝ	0.01 592	9.85 730	12	3		
58	9.84 151	13	9.98 433	2ŝ	0.01 56ŝ	9.85 718	12	2		
59	9.84 164	13	9.98 45ŝ	2ŝ	0.01 54î	9.85 70ŝ	1ẑ	1		
60	9.84 177	13	9.98 48ŝ	25	0.01 51ŝ	9.85 69ŝ	12	**0**		
	Log. Cos.	d.	Log. Cot.	c. d.	Log. Tan.	Log. Sin.	d.	′		P. P.

TABLE VII.—LOGARITHMIC SINES, COSINES, TANGENTS, AND COTANGENTS.
44°

′	Log. Sin.	d.	Log. Tan.	c. d.	Log. Cot.	Log. Cos.	d.	′
0	9.84 177	13	9.98 483̂	25	0.01 516̂	9.85 693̂	12̂	60
1	9.84 190	13̂	9.98 509	2̂5	0.01 491	9.85 681	12	59
2	9.84 203̂	13	9.98 534̂	25	0.01 465̂	9.85 669	12	58
3	9.84 216̂	13̂	9.98 559̂	2̂5	0.01 440̂	9.85 657	12	57
4	9.84 229̂	13	9.98 585	25	0.01 415	9.85 644̂	12	56
5	9.84 242̂	13	9.98 610	2̂5	0.01 390	9.85 632̂	12	55
6	9.84 255̂	13	9.98 635̂	2̂5	0.01 364̂	9.85 620	12	54
7	9.84 268̂	13	9.98 660̂	25	0.01 339̂	9.85 608	12̂	53
8	9.84 281̂	13	9.98 686	25	0.01 314	9.85 595̂	12̂	52
9	9.84 294̂	13	9.98 711	2̂5	0.01 289	9.85 583̂	12	51
10	9.84 307̂	13	9.98 736̂	2̂5	0.01 263̂	9.85 571	12	50
11	9.84 320̂	13	9.98 762	25	0.01 238	9.85 559	12̂	49
12	9.84 333̂	13	9.98 787̂	25	0.01 213	9.85 546̂	12	48
13	9.84 346̂	13	9.98 812̂	2̂5	0.01 187̂	9.85 534̂	12̂	47
14	9.84 359̂	13	9.98 837̂	25	0.01 162̂	9.85 522	12	46
15	9.84 372̂	13	9.98 863	2̂5	0.01 137	9.85 509̂	12	45
16	9.84 385̂	13	9.98 888	25	0.01 112	9.85 497	12	44
17	9.84 398̂	13	9.98 913̂	2̂5	0.01 086̂	9.85 485	12̂	43
18	9.84 411̂	13	9.98 938̂	25	0.01 061̂	9.85 472̂	12	42
19	9.84 424̂	12̂	9.98 964	2̂5	0.01 036	9.85 460	12̂	41
20	9.84 437	13	9.98 989̂	2̂5	0.01 010̂	9.85 448	12̂	40
21	9.84 450	13	9.99 014̂	25	0.00 985̂	9.85 435̂	12	39
22	9.84 463	13	9.99 040	2̂5	0.00 960	9.85 423̂	12	38
23	9.84 476	13	9.99 065	25	0.00 935	9.85 411	12	37
24	9.84 489	13	9.99 090̂	2̂5	0.00 909̂	9.85 398	12	36
25	9.84 502	12̂	9.99 115̂	25	0.00 884̂	9.85 386	12	35
26	9.84 514̂	13	9.99 141	2̂5	0.00 859	9.85 374	12̂	34
27	9.84 527̂	13	9.99 166	25	0.00 834	9.85 361̂	12̂	33
28	9.84 540̂	13	9.99 191̂	2̂5	0.00 808̂	9.85 349	12̂	32
29	9.84 553̂	12̂	9.99 216̂	2̂5	0.00 783̂	9.85 336	12	31
30	9.84 566	13	9.99 242	25	0.00 758	9.85 324	12	30
31	9.84 579	13	9.99 267	2̂5	0.00 733	9.85 312	12̂	29
32	9.84 592	12̂	9.99 292̂	25	0.00 707̂	9.85 299̂	12̂	28
33	9.84 604̂	13	9.99 318	25	0.00 682	9.85 287	12̂	27
34	9.84 617̂	13	9.99 343	25	0.00 657	9.85 274̂	12̂	26
35	9.84 630̂	12̂	9.99 368	25	0.00 631̂	9.85 262	12̂	25
36	9.84 643	13	9.99 393	2̂5	0.00 606̂	9.85 249̂	12̂	24
37	9.84 656	13	9.99 419	25	0.00 581	9.85 237	12̂	23
38	9.84 669	12̂	9.99 444	2̂5	0.00 556	9.85 224̂	12̂	22
39	9.84 681̂	13	9.99 469̂	25	0.00 530̂	9.85 212	12̂	21
40	9.84 694̂	12̂	9.99 494̂	2̂5	0.00 505̂	9.85 199̂	12̂	20
41	9.84 707	13	9.99 520	25	0.00 480	9.85 187	12̂	19
42	9.84 720	12̂	9.99 545	2̂5	0.00 455	9.85 174̂	12̂	18
43	9.84 732̂	13	9.99 570̂	2̂5	0.00 429̂	9.85 162	12̂	17
44	9.84 745̂	12̂	9.99 595̂	25	0.00 404̂	9.85 149̂	12̂	16
45	9.84 758	13	9.99 621	2̂5	0.00 379	9.85 137	12̂	15
46	9.84 771	12̂	9.99 646	2̂5	0.00 353̂	9.85 124̂	12̂	14
47	9.84 783̂	13	9.99 671̂	25	0.00 328̂	9.85 112	12̂	13
48	9.84 796̂	12̂	9.99 697	2̂5	0.00 303	9.85 099̂	12̂	12
49	9.84 809	13	9.99 722	2̂5	0.00 278	9.85 087	12̂	11
50	9.84 822	12̂	9.99 747̂	25	0.00 252̂	9.85 074̂	12̂	10
51	9.84 834̂	12̂	9.99 772̂	25	0.00 227̂	9.85 062	12̂	9
52	9.84 847	13	9.99 798	2̂5	0.00 202	9.85 049̂	12̂	8
53	9.84 860	12̂	9.99 823	2̂5	0.00 177	9.85 037	13	7
54	9.84 872̂	12̂	9.99 848̂	25	0.00 151̂	9.85 024	12̂	6
55	9.84 885	13	9.99 873̂	2̂5	0.00 126̂	9.85 011̂	12̂	5
56	9.84 898	12̂	9.99 899	25	0.00 101	9.84 999	12̂	4
57	9.84 910̂	13	9.99 924	2̂5	0.00 076	9.84 986̂	12̂	3
58	9.84 923	13	9.99 949̂	2̂5	0.00 050̂	9.84 974	13	2
59	9.84 936̂	12̂	9.99 974̂	25	0.00 025̂	9.84 961̂	12̂	1
60	9.84 948̂		0.00 000	2̂5	0.00 000	9.84 948̂		0
	Log. Cos.	d.	Log. Cot.	c. d.	Log. Tan.	Log. Sin.	d.	′

P. P.

	25̂	25
6	2.5	2.5
7	3.0	2.9
8	3.4	3.3̂
9	3.8	3.7̂
10	4.2̂	4.1̂
20	8.5	8.3̂
30	12.7̂	12.5
40	17.0	16.6̂
50	21.2̂	20.8̂

	13̂	13
6	1.3̂	1.3
7	1.6	1.5
8	1.8	1.7̂
9	2.0	1.9̂
10	2.2̂	2.1̂
20	4.5	4.3̂
30	6.7̂	6.5
40	9.0	8.6̂
50	11.2̂	10.8̂

	12̂	12
6	1.2̂	1.2
7	1.4̂	1.4
8	1.6̂	1.6
9	1.9	1.8
10	2.1̂	2.0
20	4.1̂	4.0
30	6.2̂	6.0
40	8.3̂	8.0
50	10.4̂	10.0

45°

TABLE VIII.

LOGARITHMIC VERSED SINES AND EXTERNAL SECANTS.

TABLE VIII.—LOGARITHMIC VERSED SINES AND EXTERNAL SECANTS.

0°　　　　　　　1°

'	Log. Vers.	D	Log. Exsec.	D	Log. Vers.	D	Log. Exsec.	D	'
0	−∞		−∞		6.1827î	1435	6.18278	1436	0
I	2.62642	60206	2.62642	60206	.19707	1412	.19714	1412	I
2	3.22848	35218	3.22848	35218	.2111ĵ	138ô	.21126	1390	2
3	3.5806ĵ	24987	3.5806ĵ	24987	.22509	1368	.22516	1368	3
4	3.83054	19382	3.83054	1938²	.23877	1346	.2388â	1347	4
5	4.02436	15836	4.02436	15836	6.2522ß	1326	6.2523î	1326	5
6	.1827²	13389	.1827²	13389	.2654ĵ	1306	.26557	1306	6
7	.31662	11598	.31662	1159ß	.27856	128ß	.27864	1287	7
8	.43260	1023ô	.4326ô	1023ô	.2914²	1268	.29151	1268	8
9	.5349ô	915î	.53491	9151	.3041ô	1250	.3041ĵ	1250	9
10	4.62642	8278	4.62642	8279	6.3166ĵ	1232	6.3166ĵ	1232	10
II	.7092ô	7558	.70921	7557	.3289²	1214	.3290î	1215	II
12	.78478	6953	.78478	695²	.34107	1198	.3411ĵ	1198	12
13	.85431	6437	.85431	6437	.35305	1182	.35315	1182	13
14	.91868	5992	.91868	5993	.36487	1166	.36497	1166	14
15	4.9786ô	560ß	4.97861	560ß	6.37653	1150	6.37663	1151	15
16	5.0346ĵ	5266	5.0346ĵ	5266	.38803	113ß	.38814	1135	16
17	.08732	496â	.0873²	496â	.3993ß	1121	.3994ĵ	1121	17
18	.1369ĵ	4696	.13697	4696	.4105ĵ	1106	.4107ô	1106	18
19	.18393	4455	.18393	4456	.4216ß	1093	.42177	1093	19
20	5.22848	4238	5.22849	4238	6.4325ß	107ß	6.43270	1079	20
21	.27086	404ô	.27087	404ô	.44337	1066	.4434ĵ	1066	21
22	.3112ĵ	3861	.3112ĵ	386î	.45403	105²	.4541ß	1053	22
23	.34987	3697	.3498ß	3697	.46455	104ô	.4646ß	1040	23
24	.3868â	3545	.3868ß	3545ß	.47496	1028	.47509	1028	24
25	5.42230	340ß	5.42231	3407	6.48524	101ß	6.48537	1016	25
26	.4563ß	3278	.45638	3278	.4953ĵ	1004	.49553	1004	26
27	.48915	315ß	.48916	3159	.50544	99²	.5055ĵ	993	27
28	.5207ß	3048	.52075	3048	.5153ß	98î	.5155ô	982	28
29	.5512î	2944	.55123	2945	.52518	970	.5253²	976ô	29
30	5.58066	2848	5.58068	2848	6.53488	960	6.53503	960	30
31	.6091â	2757	.60916	2758	.54448	949	.54463	950	31
32	.63672	267²	.63674	267²	.55397	939	.55413	939	32
33	.66344	2593	.66346	2593	.56336	929	.56352	929	33
34	.68937	2518	.68940	251ĵ	.57265	919	.5728î	919ô	34
35	5.7145ß	2447	5.7145ĵ	2447	6.58184	90ß	6.58201	90ĵ	35
36	.7390²	2379	.7390â	2380	.5909ß	900	.5911ô	90ß	36
37	.76282	231ß	.7628â	231ß	.5999ß	891	.60011	891	37
38	.7859ß	2256	.78601	2256	.60884	882	.60902	882	38
39	.80854	2199	.80857	2199	.61766	87²	.61784	873â	39
40	5.8305ß	2145	5.8305ß	2145	6.62639	864	6.62657	864	40
41	.8519ß	2093	.8520î	209ß	.6350ß	85ß	.63522	856	41
42	.8729î	2044	.87295	2043ß	.64359	847	.64378	848	42
43	.89335	1996	.89338	1997	.6520ß	839	.65226	839ô	43
44	.91332	1952	.9133ß	195²	.6604ß	831	.6606ß	83î	44
45	5.93284	1909	5.93288	1909	6.6687ß	82ß	6.66897	82ß	45
46	.95193	1868	.95197	1868	.67700	81ß	.6772ô	816	46
47	.97061	1829	.97065	1829	.6851ß	808	.68536	80ß	47
48	5.98890	1790	5.98894	1791	.6932ß	80ô	.69345	800	48
49	6.0068ô	1755	6.00685	1755	.70124	793	.7014ß	794	49
50	6.0243ß	1720	6.02440	1720	6.70917	786	6.70939	786	50
51	.0415ß	168ß	.04160	1687	.71703	779	.7172ß	779ô	51
52	.05842	1654	.05847	1654	.72482	772	.72505	772	52
53	.0749ß	162ß	.0750î	162ß	.73254	765	.73277	765ß	53
54	.09120	1594	.09125	1594â	.74019	758ß	.74043	759	54
55	6.10714	1565	6.1071ĵ	1565	6.74777	752	6.74802	75²	55
56	.12279	1537	.1228â	1537	.7552ĵ	745ß	.7555â	746	56
57	.13816	1511	.13822	1511	.76275	739ô	.7630ô	739ô	57
58	.15327	148â	.15333	1485	.7701â	733	.77040	733	58
59	.1681î	1460	.16818	1460	.77747	72ß	.7777ß	727	59
60	6.1827î		6.18278		6.78474		6.78506		60
'	Log. Vers.	D	Log. Exsec.	D	Log. Vers.	D	Log. Exsec.	D	'

TABLE VIII.—LOGARITHMIC VERSED SINES AND EXTERNAL SECANTS.

2°   3°

′	Log. Vers.	D	Log. Exsec.	D	Log. Vers.	D	Log. Exsec.	D	′
0	6.78474	721	6.78500	721	7.13687	481	7.13746	481	0
1	.79195	714	.79221	715	.14168	478	.14228	479	1
2	.79909	709	.79937	709	.14646	475	.14707	476	2
3	.80618	703	.80646	703	.15122	473	.15183	474	3
4	.81322	697	.81350	698	.15595	470	.15657	471	4
5	6.82019	692	6.82048	692	7.16066	468	7.16129	469	5
6	.82711	686	.82740	687	.16534	466	.16598	466	6
7	.83398	681	.83427	682	.17000	463	.17064	464	7
8	.84079	676	.84100	676	.17463	460	.17528	461	8
9	.84755	670	.84785	670	.17923	458	.17989	459	9
10	6.85425	666	6.85457	667	7.18382	455	7.18448	456	10
11	.86091	660	.86123	660	.18837	453	.18905	454	11
12	.86751	656	.86783	656	.19291	451	.19359	452	12
13	.87407	650	.87439	651	.19742	448	.19811	449	13
14	.88057	646	.88090	646	.20191	446	.20260	447	14
15	6.88703	641	6.88737	641	7.20637	444	7.20707	445	15
16	.89344	636	.89378	636	.21081	442	.21152	442	16
17	.89980	631	.90015	632	.21523	440	.21595	440	17
18	.90612	627	.90647	628	.21963	437	.22035	438	18
19	.91239	622	.91275	623	.22400	435	.22473	436	19
20	6.91862	618	6.91898	618	7.22836	433	7.22909	434	20
21	.92480	613	.92516	614	.23269	431	.23343	431	21
22	.93093	609	.93131	610	.23700	429	.23775	429	22
23	.93703	605	.93741	605	.24129	426	.24204	427	23
24	.94308	601	.94346	601	.24555	424	.24632	425	24
25	6.94909	597	6.94948	597	7.24980	422	7.25057	423	25
26	.95506	592	.95545	593	.25402	420	.25480	421	26
27	.96099	589	.96139	589	.25823	418	.25902	419	27
28	.96688	584	.96728	585	.26241	416	.26321	417	28
29	.97272	581	.97313	581	.26658	414	.26738	415	29
30	6.97853	577	6.97895	577	7.27072	412	7.27153	413	30
31	.98430	573	.98472	574	.27485	410	.27567	411	31
32	.99004	569	.99046	570	.27895	409	.27978	409	32
33	6.99573	565	6.99616	566	.28304	406	.28387	407	33
34	7.00139	562	7.00182	563	.28711	405	.28795	405	34
35	7.00701	558	7.00745	559	7.29116	402	7.29200	404	35
36	.01259	555	.01304	555	.29518	401	.29604	402	36
37	.01814	551	.01860	552	.29919	399	.30006	400	37
38	.02366	548	.02412	548	.30319	397	.30406	398	38
39	.02914	544	.02960	545	.30716	395	.30804	396	39
40	7.03458	541	7.03505	541	7.31112	393	7.31201	394	40
41	.03999	537	.04047	538	.31505	392	.31595	393	41
42	.04537	534	.04585	535	.31897	390	.31988	391	42
43	.05071	531	.05120	535	.32288	388	.32379	389	43
44	.05603	527	.05652	528	.32676	386	.32768	388	44
45	7.06130	525	7.06180	525	7.33063	385	7.33156	385	45
46	.06655	521	.06706	522	.33448	383	.33542	384	46
47	.07177	518	.07228	519	.33831	382	.33926	382	47
48	.07695	515	.07747	516	.34213	380	.34309	380	48
49	.08211	512	.08263	513	.34593	378	.34689	379	49
50	7.08723	509	7.08776	509	7.34971	377	7.35069	377	50
51	.09232	506	.09286	507	.35348	375	.35446	376	51
52	.09739	503	.09793	503	.35723	373	.35822	374	52
53	.10242	500	.10297	501	.36097	371	.36196	373	53
54	.10743	497	.10798	498	.36468	370	.36569	371	54
55	7.11240	495	7.11297	495	7.36839	368	7.36940	369	55
56	.11735	492	.11792	493	.37207	367	.37310	368	56
57	.12227	489	.12285	490	.37574	366	.37678	366	57
58	.12716	486	.12775	487	.37940	364	.38044	365	58
59	.13203	484	.13262	484	.38304	362	.38409	363	59
60	7.13687		7.13746		7.38667		7.38773		60
′	Log. Vers.	D	Log. Exsec.	D	Log. Vers.	D	Log. Exsec.	D	′

TABLE VIII.—LOGARITHMIC VERSED SINES AND EXTERNAL SECANTS.

4° 5°

′	Log. Vers.	D	Log. Exsec.	D	Log. Vers.	D	Log. Exsec.	D	′
0	7.38667	361	7.38773	36î	7.58039	289	7.58204	290	0
1	.39028	35ô	.39134	36ô	.58328	28ŷ	.58494	289	1
2	.39387	358	.39495	359	.58615	287	.58783	288	2
3	.39745	356	.39854	357	.58902	286	.59071	287	3
4	.40102	35ô	.4021î	356	.59188	285	.59358	28ô	4
5	7.40457	35ŝ	7.40567	354	7.59473	284	7.59645	285	5
6	.40816	352	.40922	353	.59758	283	.59930	284	6
7	.41163	35ô	.41275	352	.60041	28ŝ	.60214	28ŝ	7
8	.41513	349	.41627	350	.60323	281	.60498	28ŝ	8
9	.41863	348	.41977	349	.60604	28ô	.60780	28î	9
10	7.42211	346	7.42326	347	7.60885	280	7.61062	28ô	10
11	.42557	345	.42673	346	.61164	279	.61342	280	11
12	.42903	343	.43019	345	.61443	277	.61622	279	12
13	.43246	342	.43364	343	.61721	277	.61901	278	13
14	.43589	34î	.43708	342	.61998	276	.62179	277	14
15	7.43936	339	7.44050	34ô	7.62274	275	7.62456	276	15
16	.44270	338	.44390	339	.62549	274	.62733	275	16
17	.44608	337	.44730	338	.62823	273	.63008	274	17
18	.44946	335	.45068	337	.63096	27ŝ	.63282	27â	18
19	.45281	334	.45405	335	.63369	27ŝ	.63556	274	19
20	7.45616	333	7.45740	33ŝ	7.63641	272	7.63829	273	20
21	.45949	332	.46075	33â	.63911	27ô	.64101	272	21
22	.46281	33ô	.46407	33ŝ	.64181	270	.64372	271	22
23	.46612	329	.46739	332	.64451	269	.64643	27ô	23
24	.46941	328	.47070	33ô	.64719	268	.64912	269	24
25	7.47270	327	7.47399	329	7.64986	267	7.65181	269	25
26	.47597	32ŝ	.47727	328	.65253	26ŝ	.65449	268	26
27	.47922	32â	.48054	327	.65519	266	.65716	267	27
28	.48247	32ŝ	.48379	32ŝ	.65784	265	.65982	266	28
29	.48570	323	.48703	324	.66048	26â	.66247	26ŝ	29
30	7.48892	322	7.49026	323	7.66311	263	7.66512	264	30
31	.49213	321	.49348	322	.66574	263	.66776	264	31
32	.49533	320	.49669	321	.66836	26î	.67039	263	32
33	.49852	31ŝ	.49989	31ŷ	.67097	261	.67301	262	33
34	.50169	31ŷ	.50307	31ŝ	.67357	26ô	.67562	26î	34
35	7.50485	316	7.50624	317	7.67617	25ŷ	7.67823	261	35
36	.50800	315	.50941	31ô	.67875	258	.68083	260	36
37	.51114	314	.51256	315	.68133	258	.68342	259	37
38	.51427	313	.51569	313	.68390	257	.68601	25ŝ	38
39	.51739	31î	.51882	313	.68647	256	.68858	257	39
40	7.52050	31î	7.52194	31î	7.68902	25ŝ	7.69115	257	40
41	.52359	30ŷ	.52504	31ô	.69157	255	.69371	256	41
42	.52667	308	.52814	30ŷ	.69411	254	.69627	25ŝ	42
43	.52975	307	.53122	308	.69665	253	.69881	254	43
44	.53281	306	.53429	307	.69917	25ŝ	.70135	254	44
45	7.53586	30ŝ	7.53735	306	7.70169	252	7.70388	253	45
46	.53890	304	.54041	30ŝ	.70421	25î	.70641	25ŝ	46
47	.54193	303	.54345	304	.70671	250	.70893	252	47
48	.54495	302	.54648	303	.70921	250	.71144	251	48
49	.54796	30ô	.54950	302	.71170	249	.71394	250	49
50	7.55096	300	7.55251	301	7.71418	248	7.71644	250	50
51	.55395	299	.55556	29ŷ	.71666	247	.71892	24ŝ	51
52	.55692	29ŷ	.55849	299	.71913	247	.72141	24ŷ	52
53	.55989	297	.56147	298	.72159	246	.72388	24ŝ	53
54	.56285	295	.56444	29ô	.72404	24ŝ	.72635	246	54
55	7.56580	295	7.56740	296	7.72649	245	7.72881	245	55
56	.56873	29ŝ	.57035	295	.72893	24ŝ	.73126	245	56
57	.57166	292	.57329	294	.73137	24â	.73371	24ŝ	57
58	.57458	29ô	.57621	29ŝ	.73379	24ŝ	.73615	24â	58
59	.57749	290	.57913	291	.73621	24î	.73859	24ŝ	59
60	7.58039		7.58204		7.73863		7.74101		60

Bottom column labels: ′ | Log. Vers. | D | Log. Exsec. | D | Log. Vers. | D | Log. Exsec. | D | ′

P. P.

	360	350	340
6	36.0	35.0	34.0
7	42.0	40.8	39.6
8	48.0	46.6	45.3
9	54.0	51.5	51.0
10	60.0	58.3	56.6
20	120.0	116.6	113.3
30	180.0	175.0	170.0
40	240.0	233.3	226.6
50	300.0	291.6	283.3

	330	320	310
6	33.0	32.0	31.0
7	38.5	37.3	36.1
8	44.0	42.6	41.3
9	49.5	48.0	46.5
10	55.0	53.3	51.6
20	110.0	106.6	103.3
30	165.0	160.0	155.0
40	220.0	213.3	206.6
50	275.0	266.6	258.3

	300	290	280
6	30.0	29.0	28.0
7	35.0	33.8	32.6
8	40.0	38.6	37.3
9	45.0	43.5	42.0
10	50.0	48.3	46.6
20	100.0	96.6	93.3
30	150.0	145.0	140.0
40	200.0	193.3	186.6
50	250.0	241.6	233.3

	270	260	250
6	27.0	26.0	25.0
7	31.5	30.3	29.1
8	36.0	34.6	33.3
9	40.5	39.0	37.5
10	45.0	43.3	41.6
20	90.0	87.6	83.3
30	135.0	130.0	125.0
40	180.0	173.3	166.6
50	225.0	216.6	208.3

	240	230	220
6	24.0	23.0	22.0
7	28.0	26.8	25.6
8	32.0	30.6	29.3
9	36.0	34.5	33.0
10	40.0	38.3	36.6
20	80.0	76.6	73.3
30	120.0	115.0	110.0
40	160.0	153.3	146.6
50	200.0	191.6	183.3

	210	200	190
6	21.0	20.0	19.0
7	24.5	23.3	22.1
8	28.0	26.6	25.3
9	31.5	30.0	28.5
10	35.0	33.3	31.6
20	70.0	66.6	63.3
30	105.0	100.0	95.0
40	140.0	133.3	126.6
50	175.0	166.6	158.3

TABLE VIII.—LOGARITHMIC VERSED SINES AND EXTERNAL SECANTS.

6°   7°

′	Log. Vers.	D	Log. Exsec.	D	Log. Vers.	D	Log. Exsec.	D	′
0	7.73863	241	7.74101	242	7.87238	206	7.87563	208	0
1	.74104	240	.74343	241	.87444	205	.87771	207	1
2	.74344	239	.74585	241	.87650	205	.87978	207	2
3	.74583	239	.74826	241	.87855	204	.88185	207	3
4	.74822	238	.75066	240	.88060	204	.88391	206	4
5	7.75060	237	7.75305	239	7.88264	204	7.88597	206	5
6	.75297	236	.75544	239	.88468	204	.88803	205	6
7	.75534	236	.75782	238	.88672	203	.89008	205	7
8	.75770	235	.76019	237	.88875	203	.89212	204	8
9	.76006	235	.76256	237	.89077	202	.89416	204	9
10	7.76240	234	7.76492	236	7.89279	202	7.89620	203	10
11	.76475	234	.76728	235	.89481	201	.89823	203	11
12	.76708	233	.76963	235	.89682	201	.90025	202	12
13	.76941	233	.77197	234	.89882	200	.90228	202	13
14	.77173	232	.77431	233	.90082	200	.90426	201	14
15	7.77405	232	7.77664	233	7.90282	199	7.90630	201	15
16	.77636	231	.77897	232	.90481	199	.90831	201	16
17	.77867	230	.78128	231	.90680	198	.91032	200	17
18	.78097	230	.78360	231	.90878	198	.91231	199	18
19	.78326	229	.78590	230	.91076	197	.91431	199	19
20	7.78554	228	7.78820	230	7.91273	197	7.91630	199	20
21	.78783	228	.79050	229	.91470	197	.91828	198	21
22	.79010	227	.79279	229	.91667	196	.92027	198	22
23	.79237	227	.79507	228	.91863	196	.92224	197	23
24	.79463	226	.79735	228	.92058	195	.92421	197	24
25	7.79689	225	7.79962	227	7.92253	195	7.92618	197	25
26	.79914	225	.80188	226	.92448	195	.92815	196	26
27	.80138	224	.80414	226	.92642	194	.93010	195	27
28	.80362	224	.80639	225	.92836	194	.93206	195	28
29	.80586	223	.80864	225	.93029	193	.93401	195	29
30	7.80808	222	7.81088	224	7.93222	193	7.93596	195	30
31	.81031	222	.81312	224	.93415	192	.93790	194	31
32	.81253	221	.81535	223	.93607	192	.93984	194	32
33	.81473	221	.81758	222	.93799	191	.94177	193	33
34	.81694	220	.81980	222	.93991	191	.94370	193	34
35	7.81914	220	7.82201	221	7.94181	190	7.94562	192	35
36	.82133	219	.82422	221	.94371	190	.94754	192	36
37	.82352	219	.82642	220	.94561	190	.94946	192	37
38	.82570	218	.82862	219	.94751	189	.95137	191	38
39	.82788	217	.83081	219	.94940	189	.95328	191	39
40	7.83005	217	7.83300	219	7.95129	189	7.95519	190	40
41	.83222	217	.83518	218	.95317	188	.95709	190	41
42	.83438	216	.83735	217	.95505	187	.95898	189	42
43	.83653	215	.83952	217	.95693	188	.96088	189	43
44	.83868	215	.84169	216	.95880	187	.96277	188	44
45	7.84083	214	7.84385	216	7.96066	186	7.96465	188	45
46	.84297	214	.84600	215	.96253	186	.96653	188	46
47	.84510	213	.84815	215	.96439	186	.96841	188	47
48	.84723	213	.85030	214	.96624	185	.97028	187	48
49	.84935	212	.85243	213	.96809	185	.97215	187	49
50	7.85147	212	7.85457	213	7.96994	184	7.97401	186	50
51	.85359	211	.85670	213	.97178	184	.97587	186	51
52	.85570	211	.85882	212	.97362	184	.97773	185	52
53	.85780	210	.86094	211	.97546	183	.97958	185	53
54	.85990	210	.86305	211	.97729	183	.98143	184	54
55	7.86199	209	7.86516	210	7.97912	182	7.98327	184	55
56	.86408	209	.86726	210	.98094	182	.98512	183	56
57	.86616	208	.86936	209	.98276	182	.98695	183	57
58	.86824	208	.87146	208	.98458	181	.98879	183	58
59	.87031	207	.87354	208	.98639	181	.99062	182	59
60	7.87238	206	7.87563	208	7.98820		7.99244	182	60
′	Log. Vers.	D	Log. Exsec.	D	Log. Vers.	D	Log. Exsec.	D	′

P. P.

	180	9	9
6	18.0	0.9	0.9
7	21.0	1.1	1.0
8	24.0	1.2	1.2
9	27.0	1.4	1.3
10	30.0	1.6	1.5
20	60.0	3.1	3.0
30	90.0	4.7	4.5
40	120.0	6.3	6.0
50	150.0	7.9	7.5

	8	8	7
6	0.8	0.8	0.7
7	1.0	1.0	1.0
8	1.1	1.0	1.0
9	1.3	1.2	1.1
10	1.4	1.3	1.2
20	2.8	2.6	2.5
30	4.2	4.0	3.7
40	5.6	5.3	5.0
50	7.1	6.6	6.2

	7	6	6
6	0.7	0.6	0.6
7	0.8	0.7	0.7
9	0.9	0.8	0.8
10	1.1	1.1	1.0
20	2.3	2.1	2.0
30	3.5	3.2	3.0
40	4.6	4.3	4.0
50	5.8	5.4	5.0

	5	5	4
6	0.5	0.5	0.4
7	0.5	0.6	0.4
8	0.7	0.6	0.6
9	0.8	0.7	0.7
10	0.9	0.8	0.7
20	1.8	1.6	1.5
30	2.7	2.5	2.2
40	3.6	3.3	3.0
50	4.6	4.1	3.7

	4	3	3
6	0.4	0.3	0.3
7	0.4	0.4	0.3
8	0.5	0.4	0.4
9	0.6	0.5	0.4
10	0.6	0.6	0.5
20	1.3	1.1	1.0
30	2.0	1.7	1.5
40	2.6	2.3	2.0
50	3.3	2.9	2.5

	2	2	1
6	0.2	0.2	0.1
7	0.3	0.2	0.2
8	0.3	0.2	0.2
9	0.4	0.3	0.2
10	0.4	0.3	0.2
20	0.8	0.6	0.5
30	1.2	1.0	0.7
50	2.1	1.6	1.2

	1	0
6	0.1	0.0
7	0.1	0.0
8	0.1	0.0
9	0.1	0.1
10	0.1	0.1
20	0.3	0.1
30	0.5	0.2
40	0.6	0.3
50	0.8	0.4

TABLE VIII.—LOGARITHMIC VERSED SINES AND EXTERNAL SECANTS.

8°   9°

′	Log. Vers.	D	Log. Exsec.	D	Log. Vers.	D	Log. Exsec.	D	′
0	7.98820	180	7.99244	182	8.09031	160	8.09569	162	0
1	.99000	180	.99427	182	.09192	160	.09732	162	1
2	.99180	179	.99609	181	.09352	160	.09894	162	2
3	.99360	179	.99790	181	.09512	159	.10056	161	3
4	.99539	179	7.99971	180	.09671	159	.10217	161	4
5	7.99718	178	8.00152	180	8.09830	159	8.10378	161	5
6	7.99897	178	.00332	180	.09989	158	.10539	160	6
7	8.00075	178	.00512	180	.10148	158	.10700	160	7
8	.00253	177	.00692	179	.10306	158	.10860	160	8
9	.00431	178	.00871	179	.10464	158	.11020	160	9
10	8.00608	177	8.01050	179	8.10622	157	8.11180	159	10
11	.00784	176	.01229	178	.10779	157	.11340	159	11
12	.00961	176	.01407	178	.10936	157	.11499	159	12
13	.01137	176	.01585	178	.11093	157	.11658	159	13
14	.01313	175	.01763	177	.11250	156	.11816	158	14
15	8.01488	175	8.01940	177	8.11406	156	8.11975	158	15
16	.01663	175	.02117	177	.11562	156	.12133	158	16
17	.01838	174	.02293	176	.11718	155	.12291	157	17
18	.02012	174	.02469	176	.11873	155	.12448	157	18
19	.02186	173	.02645	175	.12029	155	.12605	157	19
20	8.02359	173	8.02820	175	8.12184	155	8.12762	157	20
21	.02533	173	.02995	175	.12338	154	.12919	157	21
22	.02706	172	.03170	175	.12492	154	.13075	156	22
23	.02878	172	.03345	174	.12647	154	.13232	156	23
24	.03050	172	.03519	174	.12800	153	.13387	155	24
25	8.03222	171	8.03692	173	8.12954	153	8.13543	155	25
26	.03394	171	.03866	173	.13107	153	.13698	155	26
27	.03565	171	.04039	173	.13260	153	.13854	154	27
28	.03736	170	.04212	173	.13413	152	.14008	154	28
29	.03906	170	.04384	172	.13565	152	.14163	154	29
30	8.04076	170	8.04556	172	8.13717	152	8.14317	154	30
31	.04246	170	.04728	171	.13869	152	.14471	154	31
32	.04416	169	.04899	171	.14021	151	.14625	153	32
33	.04585	169	.05070	171	.14172	151	.14778	153	33
34	.04754	169	.05241	170	.14323	151	.14932	153	34
35	8.04922	168	8.05411	170	8.14474	151	8.15085	153	35
36	.05090	168	.05581	170	.14625	150	.15237	152	36
37	.05258	168	.05751	170	.14775	150	.15390	152	37
38	.05426	167	.05921	169	.14925	150	.15542	152	38
39	.05593	167	.06090	169	.15075	149	.15694	152	39
40	8.05760	167	8.06259	169	8.15225	150	8.15846	152	40
41	.05926	166	.06427	168	.15374	149	.15997	151	41
42	.06093	166	.06595	168	.15523	149	.16148	151	42
43	.06259	166	.06763	168	.15672	149	.16299	151	43
44	.06424	165	.06931	167	.15820	148	.16450	150	44
45	8.06589	165	8.07098	167	8.15968	148	8.16600	150	45
46	.06754	165	.07265	167	.16116	148	.16750	150	46
47	.06919	164	.07431	166	.16264	148	.16900	150	47
48	.07083	164	.07598	166	.16412	147	.17050	149	48
49	.07247	164	.07764	166	.16559	147	.17199	149	49
50	8.07411	164	8.07929	165	8.16706	147	8.17349	149	50
51	.07575	163	.08095	165	.16852	146	.17497	148	51
52	.07738	163	.08260	165	.16999	146	.17646	149	52
53	.07900	162	.08424	164	.17145	146	.17795	148	53
54	.08063	162	.08589	164	.17291	146	.17943	148	54
55	8.08225	162	8.08753	164	8.17437	145	8.18091	148	55
56	.08387	161	.08917	163	.17582	145	.18238	147	56
57	.08549	161	.09081	164	.17728	145	.18386	147	57
58	.08710	161	.09244	163	.17873	145	.18533	147	58
59	.08871	161	.09407	163	.18017	144	.18680	147	59
60	8.09031		8.09569		8.18162	144	8.18827	146	60
′	Log. Vers.	D	Log. Exsec.	D	Log. Vers.	D	Log. Exsec.	D	′

P. P.

	180	170	160
6	18.0	17.0	16.0
7	21.0	19.8	18.6
8	24.0	22.6	21.3
9	27.0	25.5	24.0
10	30.0	28.3	26.6
20	60.0	56.6	53.3
30	90.0	85.0	80.0
40	120.0	113.3	106.6
50	150.0	141.6	133.3

	150	140
6	15.0	14.0
7	17.5	16.3
8	20.0	18.6
9	22.5	21.0
10	25.0	23.3
20	50.0	46.6
30	75.0	70.0
40	100.0	93.3
50	125.0	116.6

	9	9	8
6	0.9	0.9	0.8
7	1.1	1.0	1.1
8	1.2	1.2	1.1
9	1.4	1.3	1.3
10	1.6	1.5	1.4
20	3.1	3.0	2.8
30	4.7	4.5	4.2
40	6.3	6.0	5.6
50	7.9	7.5	7.1

	8	7	7
6	0.8	0.7	0.7
7	0.9	0.9	0.8
8	1.0	1.0	0.9
9	1.2	1.1	1.0
10	1.3	1.2	1.1
20	2.6	2.5	2.3
30	3.9	3.7	3.5
40	5.3	5.0	4.6
50	6.6	6.2	5.8

	6	6
6	0.6	0.6
7	0.7	0.7
8	0.8	0.8
9	1.0	0.9
10	1.1	1.0
20	2.1	2.0
30	3.2	3.0
40	4.3	4.0
50	5.4	5.0

	5	5
6	0.5	0.5
7	0.6	0.6
8	0.7	0.6
9	0.8	0.7
10	0.9	0.8
20	1.8	1.6
30	2.7	2.5
40	3.6	3.3
50	4.6	4.1

TABLE VIII.—LOGARITHMIC VERSED SINES AND EXTERNAL SECANTS.

10° 11°

′	Log. Vers.	D	Log. Exsec.	D	Log. Vers.	D	Log. Exsec.	D	′
0	8.18162	144½	8.18827	146	8.26417	131	8.27223	133⅔	0
1	.18306	144	.18973	146	.26548	131	.27356	133	1
2	.18450	144	.19120	146	.26679	131	.27490	133	2
3	.18594	143¾	.19266	145	.26810	130	.27623	133	3
4	.18738	143⅝	.19411	145	.26941	130	.27756	133	4
5	8.18881	143½	8.19557	145⅝	8.27071	130	8.27889	133	5
6	.19024	143½	.19702	145	.27201	130	.28021	132	6
7	.19167	143½	.19847	145	.27331	130	.28153	132	7
8	.19309	142½	.19992	145	.27461	129	.28286	132	8
9	.19452	142	.20137	144⅝	.27590	129	.28418	132	9
10	8.19594	142	8.20281	144	8.27719	129	8.28550	132	10
11	.19736	142	.20425	144	.27849	129	.28681	131	11
12	.19878	141	.20569	144	.27977	128	.28813	131	12
13	.20019	141	.20713	144	.28106	129	.28944	131	13
14	.20166	141	.20857	143	.28235	128	.29075	131	14
15	8.20301	141	8.21000	143	8.28363	128	8.29206	131	15
16	.20442	140	.21143	143	.28491	128	.29336	130	16
17	.20582	140	.21286	143	.28619	128	.29467	130	17
18	.20723	140	.21428	142	.28747	127	.29597	130	18
19	.20863	140	.21571	142	.28875	127	.29727	130	19
20	8.21003	139⅞	8.21713	142	8.29002	127	8.29857	130	20
21	.21142	139	.21855	141	.29129	127	.29987	129	21
22	.21282	139	.21996	141	.29256	127	.30117	129	22
23	.21421	139	.22138	141	.29383	126	.30246	129	23
24	.21560	138⅞	.22279	141	.29510	126	.30375	129	24
25	8.21698	138	8.22420	140	8.29636	126	8.30504	129	25
26	.21837	138	.22561	140	.29763	126	.30633	128	26
27	.21975	138	.22701	140	.29889	126	.30762	128	27
28	.22113	137	.22842	140	.30015	125	.30890	128	28
29	.22251	138	.22982	140	.30140	125	.31019	128	29
30	8.22389	137	8.23122	140	8.30266	125	8.31147	128	30
31	.22526	137	.23262	139	.30391	125	.31275	127	31
32	.22663	136	.23401	139	.30516	125	.31402	127	32
33	.22800	137	.23540	139	.30642	124	.31530	127	33
34	.22937	136	.23679	139	.30766	124	.31657	127	34
35	8.23073	136	8.23818	139	8.30891	124	8.31785	127	35
36	.23209	136	.23957	138	.31015	124	.31912	127	36
37	.23346	135	.24095	138	.31140	124	.32039	126	37
38	.23481	136	.24234	138	.31264	124	.32165	126	38
39	.23617	135	.24372	138	.31388	123	.32292	126	39
40	8.23752	135	8.24509	137	8.31511	124	8.32418	126	40
41	.23888	135	.24647	138	.31635	123	.32544	126	41
42	.24023	135	.24784	137	.31758	123	.32670	126	42
43	.24158	134	.24922	137	.31882	123	.32796	125	43
44	.24292	134	.25059	137	.32005	123	.32922	125	44
45	8.24426	134	8.25195	136	8.32128	123	8.33047	125	45
46	.24561	134	.25332	136	.32250	122	.33173	125	46
47	.24695	133¾	.25468	136	.32373	122	.33298	125	47
48	.24828	133¾	.25604	136	.32495	122	.33423	124	48
49	.24962	133¾	.25740	136	.32617	122	.33547	125	49
50	8.25095	133	8.25876	136	8.32739	122	8.33672	125	50
51	.25228	133	.26012	135	.32861	121	.33797	124	51
52	.25361	132	.26147	135	.32983	121	.33921	124	52
53	.25494	133	.26282	135	.33104	121	.34045	124	53
54	.25627	132	.26417	135	.33225	121	.34169	124	54
55	8.25759	132	8.26552	134	8.33347	121	8.34293	124	55
56	.25891	132	.26686	134	.33468	120	.34417	123	56
57	.26023	132	.26821	134	.33588	120	.34540	123	57
58	.26155	131	.26955	134	.33709	120	.34663	123	58
59	.26286	131	.27089	134	.33829	120	.34786	123	59
60	8.26417		8.27223		8.33950		8.34909		60
′	Log. Vers.	D	Log. Exsec.	D	Log. Vers.	D	Log. Exsec.	D	′

P. P.

	130	120
6	13.0	12.0
7	15.1	14.0
8	17.3	16.0
9	19.5	18.0
10	21.6	20.0
20	43.3	40.0
30	65.0	60.0
40	86.6	80.0
50	108.3	100.0

	4̂	4	3̂
6	0.4	0.4	0.3
7	0.5	0.4	0.4
8	0.6	0.5	0.4
9	0.7	0.6	0.5
10	0.7	0.6	0.6
20	1.5	1.3	1.1
30	2.2	2.0	1.7
40	3.0	2.6	2.3
50	3.7	3.3	2.9

	3	2̂
6	0.3	0.2
7	0.3	0.3
8	0.4	0.3
9	0.4	0.4
10	0.5	0.4
20	1.0	0.8
30	1.5	1.2
40	2.0	1.6
50	2.5	2.1

	2	1̂
6	0.2	0.1
7	0.2	0.2
8	0.2	0.2
9	0.3	0.2
10	0.3	0.2
20	0.6	0.5
30	1.0	0.7
40	1.3	1.0
50	1.6	1.2

	1	0̂
6	0.1	0.0
7	0.1	0.0
8	0.1	0.0
9	0.1	0.1
10	0.1	0.1
20	0.3	0.2
30	0.5	0.2
40	0.6	0.3
50	0.8	0.4

TABLE VIII.—LOGARITHMIC VERSED SINES AND EXTERNAL SECANTS.

12°   13°

′	Log. Vers.	D	Log. Exsec.	D	Log. Vers.	D	Log. Exsec.	D	′
0	8.33950	120	8.34909̂	123	8.40875	116̂	8.42002̂	113̂	0
1	.34070	120	.35032̂	122̂	.40985̂	116̂	.42116	113̂	1
2	.34190	119̂	.35155	122̂	.41096̂	116̂	.42229̂	113̂	2
3	.34309̂	120	.35277̂	122	.41206̂	116̂	.42343	113̂	3
4	.34429̂	119̂	.35399̂	122̂	.41317	116̂	.42456̂	113	4
5	8.34549	119	8.35522	122	8.41427̂	116̂	8.42569̂	113	5
6	.34668	119	.35644	122	.41537̂	110	.42682̂	113	6
7	.34787	119	.35765̂	121̂	.41647̂	110	.42795̂	113	7
8	.34906	119	.35887̂	121̂	.41757	109̂	.42908̂	112̂	8
9	.35025	118̂	.36009	121	.41867	110	.43021	112̂	9
10	8.35143̂	118̂	8.36130	121	8.41976̂	109̂	8.43133̂	112̂	10
11	.35262	118̂	.36251̂	121̂	.42086	109̂	.43246	112̂	11
12	.35380̂	118	.36372̂	121	.42195	109	.43358	112	12
13	.35498̂	118	.36493	120̂	.42304	109	.43470	112	13
14	.35616̂	117̂	.36614	121	.42413	109	.43582	112	14
15	8.35734	118	8.36734̂	120̂	8.42522	109	8.43694	111̂	15
16	.35852	117	.36855	120	.42630̂	109	.43805̂	111̂	16
17	.35969̂	117	.36975̂	120	.42739̂	108	.43917	111	17
18	.36086̂	117̂	.37095	120	.42847̂	108	.44028̂	111	18
19	.36204	117	.37215	120	.42956	108	.44139̂	111̂	19
20	8.36321	116̂	8.37335	119̂	8.43064	108	8.44251	111	20
21	.36437	117	.37454̂	119̂	.43172	108	.44362	111	21
22	.36554̂	116̂	.37574	119	.43280	108	.44473	111	22
23	.36671	116	.37693̂	119̂	.43388	107̂	.44583̂	110̂	23
24	.36787	116̂	.37812̂	119	.43495̂	108	.44694	110̂	24
25	8.36903̂	116	8.37931̂	118̂	8.43603	107̂	8.44804̂	110̂	25
26	.37019̂	116	.38050	119	.43710	107	.44915	110	26
27	.37135	115̂	.38169	118̂	.43817̂	107	.45025	110̂	27
28	.37251	115̂	.38287̂	118̂	.43924̂	107	.45135̂	110	28
29	.37366̂	115̂	.38406	118	.44031̂	107	.45245	110	29
30	8.37482	115	8.38524	118	8.44138	106̂	8.45355	110	30
31	.37597	115	.38642	118	.44245	107	.45465	109̂	31
32	.37712̂	115	.38760	118	.44351̂	106̂	.45574̂	109̂	32
33	.37827̂	115	.38878	117	.44458	106̂	.45684	109	33
34	.37942̂	114̂	.38995	117̂	.44564	106	.45793	109	34
35	8.38057	114̂	8.39113	117	8.44670	106	8.45902̂	109	35
36	.38171̂	114̂	.39230	117̂	.44776̂	105̂	.46011̂	109	36
37	.38286	114	.39347̂	117	.44882	106	.46120̂	108̂	37
38	.38400	114	.39464̂	117	.44988	105	.46229	109	38
39	.38514	114	.39581̂	116̂	.45093̂	105̂	.46338	108̂	39
40	8.38628	113̂	8.39698	116̂	8.45199	105̂	8.46446	108̂	40
41	.38741̂	114	.39814̂	116̂	.45304̂	105	.46555	108	41
42	.38855̂	113̂	.39931	116	.45409̂	105	.46663	108̂	42
43	.38969	113	.40047̂	116	.45514̂	105	.46771̂	108	43
44	.39082	113	.40163̂	116	.45619̂	105	.46879̂	108	44
45	8.39195	113	8.40279̂	116	8.45724̂	104̂	8.46987̂	107̂	45
46	.39308	113	.40395̂	115̂	.45829	105	.47095	108	46
47	.39421	113	.40511	115̂	.45934	104̂	.47203	107̂	47
48	.39534	112̂	.40626̂	115̂	.46038	104̂	.47310̂	107	48
49	.39646̂	112	.40742	115̂	.46142̂	104̂	.47417̂	107̂	49
50	8.39758̂	112̂	8.40857̂	115	8.46247	104	8.47525	107	50
51	.39871	112	.40972̂	115	.46351̂	104	.47632̂	107	51
52	.39983	112	.41087̂	115	.46455	103̂	.47739̂	106̂	52
53	.40095	112	.41202̂	114	.46558̂	104	.47846	107	53
54	.40207	111̂	.41317	114̂	.46662̂	103̂	.47953	106̂	54
55	8.40318̂	111̂	8.41431̂	114̂	8.46766	103̂	8.48060	106̂	55
56	.40430	111̂	.41546	114	.46869̂	103̂	.48166̂	106̂	56
57	.40541̂	111	.41660̂	114	.46972̂	103̂	.48273	106	57
58	.40652̂	111̂	.41774̂	114	.47076	103	.48379	106̂	58
59	.40764	111	.41888	114	.47179	103	.48485̂	106	59
60	8.40875		8.42002̂		8.47282		8.48591̂		60
′	Log. Vers.	D	Log. Exsec.	D	Log. Vers.	D	Log. Exsec.	D	′

P. P.

	120	119	118
6	12.0	11.9	11.8
7	14.0	13.9	13.7
8	16.0	15.8	15.7
9	18.0	17.8	17.7
10	20.0	19.8	19.6
20	40.0	39.6	39.3
30	60.0	59.5	59.0
40	80.0	79.2	78.6
50	100.0	99.1	98.3

	117	116	115
6	11.7	11.6	11.5
7	13.6	13.5	13.4
8	15.6	15.4	15.3
9	17.5	17.4	17.2
10	19.5	19.3	19.1
20	39.0	38.6	38.3
30	58.5	58.0	57.5
40	78.0	77.3	76.6
50	97.5	96.6	95.8

	114	113	112
6	11.4	11.3	11.2
7	13.3	13.2	13.0
8	15.2	15.0	14.9
9	17.1	16.6	16.8
10	19.0	18.8	18.6
20	38.0	37.6	37.3
30	57.0	56.5	56.0
40	76.0	75.3	74.6
50	95.0	94.1	93.3

	111	110	109
6	11.1	11.0	10.9
7	12.6	12.8	12.7
8	14.8	14.6	14.5
9	16.6	16.5	16.3
10	18.5	18.3	18.1
20	37.0	36.6	36.3
30	55.5	55.0	54.5
40	74.0	73.3	72.6
50	92.5	91.6	90.8

	108	107	106
6	10.8	10.7	10.6
7	12.6	12.5	12.3
8	14.4	14.2	14.1
9	16.2	16.0	15.9
10	18.0	17.8	17.6
20	36.0	35.6	35.3
30	54.0	53.5	53.0
40	72.0	71.3	70.6
50	90.0	89.1	88.3

	105	104	0̂
6	10.5	10.4	0.0
7	12.2	12.1	0.0
8	14.0	13.8	0.0
9	15.7	15.6	0.1
10	17.5	17.3	0.1
20	35.0	34.6	0.2
30	52.5	52.0	0.2
40	70.0	69.3	0.3
50	87.5	86.6	0.4

TABLE VIII.—LOGARITHMIC VERSED SINES AND EXTERNAL SECANTS.

14°   15°

′	Log. Vers.	D	Log. Exsec.	D	Log. Vers.	D	Log. Exsec.	D	′
0	8.47282	102	8.48591	106	8.53242	96	8.54748	99	0
1	.47384	103	.48697	106	.53338	96	54847	99	1
2	.47487	102	.48803	105	.53434	96	.54946	99	2
3	.47590	102	.48909	105	.53530	95	.55045	99	3
4	.47692	102	.49014	105	.53625	96	.55144	99	4
5	8.47795	102	8.49120	105	8.53721	95	8.55243	99	5
6	.47897	102	.49225	105	.53816	95	.55342	99	6
7	.47999	102	.49331	105	.53911	96	.55441	98	7
8	.48101	102	.49436	105	.54007	95	.55539	98	8
9	.48203	101	.49541	105	.54102	95	.55638	98	9
10	8.48304	101	8.49646	104	8.54197	94	8.55736	98	10
11	.48406	101	.49750	105	.54291	95	.55834	98	11
12	.48507	101	.49855	105	.54386	95	.55933	98	12
13	.48609	101	.49960	104	.54481	94	.56031	98	13
14	.48710	101	.50064	104	.54575	95	.56129	98	14
15	8.48811	101	8.50168	104	8.54670	94	8.56226	98	15
16	.48912	101	.50273	104	.54764	94	.56324	97	16
17	.49013	101	.50377	104	.54858	94	.56422	97	17
18	.49114	100	.50481	104	.54952	94	.56519	97	18
19	.49215	101	.50585	104	.55046	94	.56617	97	19
20	8.49315	100	8.50688	103	8.55140	94	8.56714	97	20
21	.49415	100	.50792	104	.55234	94	.56812	97	21
22	.49516	100	.50896	103	.55328	93	.56909	97	22
23	.49616	100	.50999	103	.55421	94	.57006	97	23
24	.49716	100	.51102	103	.55515	93	.57103	97	24
25	8.49816	100	8.51205	103	8.55608	93	8.57200	96	25
26	.49916	99	.51309	103	.55701	94	.57296	97	26
27	.50015	100	.51412	102	.55795	93	.57393	97	27
28	.50115	99	.51514	103	.55888	93	.57490	96	28
29	.50215	99	.51617	103	.55981	93	.57586	96	29
30	8.50314	99	8.51720	102	8.56074	92	8.57682	96	30
31	.50413	99	.51822	103	.56166	93	.57779	96	31
32	.50512	99	.51925	102	.56259	93	.57875	96	32
33	.50611	99	.52027	102	.56352	92	.57971	96	33
34	.50710	99	.52129	102	.56444	92	.58067	96	34
35	8.50809	99	8.52231	102	8.56536	93	8.58163	96	35
36	.50908	98	.52333	102	.56629	92	.58259	95	36
37	.51006	99	.52435	102	.56721	92	.58354	96	37
38	.51105	98	.52537	101	.56813	92	.58450	96	38
39	.51203	98	.52638	102	.56905	92	.58546	95	39
40	8.51301	98	8.52740	101	8.56997	92	8.58641	95	40
41	.51399	98	.52841	102	.57089	91	.58736	96	41
42	.51497	98	.52943	101	.57180	92	.58832	95	42
43	.51595	98	.53044	101	.57272	91	.58927	95	43
44	.51693	97	.53145	101	.57363	92	.59022	95	44
45	8.51791	98	8.53246	101	8.57455	91	8.59117	95	45
46	.51888	97	.53347	101	.57546	91	.59211	94	46
47	.51986	97	.53448	101	.57637	91	.59306	95	47
48	.52083	97	.53548	100	.57728	91	.59401	94	48
49	.52180	97	.53649	100	.57819	91	.59495	94	49
50	8.52277	97	8.53749	100	8.57910	91	8.59590	94	50
51	.52374	97	.53850	100	.58001	91	.59684	94	51
52	.52471	97	.53950	100	.58092	90	.59779	94	52
53	.52568	97	.54050	100	.58182	91	.59873	94	53
54	.52665	96	.54150	100	.58273	90	.59967	94	54
55	8.52761	97	8.54250	100	8.58363	90	8.60061	94	55
56	.52858	96	.54350	99	.58453	91	.60155	94	56
57	.52954	96	.54449	100	.58544	90	.60249	93	57
58	.53050	96	.54549	99	.58634	90	.60342	94	58
59	.53146	96	.54649	99	.58724	90	.60436	93	59
60	8.53242		8.54748		8.58814		8.60530		60
′	Log. Vers.	D	Log. Exsec.	D	Log. Vers.	D	Log. Exsec.	D	′

P. P.

	103	102	101
6	10.3	10.2	10.1
7	12.0	11.9	11.8
8	13.7	13.6	13.4
9	15.4	15.3	15.1
10	17.1	17.0	16.8
20	34.3	34.0	33.6
30	51.5	51.0	50.5
40	68.6	68.0	67.3
50	85.8	85.0	84.1

	100	99	98
6	10.0	9.9	9.8
7	11.6	11.5	11.4
8	13.3	13.2	13.0
9	15.0	14.8	14.7
10	16.6	16.5	16.3
20	33.3	33.0	32.6
30	50.0	49.5	49.0
40	66.6	66.0	65.3
50	83.3	82.5	81.6

	97	96	95
6	9.7	9.6	9.5
7	11.3	11.2	11.1
8	12.9	12.8	12.6
9	14.5	14.4	14.2
10	16.1	16.0	15.8
20	32.3	32.0	31.6
30	48.5	48.0	47.5
40	64.6	64.0	63.3
50	80.8	80.0	79.1

	94	93	92
6	9.4	9.3	9.2
7	10.9	10.8	10.7
8	12.5	12.4	12.2
9	14.1	13.9	13.8
10	15.6	15.5	15.3
20	31.3	31.0	30.6
30	47.0	46.5	46.0
40	62.6	62.0	61.3
50	78.3	77.5	76.6

	91	90	0
6	9.1	9.0	0.0
7	10.6	10.5	0.0
8	12.1	12.0	0.0
9	13.6	13.5	0.1
10	15.1	15.0	0.1
20	30.3	30.0	0.1
30	45.5	45.0	0.2
40	60.6	60.0	0.3
50	75.8	75.0	0.4

TABLE VIII.—LOGARITHMIC VERSED SINES AND EXTERNAL SECANTS.

16°   17°

′	Log. Vers.	D	Log. Exsec.	D	Log. Vers.	D	Log. Exsec.	D	′
0	8.58814	90	8.60530	93̂	8.64043̂	84̂	8.65984	88	0
1	.58904	89̂	.60623̂	93	.64128	84̂	.66072	88	1
2	.58993̂	90	.60716̂	93	.64212̂	84	.66160̂	88̂	2
3	.59083̂	89̂	.60810	93̂	.64296̂	84̂	.66248̂	88	3
4	.59173	89̂	.60903	93	.64381	84	.66336	88	4
5	8.59262̂	89̂	8.60996̂	93̂	8.64465	84	8.66425	88̂	5
6	.5935î	89	.61089̂	93	.64549	84	.66512	88	6
7	.59441	89	.61182̂	92̂	.64633	84	.66600̂	88	7
8	.59530	89	.61275	93	.64717	84	.66688̂	87	8
9	.59619	89	.61368	92̂	.64801	83̂	.66776	88	9
10	8.59708	89	8.61460̂	92	8.64884̂	83̂	8.66863̂	88	10
11	.59797	89	.61553	92	.64968	84	.6695î	87̂	11
12	.59886	88̂	.61645	92	.65052	83̂	.67039	87	12
13	.59974̂	89	.61738	92̂	.65135̂	83	.67126̂	87	13
14	.60063	88̂	.61830	92	.65218̂	83	.67213̂	87	14
15	8.60152	88	8.61922̂	92	8.65302	83̂	8.67301	87	15
16	.60240̂	88	.62014̂	92	.65385	83	.67388	87	16
17	.60328̂	88	.62106̂	92	.65468̂	83	.67475	87	17
18	.60417	88	.62198̂	92	.6555î	83	.67562	87	18
19	.60505	88	.62290̂	92	.65634̂	83	.67649	87	19
20	8.60593	88	8.62382	91̂	8.65717	83	8.67736	86̂	20
21	.60681	88	.62474	91̂	.65800̂	82̂	.67822̂	87	21
22	.60769	88	.62565̂	91̂	.65883	82̂	.67909̂	86̂	22
23	.60857	87	.62657	91̂	.65965̂	83	.67996̂	86̂	23
24	.60944̂	87̂	.62748	91	.66048̂	82̂	.68082̂	86̂	24
25	8.61032	87	8.62840	91	8.66131	82	8.68169	86	25
26	.61119̂	87	.62931	91̂	.66213	82̂	.68255̂	86	26
27	.61207	87	.63022	91	.66295̂	82̂	.6834î	86	27
28	.61294̂	87	.63113̂	91	.66378	82	.68428	86	28
29	.6138î	87	.63204̂	91	.66460	82	.68514	86̂	29
30	8.61469	87	8.63295	90̂	8.66542	82	8.68600	86	30
31	.61556	87	.63386	91	.66624	82	.68686̂	86	31
32	.61643	87	.63477	90̂	.66706	82	.68772	86	32
33	.61730	86̂	.63567̂	90̂	.66788	82	.68858	85̂	33
34	.6181̂6	87	.63658	90̂	.66870	81̂	.68944	86	34
35	8.61903̂	86̂	8.63748̂	90̂	8.6695î	81̂	8.69029̂	85̂	35
36	.61990	86̂	.63839	90	.67033	82	.6911̂5	85̂	36
37	.62076̂	86̂	.63929	90	.67115	81̂	.69201	85	37
38	.62163	86	.64019̂	90	.67196̂	81	.69286̂	85̂	38
39	.62249̂	86	.64109̂	90	.67277	81̂	.69372	85	39
40	8.62336	86	8.64199̂	90	8.67359	81̂	8.69457	85	40
41	.62422	86	.64289̂	90	.6744̂0	81	.69542̂	85	41
42	.62508	86	.64379̂	90	.6752î	81	.69627̂	85	42
43	.62594	86	.64469̂	89̂	.67602̂	81	.69712̂	85	43
44	.62680	86	.64559	90	.67683	81	.69798	84̂	44
45	8.62766	86	8.64649	89̂	8.67764	81	8.69883	85	45
46	.62852	85̂	.64738̂	89̂	.67845	80̂	.69967̂	85	46
47	.62937̂	85̂	.64828	89	.67926	81	.70052̂	84̂	47
48	.63023	85̂	.64917	89	.68007	80̂	.70137̂	84̂	48
49	.63108̂	85	.65006̂	89	.68087̂	80̂	.70222	84̂	49
50	8.63194	85	8.65096	89	8.68168	80̂	8.70306̂	84̂	50
51	.63279	85	.65185	89	.68248̂	80̂	.70391	84̂	51
52	.63364̂	85̂	.65274	89	.68329	80	.70475̂	84	52
53	.63449̂	85	.65363	89	.68409̂	80	.70560	84	53
54	.63534	85	.65452	89	.68489̂	80	.70644̂	84	54
55	8.63619̂	85	8.65541	88̂	8.68569̂	80	8.70728̂	84	55
56	.63704̂	85	.65629̂	88	.68650	80	.70813	84	56
57	.63789̂	84̂	.65718	89	.68730	80	.70897	84	57
58	.63874	85	.65807	88̂	.68810	79̂	.70981	84	58
59	.63959	84̂	.65895̂	88̂	.68888̂	80	.71065	84	59
60	8.64043̂		8.65984		8.68969̂		8.71149		60
′	Log. Vers.	D	Log. Exsec.	D	Log. Vers.	D	Log. Exsec.	D	′

P. P.

	93	92	91
6	9.3	9.2	9.1
7	10.8	10.7	10.6
8	12.4	12.2	12.1
9	13.9	13.8	13.6
10	15.5	15.3	15.1
20	31.0	30.6	30.3
30	46.5	46.0	45.5
40	62.0	61.3	60.6
50	77.5	76.6	75.8

	90	89	88
6	9.0	8.9	8.8
7	10.5	10.4	10.2
8	12.0	11.8	11.7
9	13.5	13.3	13.2
10	15.0	14.8	14.6
20	30.0	29.6	29.3
30	45.0	44.5	44.0
40	60.0	59.3	58.6
50	75.0	74.1	73.3

	87	86	85
6	8.7	8.6	8.5
7	10.1	10.0	9.9
8	11.6	11.4	11.3
9	13.0	12.9	12.7
10	14.5	14.3	14.1
20	29.0	28.6	28.3
30	43.5	43.0	42.5
40	58.0	57.3	56.6
50	72.5	71.6	70.8

	84	83	82
6	8.4	8.3	8.2
7	9.8	9.7	9.5
8	11.2	11.0	10.9
9	12.6	12.4	12.3
10	14.0	13.8	13.6
20	28.0	27.6	27.3
30	42.0	41.5	41.0
40	56.0	55.3	54.6
50	70.0	69.1	68.3

	81	80	79
6	8.1	8.0	7.9
7	9.4	9.3	9.2
8	10.8	10.6	10.5
9	12.1	12.0	11.8
10	13.5	13.3	13.1
20	27.0	26.6	26.3
30	40.5	40.0	39.5
40	54.0	53.3	52.6
50	67.5	66.6	65.8

	6̂
6	0.6
7	0.6
8	0.6
9	0.1
10	0.1
20	0.1
30	0.2
40	0.3
50	0.4

TABLE VIII.—LOGARITHMIC VERSED SINES AND EXTERNAL SECANTS.

18°   **19°**

′	Log. Vers.	D	Log. Exsec.	D	Log. Vers.	D	Log. Exsec.	D	′
0	8.68969	79	8.71149	83	8.73625	75	8.76058	79	0
1	.69049	80	.71232	84	.73700	75	.76137	80	1
2	.69129	79	.71316	83	.73775	75	.76217	79	2
3	.69208	79	.71400	84	.73851	75	.76297	79	3
4	.69288	79	.71484	83	.73926	75	.76376	79	4
5	8.69367	79	8.71567	83	8.74001	75	8.76456	80	5
6	.69446	79	.71651	83	.74076	75	.76536	79	6
7	.69526	79	.71734	83	.74151	75	.76615	79	7
8	.69605	79	.71817	83	.74226	75	.76694	79	8
9	.69684	79	.71901	83	.74301	75	.76774	79	9
10	8.69763	79	8.71984	83	8.74376	75	8.76853	79	10
11	.69842	79	.72067	83	.74451	74	.76932	79	11
12	.69921	79	.72150	83	.74526	75	.77011	79	12
13	.70000	78	.72233	83	.74600	74	.77090	79	13
14	.70079	79	.72316	83	.74675	74	.77169	79	14
15	8.70157	78	8.72399	83	8.74749	74	8.77248	79	15
16	.70236	78	.72481	82	.74824	74	.77327	79	16
17	.70314	78	.72564	83	.74898	74	.77406	78	17
18	.70393	78	.72647	82	.74973	74	.77485	79	18
19	.70471	78	.72729	82	.75047	74	.77563	78	19
20	8.70550	78	8.72812	82	8.75121	74	8.77642	78	20
21	.70628	78	.72894	82	.75193	74	.77720	79	21
22	.70706	78	.72977	82	.75269	74	.77799	78	22
23	.70784	78	.73059	82	.75343	74	.77877	78	23
24	.70862	78	.73141	82	.75417	74	.77956	78	24
25	8.70940	78	8.73223	82	8.75491	74	8.78034	78	25
26	.71018	77	.73306	82	.75565	73	.78112	78	26
27	.71096	78	.73388	82	.75639	73	.78191	78	27
28	.71174	77	.73470	81	.75712	73	.78269	78	28
29	.71251	77	.73551	82	.75786	73	.78347	78	29
30	8.71329	77	8.73633	82	8.75860	74	8.78425	78	30
31	.71406	77	.73715	81	.75933	73	.78503	78	31
32	.71484	77	.73797	81	.76006	74	.78581	78	32
33	.71561	77	.73878	81	.76080	73	.78659	79	33
34	.71639	77	.73960	81	.76153	73	.78736	78	34
35	8.71716	77	8.74041	81	8.76226	73	8.78814	78	35
36	.71793	77	.74123	81	.76300	73	.78892	77	36
37	.71870	77	.74204	81	.76373	73	.78969	77	37
38	.71947	77	.74286	81	.76446	73	.79047	77	38
39	.72024	77	.74367	81	.76519	73	.79124	77	39
40	8.72101	77	8.74448	81	8.76592	73	8.79202	77	40
41	.72178	76	.74529	81	.76664	72	.79279	77	41
42	.72255	77	.74610	81	.76737	73	.79357	77	42
43	.72331	76	.74691	81	.76810	73	.79434	77	43
44	.72408	77	.74772	80	.76883	72	.79511	77	44
45	8.72485	76	8.74853	81	8.76955	72	8.79588	77	45
46	.72561	76	.74934	80	.77028	72	.79665	77	46
47	.72637	76	.75014	80	.77100	72	.79742	77	47
48	.72714	76	.75095	80	.77173	72	.79819	77	48
49	.72790	76	.75175	80	.77245	72	.79896	77	49
50	8.72866	76	8.75256	80	8.77317	72	8.79973	77	50
51	.72942	76	.75336	80	.77390	72	.80050	76	51
52	.73018	76	.75417	80	.77462	72	.80126	77	52
53	.73094	76	.75497	80	.77534	72	.80203	76	53
54	.73170	76	.75577	80	.77606	72	.80280	76	54
55	8.73246	76	8.75658	80	8.77678	72	8.80356	76	55
56	.73322	76	.75738	80	.77750	72	.80433	76	56
57	.73398	75	.75818	80	.77822	71	.80509	76	57
58	.73473	75	.75898	80	.77893	72	.80586	76	58
59	.73549	75	.75978	80	.77965	71	.80662	76	59
60	8.73625		8.76058		8.78037		8.80738		60
′	Log. Vers.	D	Log. Exsec.	D	Log. Vers.	D	Log. Exsec.	D	′

P. P.

	84	83	82
6	8.4	8.3	8.2
7	9.8	9.7	9.5
8	11.2	11.6	10.9
9	12.6	12.4	12.3
10	14.0	13.8	13.6
20	28.0	27.6	27.3
30	42.0	41.5	41.0
40	56.0	55.3	54.6
50	70.0	69.1	68.3

	81	80	79
6	8.1	8.0	7.9
7	9.4	9.3	9.2
8	10.8	10.6	10.5
9	12.1	12.0	11.8
10	13.5	13.3	13.1
20	27.0	26.6	26.3
30	40.5	40.0	39.5
40	54.0	53.3	52.6
50	67.5	66.6	65.3

	78	77	76
6	7.8	7.7	7.6
7	9.1	9.0	8.8
9	11.7	11.5	11.4
10	13.0	12.8	12.6
20	26.0	25.6	25.3
30	39.0	38.5	38.0
40	52.0	51.3	50.6
50	65.0	64.1	63.3

	75	74	73
6	7.5	7.4	7.3
7	8.7	8.6	8.5
8	10.0	9.8	9.7
9	11.2	11.1	10.9
10	12.5	12.3	12.1
20	25.0	24.6	24.3
30	37.5	37.0	36.5
40	50.0	49.3	48.6
50	62.5	61.6	60.8

	72	71	ô
6	7.2	7.1	0.ô
7	8.4	8.3	0.ô
8	9.6	9.4	0.ô
9	10.8	10.6	0.1
10	12.0	11.8	0.1
20	24.0	23.6	0.î
30	36.0	35.5	0.2
40	48.0	47.3	0.3
50	60.0	59.1	0.4

P. P.

TABLE VIII.—LOGARITHMIC VERSED SINES AND EXTERNAL SECANTS.

20°   21°

′	Log. Vers.	D	Log. Exsec.	D	Log. Vers.	D	Log. Exsec.	D	′
0	8.78037	71	8.80738	76	8.82229	68	8.85214	73	0
1	.78108	71	.80814	76	.82297	68	.85287	73	1
2	.78180	71	.80891	76	.82366	68	.85360	72	2
3	.78251	71	.80967	76	.82434	68	.85433	73	3
4	.78323	71	.81043	76	.82502	68	.85506	73	4
5	8.78394	71	8.81119	76	8.82569	67	8.85579	73	5
6	.78466	71	.81195	76	.82637	68	.85651	72	6
7	.78537	71	.81271	76	.82705	68	.85724	73	7
8	.78608	71	.81346	75	.82773	68	.85797	72	8
9	.78679	71	.81422	76	.82841	68	.85869	72	9
10	8.78750	71	8.81498	75	8.82908	67	8.85942	72	10
11	.78821	71	.81573	75	.82976	67	.86014	72	11
12	.78892	71	.81649	76	.83043	67	.86087	72	12
13	.78963	71	.81725	75	.83111	67	.86159	72	13
14	.79034	70	.81800	75	.83178	67	.86231	72	14
15	8.79105	71	8.81876	75	8.83246	67	8.86304	72	15
16	.79175	70	.81951	75	.83313	67	.86376	72	16
17	.79246	71	.82026	75	.83380	67	.86448	72	17
18	.79317	70	.82102	75	.83447	67	.86520	72	18
19	.79387	70	.82177	75	.83515	67	.86592	72	19
20	8.79458	70	8.82252	75	8.83582	67	8.86664	72	20
21	.79528	70	.82327	75	.83649	67	.86736	72	21
22	.79598	70	.82402	75	.83716	67	.86808	72	22
23	.79669	70	.82477	75	.83783	67	.86880	72	23
24	.79739	70	.82552	74	.83850	67	.86952	71	24
25	8.79809	70	8.82627	75	8.83916	66	8.87024	72	25
26	.79879	70	.82702	75	.83983	66	.87095	71	26
27	.79949	70	.82776	74	.84050	67	.87167	72	27
28	.80019	70	.82851	75	.84117	66	.87239	71	28
29	.80089	70	.82926	74	.84183	66	.87310	71	29
30	8.80159	70	8.83000	74	8.84250	66	8.87382	71	30
31	.80229	70	.83075	74	.84316	66	.87453	71	31
32	.80299	70	.83149	74	.84383	66	.87525	71	32
33	.80369	69	.83224	74	.84449	66	.87596	71	33
34	.80438	69	.83298	74	.84515	66	.87668	71	34
35	8.80508	69	8.83373	74	8.84582	66	8.87739	71	35
36	.80577	69	.83447	74	.84648	66	.87810	71	36
37	.80647	69	.83521	74	.84714	66	.87881	71	37
38	.80716	69	.83595	74	.84780	66	.87953	71	38
39	.80786	69	.83670	74	.84846	66	.88024	71	39
40	8.80855	69	8.83744	74	8.84912	66	8.88095	71	40
41	.80924	69	.83818	74	.84978	66	.88166	71	41
42	.80993	69	.83892	74	.85044	66	.88237	71	42
43	.81063	69	.83966	73	.85110	66	.88308	71	43
44	.81132	69	.84039	73	.85176	66	.88378	70	44
45	8.81201	69	8.84113	74	8.85242	65	8.88449	71	45
46	.81270	69	.84187	73	.85308	66	.88520	71	46
47	.81339	68	.84261	74	.85373	65	.88591	70	47
48	.81407	69	.84334	73	.85439	66	.88661	71	48
49	.81476	68	.84408	73	.85505	66	.88732	70	49
50	8.81545	69	8.84481	73	8.85570	65	8.88803	70	50
51	.81614	68	.84555	73	.85626	65	.88873	70	51
52	.81682	69	.84628	73	.85701	65	.88944	70	52
53	.81751	68	.84702	73	.85766	65	.89014	70	53
54	.81819	68	.84775	73	.85832	65	.89085	70	54
55	8.81888	68	8.84848	73	8.85897	65	8.89155	70	55
56	.81956	68	.84922	73	.85962	65	.89225	70	56
57	.82025	68	.84995	73	.86027	65	.89295	70	57
58	.82093	68	.85068	73	.86092	65	.89366	70	58
59	.82161	68	.85141	73	.86158	65	.89436	70	59
60	8.82229		8.85214		8.86223		8.89506		60
′	Log. Vers.	D	Log. Exsec.	D	Log. Vers.	D	Log. Exsec.	D	′

P. P.

	76	75	74
6	7.6	7.5	7.4
7	8.8	8.7	8.6
8	10.1	10.0	9.8
9	11.4	11.2	11.1
20	25.3	25.0	24.6
30	38.0	37.5	37.0
40	50.6	50.0	49.3
50	63.3	62.5	61.6

	73	72	71
6	7.3	7.2	7.1
7	8.5	8.4	8.3
8	9.7	9.6	9.4
10	12.1	12.0	11.8
20	24.3	24.0	23.6
30	36.5	36.0	35.5
40	48.6	48.0	47.3
50	60.8	60.0	59.1

	70	69	68
6	7.0	6.9	6.8
7	8.1	8.0	7.9
9	10.5	10.3	10.2
10	11.6	11.5	11.3
20	23.3	23.0	22.6
30	35.0	34.5	34.0
40	46.6	46.0	45.3
50	58.3	57.5	56.6

	67	66	65
6	6.7	6.6	6.5
7	7.8	7.7	7.6
8	8.9	8.8	8.6
9	10.0	9.9	9.7
10	11.1	11.0	10.8
20	22.3	22.0	21.6
30	33.5	33.0	32.5
40	44.6	44.0	43.3
50	55.8	55.0	54.1

	0̂
6	0.6
7	0.6
8	0.6
9	0.1
10	0.1
20	0.1
30	0.2
40	0.3
50	0.4

TABLE VIII.—LOGARITHMIC VERSED SINES AND EXTERNAL SECANTS.

22°   23°

′	Log. Vers.	D	Log. Exsec.	D	Log. Vers.	D	Log. Exsec.	D	′
0	8.86223	6¼	8.89506	7ô	8.90034	62	8.9363î	67	0
1	.86287	65	.89576	70	.90096	62	.93699	6⁷	1
2	.86352	65	.89646	70	.90158	62	.93766	6⁷	2
3	.86417	65	.89716	70	.90220	62	.93833	6⁷	3
4	.86482	64	.89786	69	.90282	62	.93901	6⁷	4
5	8.86547	65	8.89856	70	8.90344	62	8.93968	67	5
6	.86612	64	.89926	69	.90406	61	.94035	6⁷	6
7	.86676	6⁴	.89995	70	.90467	62	.94102	6⁷	7
8	.86741	6⁴	.90065	69	.90529	61	.94170	67	8
9	.86805	6⁴	.90135	70	.90591	6î	.94237	67	9
10	8.86870	6⁴	8.90205	6⁹	8.90652	6î	8.94304	67	10
11	.86934	6⁴	.90274	6⁹	.90714	6î	.94371	67	11
12	.86999	6⁴	.90344	6⁹	.90776	6î	.94438	67	12
13	.87063	64	.90413	6⁹	.90837	6î	.94505	67	13
14	.87127	6⁴	.90483	6⁹	.90899	6î	.94572	67	14
15	8.87192	6⁴	8.90552	6⁹	8.90960	6î	8.94638	6⁶	15
16	.87256	6⁴	.90622	6⁹	.91021	61	.94705	67	16
17	.87320	6⁴	.90691	6⁹	.91083	6î	.94772	66	17
18	.87384	64	.90760	69	.91144	6î	.94839	6ô	18
19	.87448	64	.90830	69	.91205	6î	.94905	66	19
20	8.87512	64	8.90899	69	8.91267	61	8.94972	6⁶	20
21	.87576	64	.90968	69	.91328	61	.95039	6⁶	21
22	.87640	64	.91037	69	.91389	61	.95105	66	22
23	.87704	63	.91106	69	.91450	61	.95172	66	23
24	.87768	64	.91175	69	.91511	61	.95238	6⁶	24
25	8.87832	63	8.91244	69	8.91572	61	8.95305	6⁶	25
26	.87895	64	.91313	68	.91633	61	.95371	66	26
27	.87959	63	.91382	69	.91694	61	.95437	6ô	27
28	.88023	63	.91451	69	.91755	6ô	.95504	6ô	28
29	.88086	63	.91520	68	.91815	61	.95570	6ô	29
30	8.88150	63	8.91588	69	8.91876	6ô	8.95636	66	30
31	.88213	63	.91657	6⁸	.91937	6ô	.95703	66	31
32	.88277	63	.91726	6⁸	.91997	61	.95769	66	32
33	.88340	63	.91794	6⁸	.92058	6ô	.95835	66	33
34	.88404	63	.91863	69	.92119	6ô	.95901	66	34
35	8.88467	63	8.91932	6⁸	8.92179	6ô	8.95967	66	35
36	.88530	63	.92000	68	.92240	6ô	.96033	66	36
37	.88593	63	.92068	6⁸	.92300	6ô	.96099	66	37
38	.88656	63	.92137	6⁸	.92361	60	.96165	66	38
39	.88720	63	.92205	69	.92421	6ô	.96231	66	39
40	8.88783	63	8.92274	6⁸	8.92482	6ô	8.96297	6⁵	40
41	.88846	63	.92342	68	.92542	60	.96362	66	41
42	.88909	62	.92410	68	.92602	60	.96428	6⁵	42
43	.88971	63	.92478	68	.92662	6ô	.96494	66	43
44	.89034	63	.92546	68	.92722	60	.96560	6⁵	44
45	8.89097	62	8.92615	68	8.92782	60	8.96625	6⁵	45
46	.89160	63	.92683	68	.92842	60	.96691	66	46
47	.89223	62	.92751	68	.92902	60	.96757	6⁵	47
48	.89285	62	.92819	68	.92962	60	.96822	6⁵	48
49	.89348	63	.92887	68	.93022	60	.96888	6⁵	49
50	8.89411	62	8.92955	6⁷	8.93082	5ô	8.96953	6⁵	50
51	.89473	63	.93022	68	.93142	60	.97018	6⁵	51
52	.89536	62	.93090	6⁷	.93202	5ô	.97084	6⁵	52
53	.89598	62	.93158	68	.93261	60	.97149	65	53
54	.89660	62	.93226	6⁷	.93321	59	.97214	65	54
55	8.89723	62	8.93293	68	8.93381	5ô	8.97280	65	55
56	.89785	62	.93361	6⁷	.93440	5ô	.97345	6⁵	56
57	.89847	63	.93429	6⁷	.93500	5ô	.97410	65	57
58	.89910	62	.93496	6⁷	.93560	5ô	.97475	6⁵	58
59	.89972	62	.93564	6⁷	.93619	5ô	.97540	6⁵	59
60	8.90034		8.9363î		8.93679		8.97606		60
′	Log. Vers.	D	Log. Exsec.	D	Log. Vers.	D	Log. Exsec.	D	′

P. P.

	70	69	68
6	7.0	6.9	6.8
7	8.1	8.0	7.9
8	9.3	9.2	9.0
9	10.5	10.3	10.2
10	11.6	11.5	11.3
20	23.3	23.0	22.6
30	35.0	34.5	34.0
40	46.6	46.0	45.3
50	58.3	57.5	56.6

	67	66	65
6	6.7	6.6	6.5
7	7.8	7.7	7.6
8	8.9	8.8	8.6
9	10.0	9.9	9.7
10	11.1	11.0	10.8
20	22.3	22.0	21.6
30	33.5	33.0	32.5
40	44.6	44.0	43.3
50	55.8	55.0	54.1

	64	63	62
6	6.4	6.3	6.2
7	7.4	7.3	7.2
8	8.5	8.4	8.2
9	9.6	9.4	9.3
10	10.6	10.5	10.3
20	21.3	21.0	20.6
30	32.0	31.5	31.0
40	42.6	42.0	41.3
50	53.3	52.5	51.6

	61	60	59
6	6.1	6.0	5.9
7	7.1	7.0	6.9
8	8.1	8.0	7.8
9	9.1	9.0	8.8
10	10.1	10.0	9.8
20	20.3	20.0	19.6
30	30.5	30.0	29.5
40	40.6	40.0	39.3
50	50.8	50.0	49.1

	δ
6	0.0
7	0.0
8	0.0
9	0.1
10	0.1
20	0.1
30	0.2
40	0.3
50	0.4

TABLE VIII.—LOGARITHMIC VERSED SINES AND EXTERNAL SECANTS.

24°   25°

′	Log. Vers.	D	Log. Exsec.	D	Log. Vers.	D	Log. Exsec.	D	′
0	8.93679	59	8.97606	65	8.97170	57	9.01443	62	0
1	.93738	59	.97671	65	.97227̂	56	.01505̂	63	1
2	.93797	59	.97736	65	.97284	57	.01568̂	63	2
3	.93857	59	.97801	64	.97341̂	57	.01631	62	3
4	.93916	59	.97865̂	65	.97398	57	.01694	63	4
5	8.93975̂	59	8.97930̂	65	8.97455	56	9.01756̂	62	5
6	.94034̂	59	.97995̂	64	.97511̂	57	.01819̂	63	6
7	.94094	59	.98060	65	.97568	56	.01882	62	7
8	.94153	59	.98125	65	.97625	56	.01944̂	62	8
9	.94212	59	.98190	64	.97681̂	56	.02007̂	63	9
10	8.94271	59	8.98254̂	64	8.97738	57	9.02070	62	10
11	.94330	59	.98319	64	.97795	56	.02132̂	62	11
12	.94389	59	.98383̂	65	.97851̂	56	.02195	62	12
13	.94448	58	.98448̂	64	.97908	56	.02257̂	62	13
14	.94506̂	59	.98513	64	.97964̂	56	.02319̂	62	14
15	8.94565̂	59	8.98577̂	64	8.98020̂	56	9.02382	62	15
16	.94624̂	59	.98642	64	.98077	56	.02444̂	62	16
17	.94683	58	.98706̂	64	.98133̂	56	.02506̂	62	17
18	.94742	59	.98770̂	64	.98190	56	.02569	62	18
19	.94800̂	58	.98835	64	.98246	56	.02631	62	19
20	8.94859	58	8.98899	64	8.98302̂	56	9.02693̂	62	20
21	.94917̂	58	.98963̂	64	.98358	56	.02755̂	62	21
22	.94976	58	.99028̂	64	.98414̂	56	.02817	63	22
23	.95034̂	58	.99092	64	.98470̂	56	.02880	62	23
24	.95093	58	.99156	64	.98527	56	.02942	62	24
25	8.95151̂	58	8.99220̂	64	8.98583̂	56	9.03004	62	25
26	.95210	58	.99284̂	64	.98639	56	.03066	62	26
27	.95268	58	.99348̂	64	.98695	55	.03128	62	27
28	.95326̂	58	.99412̂	64	.98750̂	56	.03190	62	28
29	.95384̂	58	.99476̂	64	.98806̂	56	.03252	61	29
30	8.95443	58	8.99540̂	64	8.98862̂	56	9.03313̂	62	30
31	.95501	58	.99604̂	64	.98918	55	.03375̂	62	31
32	.95559	58	.99668̂	64	.98974	56	.03437̂	61	32
33	.95617̂	58	.99732̂	63	.99030	55	.03499	62	33
34	.95675̂	58	.99796	64	.99085̂	55	.03561	61	34
35	8.95733̂	58	8.99860̂	63	8.99141	56	9.03622̂	61	35
36	.95791̂	57	.99923̂	64	.99197	55	.03684	62	36
37	.95849	58	8.99987̂	63	.99252̂	55	.03746	61	37
38	.95907	58	9.00051	63	.99308	55	.03807̂	61	38
39	.95965	58	.00114̂	64	.99363̂	55	.03869	61	39
40	8.96023	57	9.00178	63	8.99419	55	9.03930̂	61	40
41	.96080̂	57	.00242	63	.99474̂	55	.03992	61	41
42	.96138	58	.00305̂	63	.99529̂	55	.04053̂	61	42
43	.96196	57	.00369	63	.99585	55	.04115	61	43
44	.96253̂	57	.00432̂	63	.99640̂	55	.04176̂	61	44
45	8.96311	57	9.00495̂	63	8.99695̂	55	9.04238	61	45
46	.96368̂	57	.00559	63	.99751	55	.04299	61	46
47	.96426	57	.00622̂	63	.99806	55	.04360̂	61	47
48	.96483̂	57	.00686	63	.99861̂	55	.04421̂	61	48
49	.96541	57	.00749	63	.99916̂	55	.04483	61	49
50	8.96598̂	57	9.00812̂	63	8.99971̂	55	9.04544	61	50
51	.96656	57	.00875̂	63	9.00026̂	55	.04605̂	61	51
52	.96713	57	.00938	63	.00081̂	55	.04666̂	61	52
53	.96770̂	57	.01002	63	.00136̂	55	.04727̂	61	53
54	.96827̂	57	.01065̂	63	.00191̂	55	.04788	61	54
55	8.96885	57	9.01128	63	9.00246̂	55	9.04850	61	55
56	.96942	57	.01191	63	.00301̂	55	.04911	61	56
57	.96999	57	.01254	63	.00356̂	54	.04972	61	57
58	.97056̂	57	.01317	63	.00411	55	.05033	60	58
59	.97113̂	57	.01380	63	.00466	54	.05093̂	61	59
60	8.97170̂		9.01443		9.00520̂		9.05154		60
′	Log. Vers.	D	Log. Exsec.	D	Log. Vers.	D	Log. Exsec.	D	′

P. P.

	65	64	63
6	6.5	6.4	6.3
7	7.6	7.4	7.3
8	8.6	8.5	8.4
9	9.7	9.6	9.4
10	10.8	10.6	10.5
30	32.5	32.0	31.5
40	43.3	42.6	42.0
50	54.1	53.3	52.5

	62	61	60
6	6.2	6.1	6.0
7	7.2	7.1	7.0
8	8.2	8.1	8.0
9	9.3	9.1	9.0
10	10.3	10.1	10.0
20	20.6	20.3	20.0
30	31.0	30.5	30.0
40	41.3	40.6	40.0
50	51.6	50.8	50.0

	59	58	57
6	5.9	5.8	5.7
7	6.9	6.7	6.6
8	7.8	7.7	7.6
9	8.8	8.7	8.5
10	9.8	9.6	9.5
20	19.6	19.3	19.0
30	29.5	29.0	28.5
40	39.3	38.6	38.0
50	49.1	48.3	47.5

	56	55	54
6	5.6	5.5	5.4
7	6.5	6.4	6.3
8	7.4	7.3	7.2
9	8.4	8.2	8.1
20	18.6	18.3	18.0
30	28.0	27.5	27.0
40	37.3	36.6	36.0
50	46.6	45.8	45.0

	δ
6	0.6
7	0.6
8	0.6
9	0.1
10	0.1
20	0.1
30	0.2
40	0.3
50	0.4

TABLE VIII.—LOGARITHMIC VERSED SINES AND EXTERNAL SECANTS.

26°   **27°**

′	Log. Vers.	D	Log. Exsec.	D	Log. Vers.	D	Log. Exsec.	D	′
0	9.00520̂	55	9.05154̂	61	9.03740	52̂	9.08752	59	0
1	.00575̂	54̂	.05215̂	61	.03792̂	52̂	.08811	59	1
2	.00630	54̂	.05276	60	.03845	53	.08870	59	2
3	.00684̂	54̂	.05337	61	.03898̂	52̂	.08929	59	3
4	.00739	55	.05398	60	.03950̂	52	.08988	59	4
5	9.00794	54̂	9.05458̂	60	9.04002̂	52̂	9.09047	59	5
6	.00848	54	.05519̂	60	.04055	52	.09106	58	6
7	.00903	54	.05580	60	.04107̂	52̂	.09164̂	59	7
8	.00957	54	.05640̂	60	.04160	52	.09223̂	59	8
9	.01011̂	54̂	.05701	61	.04212̂	52	.09282̂	59	9
10	9.01066	54̂	9.05762	60	9.04264̂	52	9.09341	58	10
11	.01120̂	54̂	.05822̂	60	.04317	52	.09400	59	11
12	.01174̂	54	.05883	60	.04369̂	52̂	.09458̂	58	12
13	.01229	54	.05943̂	60	.04421̂	52	.09517̂	59	13
14	.01283	54	.06004	60	.04473̂	52	.09576	58	14
15	9.01337̂	54̂	9.06064̂	60	9.04525̂	52	9.09634̂	58	15
16	.01391̂	54	.06124̂	60	.04577̂	52̂	.09693	59	16
17	.01445̂	54	.06185	60	.04630	52	.09752̂	58	17
18	.01499̂	54	.06245̂	60	.04682̂	52	.09810̂	58	18
19	.01554	54	.06305	60	.04734	52	.09869	58	19
20	9.01608	54	9.06366	60	9.04786	51̂	9.09927̂	58	20
21	.01662	53̂	.06426	60	.04837̂	52	.09986	58	21
22	.01715̂	54	.06486̂	60	.04889̂	52	.10044̂	58	22
23	.01769̂	54	.06546̂	60	.04941̂	52	.10102̂	58	23
24	.01823̂	54	.06606	60	.04993̂	52	.10161	58	24
25	9.01877̂	53	9.06667	60	9.05045	52	9.10219̂	58	25
26	.01931	54̂	.06727	60	.05097̂	51̂	.10278	58	26
27	.01985	53̂	.06787	60	.05148̂	52	.10336̂	58	27
28	.02038̂	54	.06847	60	.05200	52	.10394̂	58	28
29	.02092̂	53̂	.06907	60	.05252	51̂	.10452̂	58	29
30	9.02146	53̂	9.06967	60	9.05303̂	52	9.10511	58	30
31	.02199̂	54	.07027	60	.05355̂	51̂	.10569	58	31
32	.02253̂	53̂	.07087	59̂	.05407	51̂	.10627	58	32
33	.02307	53̂	.07146̂	60	.05458̂	51̂	.10685̂	58	33
34	.02360̂	53̂	.07206̂	60	.05510	51̂	.10743̂	58	34
35	9.02414	53̂	9.07266̂	59̂	9.05561̂	51̂	9.10801̂	58	35
36	.02467̂	53	.07326	60	.05613	51̂	.10859̂	58	36
37	.02521	53	.07386	59̂	.05664̂	51	.10917̂	58	37
38	.02574	53	.07445̂	60	.05715̂	51̂	.10975̂	58	38
39	.02627̂	53	.07505	59̂	.05767	51	.11033̂	58	39
40	9.02681	53	9.07565̂	59̂	9.05818̂	51	9.11091̂	58	40
41	.02734	53	.07624̂	59̂	.05869̂	51	.11149̂	57̂	41
42	.02787̂	53	.07684	59̂	.05921	51	.11207	58	42
43	.02840̂	53̂	.07743̂	60	.05972̂	51̂	.11265̂	58	43
44	.02894	53	.07803̂	59̂	.06023̂	51	.11323	57	44
45	9.02947	53	9.07863	59̂	9.06074̂	51	9.11380̂	58	45
46	.03000	53	.07922̂	59	.06125̂	51	.11438̂	58	46
47	.03053	53	.07981̂	59̂	.06176̂	51	.11496̂	57	47
48	.03106	53	.08041̂	59	.06227̂	51̂	.11554	57̂	48
49	.03159	53	.08100̂	59̂	.06279	51	.11611̂	57	49
50	9.03212	53	9.08160	59̂	9.06330	51	9.11669̂	58	50
51	.03265	53	.08219̂	59̂	.06380̂	51	.11727	57	51
52	.03318	53	.08278̂	59	.06431̂	51	.11784̂	57	52
53	.03371	52̂	.08338	59	.06482̂	51	.11842	57̂	53
54	.03423̂	53	.08397	59̂	.06533	51	.11899̂	58	54
55	9.03476̂	53	9.08456̂	59	9.06584̂	51	9.11957̂	57	55
56	.03529̂	52̂	.08515̂	59	.06635	51	.12015	57	56
57	.03582	52̂	.08574̂	59	.06686	50̂	.12072	57̂	57
58	.03634̂	53	.08634	59	.06736̂	51	.12129̂	57̂	58
59	.03687̂	52̂	.08693	59	.06787̂	50̂	.12187	57	59
60	9.03740		9.08752		9.06838		9.12244̂		60
′	Log. Vers.	D	Log. Exsec.	D	Log. Vers.	D	Log. Exsec.	D	′

P. P.

	61	60	59
6	6.1	6.0	5.9
7	7.1	7.0	6.9
8	8.1	8.0	7.8
9	9.1	9.0	8.8
10	10.1	10.0	9.8
20	20.3	20.0	19.6
30	30.5	30.0	29.5
40	40.6	40.0	39.3
50	50.8	50.0	49.1

	58	57
6	5.8	5.7
7	6.7	6.6
8	7.7	7.6
9	8.7	8.5
10	9.6	9.5
20	19.3	19.0
30	29.0	28.5
40	38.6	38.0
50	48.3	47.5

	55	54
6	5.5	5.4
7	6.4	6.3
8	7.3	7.2
9	8.2	8.1
10	9.1	9.0
20	18.3	18.0
30	27.5	27.0
40	36.6	36.0
50	45.8	45.0

	53	52
6	5.3	5.2
7	6.2	6.6
8	7.0	6.9
9	7.9	7.8
10	8.8	8.6
20	17.6	17.3
30	26.0	26.0
40	35.3	34.6
50	44.1	43.3

	51	0̂
6	5.1	0.6
7	5.9	0.6
8	6.8	0.6
9	7.6	0.1
10	8.5	0.1
20	17.0	0.1
30	25.8	0.2
40	34.0	0.3
50	42.5	0.4

TABLE VIII.—LOGARITHMIC VERSED SINES AND EXTERNAL SECANTS.

28°   29°

′	Log. Vers.	D	Log. Exsec.	D	Log. Vers.	D	Log. Exsec.	D	′
0	9.06838	50	9.12244̂	58	9.09823	49	9.15641	56	0
1	.06888	51	.12302	57	.09872	48	.15697	55	1
2	.06939̂	50	.12359̂	57	.09920̂	49	.15752̂	56	2
3	.06996̂	50	.12416̂	58	.09969̂	49	.15808	56	3
4	.07046̂	50	.12474	57	.10018	49	.15864	56	4
5	9.07091	50	9.12531	57	9.10067	48	9.15920	55	5
6	.07141̂	50	.12588̂	57	.10115̂	49	.15975̂	56	6
7	.07192	50	.12645̂	58	.10164	49	.16031̂	55	7
8	.07242̂	50	.12703	57	.10213	48	.16087	55	8
9	.07293	50	.12760	57	.10261̂	49	.16142̂	56	9
10	9.07343	50	9.12817	57	9.10310	48	9.16198̂	56	10
11	.97393̂	51	.12874	57	.10358	49	.16254	55	11
12	.07444	50	.12931̂	57	.10407	48	.16309̂	56	12
13	.07494	50	.12988̂	57	.10455̂	49	.16365	55	13
14	.07544̂	50	.13045	57	.10504	48	.16420̂	56	14
15	9.07594̂	50	9.13102̂	57	9.10552̂	49	9.16476	55	15
16	.07644̂	51	.13159	57	.10601	48	.16531̂	56	16
17	.07695	50	.13216̂	57	.10649	48	.16587	55	17
18	.07745	50	.13273	57	.10697̂	49	.16642̂	56	18
19	.07795	50	.13330	57	.10746	48	.16698	55	19
20	9.07845	50	9.13387	57	9.10794	48	9.16753	55	20
21	.07895	50	.13444	56	.10842̂	48	.16808	56	21
22	.07945	50	.13500̂	57	.10890̂	49	.16864	55	22
23	.07995	50	.13557	57	.10939	48	.16919	55	23
24	.08045	50	.13614	57	.10987	48	.16974̂	55	24
25	9.08095	50	9.13671	56	9.11035	48	9.17029̂	55	25
26	.08145	50	.13727̂	57	.11083	48	.17085	55	26
27	.08195	49	.13784̂	57	.11131̂	48	.17140	55	27
28	.08244̂	50	.13841	56	.11179̂	48	.17195̂	55	28
29	.08294̂	50	.13897	57	.11227̂	48	.17250̂	55	29
30	9.08344	50	9.13954̂	57	9.11275̂	48	9.17305̂	56	30
31	.08394	49	.14011	56	.11323	48	.17361	55	31
32	.08443̂	50	.14067̂	57	.11371̂	48	.17416̂	55	32
33	.08493	50	.14124	56	.11419	47	.17471	55	33
34	.08543	49	.14180	57	.11467	48	.17526	55	34
35	9.08592̂	50	9.14237	56	9.11515	48	9.17581	55	35
36	.08642	49	.14293̂	57	.11562̂	47	.17636	55	36
37	.08691̂	50	.14350	56	.11610̂	48	.17691	55	37
38	.08741	49	.14406̂	56	.11658̂	48	.17746	55	38
39	.08790̂	50	.14462̂	57	.11706	47	.17801	55	39
40	9.08840	49	9.14519	56	9.11754	48	9.17856	55	40
41	.08889̂	50	.14575̂	56	.11801̂	47	.17910̂	54	41
42	.08939	49	.14631̂	57	.11849	48	.17965̂	55	42
43	.08988	49	.14688	56	.11897	47	.18020̂	55	43
44	.09037̂	50	.14744	56	.11944̂	48	.18075	54	44
45	9.09087	49	9.14800̂	56	9.11992	47	9.18130	55	45
46	.09136	49	.14856̂	57	.12039̂	47	.18185	55	46
47	.09185̂	49	.14913	56	.12087	47	.18239̂	54	47
48	.09234̂	49	.14969	56	.12134̂	48	.18294	55	48
49	.09284	49	.15025	56	.12182	47	.18349	54	49
50	9.09333	49	9.15081̂	56	9.12229̂	47	9.18403̂	55	50
51	.09382	49	.15137	56	.12277	47	.18458	55	51
52	.09431̂	49	.15193̂	56	.12324	47	.18513	54	52
53	.09480̂	49	.15240̂	56	.12371̂	48	.18567̂	54	53
54	.09529̂	49	.15305̂	56	.12419	47	.18622	54	54
55	9.09578̂	49	9.15361̂	56	9.12466	47	9.18676̂	54	55
56	.09627̂	49	.15417̂	56	.12513̂	47	.18731	55	56
57	.09676̂	49	.15473̂	56	.12560̂	48	.18786	54	57
58	.09725̂	49	.15529	56	.12608	47	.18840̂	54	58
59	.09774̂	49	.15585	55	.12655	47	.18890̂	54	59
60	9.09823		9.15641		9.12702		9.18949		60
′	Log. Vers.	D	Log. Exsec.	D	Log. Vers.	D	Log. Exsec.	D	′

P. P.

	57	57	56
6	5.7	5.7	5.6
7	6.7	6.6	6.6
8	7.6	7.6	7.5
9	8.6	8.5	8.5
20	19.1	19.0	18.8
30	28.7	28.5	28.2
40	38.3	38.0	37.6
50	47.9	47.5	47.1

	56	55	55
6	5.6	5.5	5.5
7	6.5	6.5	6.4
8	7.4	7.4	7.3
9	8.4	8.3	8.2
10	9.3	9.2	9.1
20	18.6	18.5	18.3
30	28.0	27.7	27.5
40	37.3	37.0	36.6
50	46.6	46.2	45.8

	54	54
6	5.4	5.4
7	6.3	6.3
8	7.2	7.2
9	8.2	8.1
10	9.1	9.0
20	18.1	18.0
30	27.2	27.0
40	36.3	36.0
50	45.4	45.0

	51	50	50
6	5.1	5.0	5.0
7	5.9	5.9	5.8
8	6.8	6.7	6.6
9	7.6	7.6	7.5
10	8.5	8.4	8.3
20	17.0	16.8	16.6
30	25.5	25.2	25.0
40	34.0	33.6	33.3
50	42.5	42.1	41.6

	49	49	48
6	4.9	4.9	4.8
7	5.8	5.7	5.6
8	6.6	6.5	6.4
9	7.4	7.3	7.3
10	8.2	8.1	8.1
20	16.5	16.3	16.1
30	24.7	24.5	24.2
40	33.0	32.6	32.3
50	41.2	40.8	40.4

	48	47	47
6	4.8	4.7	4.7
7	5.6	5.5	5.5
8	6.4	6.3	6.2
9	7.2	7.1	7.0
10	8.0	7.9	7.8
20	16.0	15.8	15.6
30	24.0	23.7	23.5
40	32.0	31.6	31.3
50	40.0	39.6	39.1

TABLE VIII.—LOGARITHMIC VERSED SINES AND EXTERNAL SECANTS.

30°   31°

′	Log. Vers.	D	Log. Exsec.	D	Log. Vers.	D	Log. Exsec.	D	′
0	9.12702	47	9.18949	54	9.15483	45	9.22176	53	0
1	.12749	47	.19003	54	.15528	45	.22229	53	1
2	.12796	47	.19058	54	.15574	45	.22282	53	2
3	.12843	47	.19112	54	.15619	45	.22335	53	3
4	.12890	47	.19167	54	.15665	45	.22388	53	4
5	9.12937	47	9.19221	54	9.15710	45	9.22441	53	5
6	.12984	47	.19275	54	.15755	45	.22494	53	6
7	.13031	47	.19329	54	.15801	45	.22547	53	7
8	.13078	47	.19384	54	.15846	45	.22600	53	8
9	.13125	47	.19438	54	.15891	45	.22653	53	9
10	9.13172	46	9.19492	54	9.15937	45	9.22706	53	10
11	.13219	47	.19546	54	.15982	45	.22759	53	11
12	.13266	47	.19601	54	.16027	45	.22812	53	12
13	.13313	46	.19655	54	.16073	45	.22865	52	13
14	.13359	47	.19709	54	.16118	45	.22918	52	14
15	9.13406	46	9.19763	54	9.16163	45	9.22971	53	15
16	.13453	47	.19817	54	.16208	45	.23024	52	16
17	.13500	46	.19871	54	.16253	45	.23076	53	17
18	.13546	47	.19925	54	.16298	45	.23129	53	18
19	.13593	46	.19970	54	.16343	45	.23182	52	19
20	9.13639	46	9.20033	54	9.16388	45	9.23235	53	20
21	.13686	47	.20087	54	.16434	45	.23287	53	21
22	.13733	46	.20141	54	.16479	44	.23340	53	22
23	.13779	46	.20195	54	.16523	45	.23393	53	23
24	.13826	46	.20249	54	.16568	45	.23446	52	24
25	9.13872	46	9.20303	53	9.16613	45	9.23498	53	25
26	.13919	46	.20357	54	.16658	45	.23551	52	26
27	.13965	46	.20411	54	.16703	45	.23603	53	27
28	.14011	46	.20465	53	.16748	45	.23656	53	28
29	.14058	46	.20518	54	.16793	44	.23709	52	29
30	9.14104	46	9.20572	54	9.16838	45	9.23761	53	30
31	.14151	46	.20626	54	.16882	44	.23814	52	31
32	.14197	46	.20680	53	.16927	45	.23866	53	32
33	.14243	46	.20733	54	.16972	45	.23919	52	33
34	.14289	46	.20787	54	.17017	44	.23971	53	34
35	9.14336	46	9.20841	53	9.17061	45	9.24024	52	35
36	.14382	46	.20894	54	.17106	45	.24076	52	36
37	.14428	46	.20948	54	.17151	44	.24128	53	37
38	.14474	46	.21002	53	.17195	45	.24181	52	38
39	.14520	46	.21056	53	.17240	44	.24233	52	39
40	9.14566	46	9.21109	53	9.17284	45	9.24285	53	40
41	.14612	46	.21162	53	.17329	44	.24338	52	41
42	.14658	46	.21216	53	.17373	45	.24390	52	42
43	.14704	46	.21269	53	.17418	44	.24442	53	43
44	.14750	46	.21323	53	.17462	45	.24495	52	44
45	9.14796	46	9.21376	53	9.17507	44	9.24547	52	45
46	.14842	46	.21430	53	.17551	44	.24599	52	46
47	.14888	46	.21483	53	.17596	44	.24651	53	47
48	.14934	45	.21537	53	.17640	44	.24704	52	48
49	.14980	46	.21590	53	.17684	44	.24756	52	49
50	9.15026	46	9.21643	53	9.17729	44	9.24808	52	50
51	.15071	45	.21697	53	.17773	44	.24860	52	51
52	.15117	46	.21750	53	.17817	44	.24912	52	52
53	.15163	46	.21803	53	.17861	45	.24964	52	53
54	.15209	45	.21857	53	.17906	44	.25016	52	54
55	9.15254	46	9.21910	53	9.17950	44	9.25068	52	55
56	.15300	46	.21963	53	.17994	44	.25120	52	56
57	.15346	45	.22016	53	.18038	44	.25172	52	57
58	.15391	46	.22070	53	.18082	44	.25224	52	58
59	.15437	46	.22123	53	.18126	44	.25276	52	59
60	9.15483		9.22176		9.18170		9.25328		60
′	Log. Vers.	D	Log. Exsec.	D	Log. Vers.	D	Log. Exsec.	D	′

P. P.

	54	54	53
6	5.4	5.4	5.3
7	6.3	6.3	6.2
8	7.2	7.2	7.1
9	8.2	8.1	8.0
10	9.1	9.0	8.9
20	18.1	18.0	17.8
30	27.2	27.0	26.7
40	36.3	36.0	35.6
50	45.4	45.0	44.6

	53	52	52
6	5.3	5.2	5.2
7	6.2	6.1	6.0
8	7.0	7.0	6.9
9	7.9	7.9	7.8
10	8.8	8.7	8.6
20	17.6	17.5	17.3
30	26.5	26.2	26.0
40	35.3	35.0	34.6
50	44.1	43.7	43.3

	47	47	46
6	4.7	4.7	4.6
7	5.5	5.5	5.4
8	6.3	6.2	6.2
9	7.1	7.0	7.0
10	7.9	7.8	7.7
20	15.8	15.6	15.5
30	23.7	23.5	23.2
40	31.6	31.2	31.0
50	39.6	39.1	38.7

	46	45	45
6	4.6	4.5	4.5
7	5.3	5.3	5.2
8	6.1	6.0	6.0
9	6.9	6.8	6.7
10	7.6	7.6	7.5
20	15.3	15.1	15.0
30	23.0	22.7	22.5
40	30.6	30.3	30.0
50	38.3	37.9	37.5

	44	44
6	4.4	4.4
7	5.2	5.1
8	5.9	5.8
9	6.7	6.6
10	7.4	7.3
20	14.8	14.6
30	22.2	22.0
40	29.6	29.3
50	37.1	36.6

TABLE VIII.—LOGARITHMIC VERSED SINES AND EXTERNAL SECANTS.

32°   **33°**

′	Log. Vers.	D	Log. Exsec.	D	Log. Vers.	D	Log. Exsec.	D	′
0	9.18170		9.25328		9.20771		9.28412		0
1	.18214	44	.25380	52	.20814	42	.28463	51	1
2	.18258	44	.25432	52	.20856	42	.28514	51	2
3	.18302	44	.25484	52	.20899	43	.28564	50	3
4	.18346	44	.25536	51	.20942	43	.28615	51	4
5	9.18390	44	9.25588	52	9.20984	42	9.28666	51	5
6	.18434	44	.25640	52	.21027	42	.28717	51	6
7	.18478	44	.25692	51	.21069	42	.28768	50	7
8	.18522	44	.25743	52	.21112	42	.28818	51	8
9	.18566	43	.25795	52	.21154	42	.28869	50	9
10	9.18610	44	9.25847	52	9.21196	42	9.28920	51	10
11	.18654	44	.25899	51	.21239	42	.28970	50	11
12	.18697	43	.25950	52	.21281	42	.29021	51	12
13	.18741	44	.26002	51	.21324	42	.29072	50	13
14	.18785	43	.26054	51	.21366	42	.29122	50	14
15	9.18829	44	9.26105	52	9.21408	42	9.29173	51	15
16	.18872	43	.26157	51	.21451	42	.29223	50	16
17	.18916	43	.26209	51	.21493	42	.29274	50	17
18	.18959	44	.26260	51	.21535	42	.29324	51	18
19	.19003	44	.26312	52	.21577	42	.29375	51	19
20	9.19047	43	9.26364	51	9.21620	42	9.29426	50	20
21	.19090	43	.26415	51	.21662	42	.29476	51	21
22	.19134	43	.26467	51	.21704	42	.29527	50	22
23	.19177	43	.26518	51	.21746	42	.29577	50	23
24	.19221	43	.26570	51	.21788	42	.29627	50	24
25	9.19264	43	9.26621	51	9.21830	42	9.29678	50	25
26	.19308	43	.26673	51	.21872	42	.29728	50	26
27	.19351	43	.26724	51	.21914	42	.29779	50	27
28	.19395	43	.26776	51	.21956	42	.29829	50	28
29	.19438	43	.26827	51	.21998	42	.29879	50	29
30	9.19481	43	9.26878	51	9.22040	42	9.29930	50	30
31	.19525	43	.26930	51	.22082	42	.29980	50	31
32	.19568	43	.26981	51	.22124	42	.30030	50	32
33	.19611	43	.27032	51	.22166	42	.30081	50	33
34	.19654	43	.27084	51	.22208	42	.30131	50	34
35	9.19698	43	9.27135	51	9.22250	42	9.30181	50	35
36	.19741	43	.27186	51	.22292	41	.30231	50	36
37	.19784	43	.27238	51	.22334	42	.30282	50	37
38	.19827	43	.27289	51	.22376	42	.30332	50	38
39	.19870	43	.27340	51	.22417	41	.30382	50	39
40	9.19914	43	9.27391	51	9.22459	42	9.30432	50	40
41	.19957	43	.27443	51	.22501	42	.30482	50	41
42	.20000	43	.27494	51	.22543	41	.30533	50	42
43	.20043	43	.27545	51	.22584	42	.30583	50	43
44	.20086	43	.27596	51	.22626	42	.30633	50	44
45	9.20129	43	9.27647	51	9.22668	41	9.30683	50	45
46	.20172	43	.27698	51	.22709	42	.30733	50	46
47	.20215	43	.27749	51	.22751	41	.30783	50	47
48	.20258	43	.27800	51	.22792	42	.30833	50	48
49	.20301	43	.27852	51	.22834	41	.30883	50	49
50	9.20343	42	9.27903	51	9.22876	41	9.30933	50	50
51	.20386	43	.27954	51	.22917	41	.30983	50	51
52	.20429	43	.28005	51	.22959	41	.31033	50	52
53	.20472	43	.28056	51	.23000	41	.31083	50	53
54	.20515	42	.28107	51	.23042	41	.31133	50	54
55	9.20558	43	9.28157	50	9.23083	41	9.31183	49	55
56	.20600	42	.28208	51	.23124	41	.31233	50	56
57	.20643	43	.28259	51	.23166	41	.31283	50	57
58	.20686	43	.28310	51	.23207	41	.31333	50	58
59	.20728	42	.28361	50	.23248	41	.31383	50	59
60	9.20771	43	9.28412		9.23290		9.31432	49	60
′	Log. Vers.	D	Log. Exsec.	D	Log. Vers.	D	Log. Exsec.	D	′

P. P.

	52	5î	51
6	5.2	5.î	5.1
7	6.ô	6.0	5.9
8	6.9	6.8	6.8
9	7.8	7.7	7.6
10	8.6	8.6	8.5
20	17.3	17.î	17.0
30	26.0	25.7	25.5
40	34.6	34.3	34.0
50	43.3	42.9	42.5

	5ô	50	4ô
6	5.0	5.0	4.9
7	5.9	5.8	5.8
8	6.7	6.6	6.6
9	7.6	7.5	7.4
10	8.4	8.3	8.2
20	16.8	16.6	16.5
30	25.2	25.0	24.7
40	33.6	33.3	33.0
50	42.1	41.6	41.2

	44	43	43
6	4.4	4.3	4.3
7	5.1	5.1	5.0
8	5.8	5.8	5.7
9	6.6	6.5	6.4
10	7.3	7.2	7.1
20	14.6	14.5	14.3
30	22.0	21.7	21.5
40	29.3	29.0	28.6
50	36.6	36.2	35.8

	4î	42	4î
6	4.2	4.2	4.1
7	4.9	4.9	4.8
8	5.6	5.6	5.5
9	6.4	6.3	6.2
10	7.1	7.0	6.9
20	14.1	14.0	13.8
30	21.2	21.0	20.7
40	28.3	28.0	27.6
50	35.4	35.0	34.6

	41
6	4.1
7	4.8
8	5.4
9	6.1
10	6.8
20	13.6
30	20.5
40	27.3
50	34.1

TABLE VIII.—LOGARITHMIC VERSED SINES AND EXTERNAL SECANTS.

34°   35°

′	Log. Vers.	D	Log. Exsec.	D	Log. Vers.	D	Log. Exsec.	D	′
0	9.23290	41̂	9.31432̂	50	9.25731̂	40	9.34395	49	0
1	.23331̂	41	.31482̂	50	.25771̂	40	.34444	48	1
2	.23372̂	41̂	.31532̂	49	.25811̂	40	.34492̂	49	2
3	.23414	41	.31582	50	.25851̂	40	.34541̂	49	3
4	.23455	41	.31632	49	.25891̂	40	.34590̂	49	4
5	9.23496̂	41̂	9.31681̂	50	9.25931̂	40	9.34639̂	49	5
6	.23537	41	.31731̂	49	.25971̂	40	.34688	49	6
7	.23579	41̂	.31781	50	.26011̂	40	.34737	48	7
8	.23620	41	.31831	49	.26051̂	39̂	.34785̂	49	8
9	.23661	41	.31880̂	50	.26091	40	.34834̂	49	9
10	9.23702	41	9.31930̂	50	9.26131	40	9.34883̂	49	10
11	.23743̂	41̂	.31980	49	.26171	40	.34932	48	11
12	.23784̂	41	.32029̂	50	.26210̂	39̂	.34980̂	49	12
13	.23825̂	41	.32079	50	.26250̂	40	.35029̂	49	13
14	.23866̂	41	.32129	49	.26290̂	40	.35078	48	14
15	9.23907	41	9.32178̂	49	9.26330	39̂	9.35127	49	15
16	.23948̂	41	.32228	49	.26370	40	.35175̂	48	16
17	.23989̂	41	.32277̂	49	.26409̂	39̂	.35224̂	49	17
18	.24030̂	41	.32327	50	.26449̂	40	.35273	48	18
19	.24071̂	41	.32377	49	.26489	39̂	.35321̂	49	19
20	9.24112	40̂	9.32426̂	49	9.26528̂	39̂	9.35370	48	20
21	.24153	41	.32476	49	.26568̂	40	.35419	49	21
22	.24194	41	.32525̂	49	.26608	39̂	.35467̂	48	22
23	.24235	41	.32575	49	.26647̂	39̂	.35516	48	23
24	.24275̂	40̂	.32624	49	.26687	39̂	.35564̂	48	24
25	9.24316̂	41	9.32673̂	49	9.26726̂	39̂	9.35613	48	25
26	.24357	40̂	.32723	49	.26766̂	40	.35661̂	48	26
27	.24398	41	.32772̂	49	.26806	39̂	.35710	48	27
28	.24438̂	40̂	.32822	49	.26845̂	39̂	.35758̂	48	28
29	.24479̂	41	.32871̂	49	.26885	39̂	.35807	48	29
30	9.24520	40̂	9.32920̂	49	9.26924̂	39̂	9.35855̂	48	30
31	.24561	41	.32970	49	.26964	39̂	.35904	48	31
32	.24601̂	40̂	.33019̂	49	.27003̂	39	.35952̂	48	32
33	.24642	40̂	.33069	49	.27042̂	39̂	.36001	48	33
34	.24682̂	40̂	.33118̂	49	.27082	39̂	.36049̂	48	34
35	9.24723̂	41	9.33167̂	49	9.27121̂	39̂	9.36098	48	35
36	.24764	40̂	.33216̂	49	.27161	39̂	.36146̂	48	36
37	.24804̂	40̂	.33266	49	.27200	39̂	.36194̂	48	37
38	.24845	40̂	.33315	49	.27239̂	39	.36243	48	38
39	.24885̂	40̂	.33364̂	49	.27278	39̂	.36291̂	48	39
40	9.24926	40̂	9.33413̂	49	9.27318	39	9.36340	48	40
41	.24966̂	40̂	.33463	49	.27357	39̂	.36388	48	41
42	.25007	40̂	.33512	49	.27396̂	39̂	.36436̂	48	42
43	.25047̂	40̂	.33561	49	.27435̂	39	.36484̂	48	43
44	.25087̂	40	.33610̂	49	.27475	39	.36533	48	44
45	9.25128	40̂	9.33659̂	49	9.27514	39	9.36581̂	48	45
46	.25168̂	40̂	.33708̂	49	.27553	39̂	.36629̂	48	46
47	.25209	40	.33758	49	.27592̂	39	.36678	48	47
48	.25249	40	.33807	49	.27631̂	39	.36726	48	48
49	.25289̂	40̂	.33856	49	.27670̂	39	.36774	48	49
50	9.25329̂	40	9.33905	49	9.27709̂	39	9.36822̂	48	50
51	.25370	40	.33954	49	.27749	39	.36870̂	48	51
52	.25410	40̂	.34003	49	.27788	39	.36919	48	52
53	.25450̂	40	.34052̂	49	.27827	39	.36967	48	53
54	.25490̂	40̂	.34101	49	.27866	39	.37015	48	54
55	9.25531	40	9.34150	49	9.27905	39	9.37063	48	55
56	.25571	40	.34199	49	.27944	38	.37111̂	48	56
57	.25611	40	.34248	49	.27982̂	39	.37159̂	48	57
58	.25651	40̂	.34297	49	.28021̂	39	.37207	48	58
59	.25691̂	40	.34346̂	49	.28060̂	39	.37255̂	48	59
60	9.25731̂		9.34395		9.28099̂		9.37303̂		60
′	Log. Vers.	D	Log. Exsec.	D	Log. Vers.	D	Log. Exsec.	D	′

P. P.

	50	49̂	49
6	5.0	4.9̂	4.9
7	5.8	5.8	5.7
8	6.6	6.6	6.5
9	7.5	7.4	7.3
10	8.3	8.2	8.1
20	16.6	16.5	16.3
30	25.0	24.7	24.5
40	33.3	33.0	32.6
50	41.6	41.2	40.8

	48̂	48
6	4.8̂	4.8
7	5.6	5.6
8	6.4	6.4
9	7.3	7.2
10	8.1	8.0
20	16.1̂	16.0
30	24.2̂	24.0
40	32.3̂	32.0
50	40.4	40.0

	41̂	41
6	4.1̂	4.1
7	4.8̂	4.8
8	5.5	5.4̂
9	6.2	6.1̂
10	6.9	6.8
20	13.8	13.6
30	20.7	20.5
40	27.6	27.3̂
50	34.6	34.1

	40̂	40
6	4.0	4.0
7	4.7	4.6̂
8	5.4	5.3
9	6.1	6.0
10	6.7̂	6.6̂
20	13.5	13.3
30	20.2	20.0
40	27.0	26.6̂
50	33.7	33.3

	39̂	39
6	3.9̂	3.9
7	4.6	4.5̂
8	5.2̂	5.2
9	5.9	5.8̂
10	6.6	6.5
20	13.1̂	13.0
30	19.7	19.5
40	26.3̂	26.0
50	32.9	32.5

	38̂
6	3.8̂
7	4.5
8	5.1̂
9	5.8
10	6.4
20	12.8̂
30	19.2
40	25.6̂
50	32.1

TABLE VIII.—LOGARITHMIC VERSED SINES AND EXTERNAL SECANTS.

36°   37°

′	Log. Vers.	D	Log. Exsec.	D	Log. Vers.	D	Log. Exsec.	D	′
0	9.28099		9.37303		9.30398		9.40163		0
1	.28138	39	.37352	48	.30436	37	.40210	47	1
2	.28177	38	.37400	48	.30474	38	.40258	47	2
3	.28216	39	.37448	48	.30511	37	.40305	47	3
4	.28255	39	.37496	48	.30549	37	.40352	47	4
5	9.28293	38	9.37544	48	9.30587	38	9.40399	47	5
6	.28332	39	.37592	48	.30624	37	.40447	47	6
7	.28371	38	.37640	48	.30662	38	.40494	47	7
8	.28410	39	.37687	47	.30700	37	.40541	47	8
9	.28448	38	.37735	48	.30737	37	.40588	47	9
10	9.28487	39	9.37783	48	9.30775	37	9.40635	47	10
11	.28526	38	.37831	48	.30812	37	.40682	47	11
12	.28564	38	.37879	48	.30850	37	.40730	47	12
13	.28603	39	.37927	48	.30887	37	.40777	47	13
14	.28642	38	.37975	47	.30925	37	.40824	47	14
15	9.28680	38	9.38023	48	9.30962	37	9.40871	47	15
16	.28719	38	.38071	48	.31000	37	.40918	47	16
17	.28757	38	.38119	47	.31037	37	.40965	47	17
18	.28796	39	.38166	48	.31075	37	.41012	47	18
19	.28835	38	.38214	48	.31112	37	.41059	47	19
20	9.28873	38	9.38262	48	9.31150	37	9.41106	47	20
21	.28912	38	.38310	47	.31187	37	.41153	47	21
22	.28950	38	.38357	48	.31224	38	.41200	47	22
23	.28988	39	.38405	48	.31262	37	.41247	47	23
24	.29027	38	.38453	48	.31299	37	.41294	47	24
25	9.29065	38	9.38501	47	9.31336	37	9.41341	47	25
26	.29104	38	.38548	48	.31374	37	.41388	47	26
27	.29142	38	.38596	48	.31411	37	.41435	47	27
28	.29180	38	.38644	48	.31448	37	.41482	47	28
29	.29219	38	.38692	47	.31485	38	.41529	47	29
30	9.29257	38	9.38739	48	9.31523	37	9.41576	46	30
31	.29295	38	.38787	47	.31560	37	.41623	47	31
32	.29334	38	.38834	48	.31597	37	.41670	47	32
33	.29372	38	.38882	48	.31634	37	.41717	46	33
34	.29410	38	.38930	47	.31671	37	.41763	47	34
35	9.29448	38	9.38977	48	9.31708	38	9.41810	47	35
36	.29487	38	.39025	47	.31746	37	.41857	47	36
37	.29525	38	.39072	48	.31783	37	.41904	46	37
38	.29563	38	.39120	48	.31820	37	.41951	47	38
39	.29601	38	.39168	47	.31857	37	.41998	46	39
40	9.29639	38	9.39215	48	9.31894	37	9.42044	47	40
41	.29677	38	.39263	47	.31931	37	.42091	47	41
42	.29715	38	.39310	48	.31968	37	.42138	46	42
43	.29754	38	.39358	47	.32005	37	.42185	47	43
44	.29792	38	.39405	48	.32042	37	.42231	46	44
45	9.29830	38	9.39453	47	9.32079	37	9.42278	47	45
46	.29868	38	.39500	48	.32116	37	.42325	46	46
47	.29906	38	.39548	47	.32153	37	.42372	46	47
48	.29944	38	.39595	47	.32190	37	.42418	47	48
49	.29982	38	.39642	48	.32227	37	.42465	47	49
50	9.30020	38	9.39690	47	9.32263	36	9.42512	46	50
51	.30057	37	.39737	48	.32300	37	.42558	46	51
52	.30095	38	.39785	47	.32337	37	.42605	47	52
53	.30133	38	.39832	47	.32374	37	.42652	46	53
54	.30171	38	.39879	47	.32411	36	.42698	47	54
55	9.30209	38	9.39927	47	9.32447	37	9.42745	47	55
56	.30247	37	.39974	47	.32484	37	.42792	46	56
57	.30285	38	.40021	47	.32521	37	.42838	47	57
58	.30322	37	.40069	47	.32558	36	.42885	46	58
59	.30360	38	.40116	47	.32594	37	.42931	46	59
60	9.30398		9.40163		9.32631		9.42978		60
′	Log. Vers.	D	Log. Exsec.	D	Log. Vers.	D	Log. Exsec.	D	′

P. P.

	48	48
6	4.8	4.8
7	5.6	5.6
8	6.4	6.4
9	7.3	7.2
10	8.1	8.0
20	16.1	16.0
30	24.2	24.0
40	32.3	32.0
50	40.4	40.0

	47	47
6	4.7	4.7
7	5.5	5.5
8	6.3	6.2
9	7.1	7.0
10	7.9	7.8
20	15.8	15.6
30	23.7	23.5
40	31.6	31.3
50	39.6	39.1

	46
6	4.6
7	5.4
8	6.2
9	7.0
10	7.7
20	15.5
30	23.2
40	31.0
50	38.7

	39	38
6	3.9	3.8
7	4.5	4.5
8	5.2	5.1
9	5.8	5.8
10	6.5	6.4
20	13.0	12.8
30	19.5	19.2
40	26.0	25.6
50	32.5	32.1

	38	37
6	3.8	3.7
7	4.4	4.4
8	5.0	5.0
9	5.7	5.6
10	6.3	6.2
20	12.6	12.5
30	19.0	18.7
40	25.3	25.0
50	31.6	31.2

	37	36
6	3.7	3.6
7	4.3	4.2
8	4.9	4.8
9	5.5	5.5
10	6.1	6.1
20	12.3	12.1
30	18.5	18.2
40	24.6	24.3
50	30.8	30.4

TABLE VIII.—LOGARITHMIC VERSED SINES AND EXTERNAL SECANTS.

38°   39°

′	Log. Vers.	D	Log. Exsec.	D	Log. Vers.	D	Log. Exsec.	D	′
0	9.32631	36	9.42978	46	9.34802	35	9.45752	45	0
1	.32668	36	.43024	47	.34837	36	.45797	46	1
2	.32704	37	.43071	46	.34873	35	.45843	46	2
3	.32741	36	.43118	46	.34909	35	.45889	46	3
4	.32778	36	.43164	46	.34944	35	.45935	46	4
5	9.32814	36	9.43211	46	9.34980	36	9.45981	45	5
6	.32851	36	.43257	46	.35016	35	.46027	46	6
7	.32888	37	.43304	46	.35051	35	.46073	46	7
8	.32924	36	.43350	46	.35087	35	.46118	45	8
9	.32961	36	.43396	46	.35122	35	.46164	46	9
10	9.32997	36	9.43443	46	9.35158	35	9.46210	46	10
11	.33034	36	.43489	46	.35193	35	.46256	45	11
12	.33070	36	.43536	46	.35229	35	.46302	45	12
13	.33107	36	.43582	46	.35264	35	.46347	45	13
14	.33143	36	.43629	46	.35300	35	.46393	46	14
15	9.33180	36	9.43675	46	9.35335	35	9.46439	45	15
16	.33216	36	.43721	46	.35370	35	.46485	46	16
17	.33252	36	.43768	46	.35406	35	.46530	45	17
18	.33289	36	.43814	46	.35441	35	.46576	46	18
19	.33325	36	.43861	46	.35477	35	.46622	46	19
20	9.33361	36	9.43907	46	9.35512	35	9.46668	45	20
21	.33398	36	.43953	46	.35547	35	.46713	46	21
22	.33434	36	.43999	46	.35583	35	.46759	46	22
23	.33470	36	.44046	46	.35618	35	.46805	45	23
24	.33507	36	.44092	46	.35653	35	.46850	45	24
25	9.33543	36	9.44138	46	9.35689	35	9.46896	46	25
26	.33579	36	.44185	46	.35724	35	.46942	45	26
27	.33615	36	.44231	46	.35759	35	.46987	45	27
28	.33652	36	.44277	46	.35794	35	.47033	45	28
29	.33688	36	.44323	46	.35829	35	.47078	45	29
30	9.33724	36	9.44370	46	9.35865	35	9.47124	46	30
31	.33760	36	.44416	46	.35900	35	.47170	45	31
32	.33796	36	.44462	46	.35935	35	.47215	45	32
33	.33833	36	.44508	46	.35970	35	.47261	45	33
34	.33869	36	.44554	46	.36005	35	.47306	45	34
35	9.33905	36	9.44601	46	9.36040	35	9.47352	46	35
36	.33941	36	.44647	46	.36076	35	.47398	45	36
37	.33977	36	.44693	46	.36111	35	.47443	45	37
38	.34013	36	.44739	46	.36146	35	.47489	45	38
39	.34049	36	.44785	46	.36181	35	.47534	45	39
40	9.34085	36	9.44831	46	9.36216	35	9.47580	45	40
41	.34121	36	.44877	46	.36251	35	.47625	45	41
42	.34157	36	.44924	46	.36286	35	.47671	45	42
43	.34193	36	.44970	46	.36321	35	.47716	45	43
44	.34229	36	.45016	46	.36356	35	.47762	45	44
45	9.34265	36	9.45062	46	9.36391	35	9.47807	45	45
46	.34301	36	.45108	46	.36426	35	.47853	45	46
47	.34337	36	.45154	46	.36461	34	.47898	45	47
48	.34373	35	.45200	46	.36495	35	.47943	45	48
49	.34408	36	.45246	46	.36530	35	.47989	45	49
50	9.34444	36	9.45292	46	9.36565	35	9.48034	45	50
51	.34480	36	.45338	46	.36600	35	.48080	45	51
52	.34516	35	.45384	46	.36635	34	.48125	45	52
53	.34552	36	.45430	46	.36670	35	.48170	45	53
54	.34587	35	.45476	46	.36705	34	.48216	45	54
55	9.34623	36	9.45522	46	9.36739	34	9.48261	45	55
56	.34659	36	.45568	46	.36774	35	.48306	45	56
57	.34695	36	.45614	46	.36809	34	.48352	45	57
58	.34730	35	.45660	46	.36844	35	.48397	45	58
59	.34766	36	.45706	46	.36878	34	.48442	45	59
60	9.34802		9.45752		9.36913		9.48488		60
′	Log. Vers.	D	Log. Exsec.	D	Log. Vers.	D	Log. Exsec.	D	′

P. P.

	47	46
6	4.7	4.6
7	5.5	5.4
8	6.2	6.2
9	7.0	7.0
10	7.8	7.7
20	15.6	15.5
30	23.5	23.2
40	31.3	31.0
50	39.1	38.7

	46	45
6	4.6	4.5
7	5.3	5.3
8	6.1	6.0
9	6.9	6.8
10	7.6	7.6
20	15.3	15.1
30	23.0	22.7
40	30.6	30.3
50	38.3	37.9

	45
6	4.5
7	5.2
8	6.0
9	6.7
10	7.5
20	15.0
30	22.5
40	30.0
50	37.5

	37	36
6	3.7	3.6
7	4.3	4.2
8	4.9	4.8
9	5.5	5.5
10	6.1	6.1
20	12.3	12.1
30	18.5	18.2
40	24.6	24.3
50	30.8	30.4

	36	35
6	3.6	3.5
7	4.2	4.1
8	4.8	4.7
9	5.4	5.3
10	6.0	5.9
20	12.0	11.8
30	18.0	17.7
40	24.0	23.6
50	30.0	29.6

	35	34
6	3.5	3.4
7	4.1	4.0
8	4.6	4.6
9	5.2	5.2
10	5.8	5.7
20	11.6	11.5
30	17.5	17.2
40	23.3	23.0
50	29.1	28.7

TABLE VIII.—LOGARITHMIC VERSED SINES AND EXTERNAL SECANTS.

40°   41°

′	Log. Vers.	D	Log. Exsec.	D	Log. Vers.	D	Log. Exsec.	D	′
0	9.36913̂	34̂	9.48488	45	9.38968	34	9.51190	45	0
1	.36948	34̂	.48533	45	.39002	33̂	.51235	44̂	1
2	.36982̂	35	.48578̂	45̂	.39035̂	34	.51279̂	45	2
3	.37017̂	34̂	.48624	45̂	.39069̂	33̂	.51324̂	44̂	3
4	.37052	34̂	.48669	45	.39103	34	.51369	44̂	4
5	9.37086̂	35	9.48714̂	45̂	9.39137	33̂	9.51414	45	5
6	.37121̂	34̂	.48759̂	45̂	.39170̂	33̂	.51458̂	45	6
7	.37156	34̂	.48805	45	.39204	34	.51503̂	44̂	7
8	.37190̂	34̂	.48850	45	.39238̂	33̂	.51548	44̂	8
9	.37225	34̂	.48895̂	45	.39271̂	33̂	.51592̂	45	9
10	9.37259̂	34̂	9.48940̂	45̂	9.39305	33̂	9.51637̂	44̂	10
11	.37294	34̂	.48986	45̂	.39339	33̂	.51682	44̂	11
12	.37328̂	34̂	.49031	45	.39372̂	33̂	.51726̂	45	12
13	.37363	34̂	.49076	45̂	.39406	33̂	.51771̂	44̂	13
14	.37397̂	34̂	.49121̂	45	.39439̂	33̂	.51816	44̂	14
15	9.37432	34̂	9.49166̂	45	9.39473	33̂	9.51860̂	44̂	15
16	.37466̂	34̂	.49211̂	45̂	.39507	34	.51905̂	45	16
17	.37501	34̂	.49257	45̂	.39540̂	33̂	.51950	44̂	17
18	.37535̂	34̂	.49302	45	.39574̂	33̂	.51994̂	44̂	18
19	.37570	34̂	.49347	45̂	.39607	33̂	.52039	45	19
20	9.37604̂	34̂	9.49392̂	45̂	9.39641	33̂	9.52084	44̂	20
21	.37639	34̂	.49437̂	45	.39674̂	33̂	.52128̂	44̂	21
22	.37673̂	34	.49482̂	45	.39708	33̂	.52173	44̂	22
23	.37707̂	34	.49527̂	45	.39741̂	33̂	.52217̂	44̂	23
24	.37742	34	.49572̂	45̂	.39774̂	33	.52262	44̂	24
25	9.37776	34̂	9.49618	45	9.39808	33̂	9.52306̂	44̂	25
26	.37810̂	34̂	.49663	45	.39841̂	33̂	.52351	44̂	26
27	.37845	34̂	.49708	45	.39875	33̂	.52396	44̂	27
28	.37879̂	34	.49753	45	.39908̂	33	.52440̂	44̂	28
29	.37913̂	34	.49798	45̂	.39941̂	33	.52485	44̂	29
30	9.37947̂	34̂	9.49843	45	9.39975	33̂	9.52529̂	44̂	30
31	.37982	34̂	.49888	45	.40008	33	.52574	44̂	31
32	.38016	34̂	.49933	45	.40041̂	33	.52618̂	44̂	32
33	.38050̂	34	.49978	45	.40075̂	33	.52663	44̂	33
34	.38084̂	34	.50023	45	.40108	33	.52707̂	44̂	34
35	9.38118̂	34̂	9.50068	45	9.40141̂	33	9.52752	44̂	35
36	.38153	34̂	.50113	45	.40175	33	.52796̂	44̂	36
37	.38187	34	.50158	45	.40208	33	.52841	44̂	37
38	.38221	34	.50203	45	.40241̂	33	.52885̂	44̂	38
39	.38255	34̂	.50248	45	.40274̂	33	.52930	44̂	39
40	9.38289	34̂	9.50293	45	9.40307	33	9.52974	44̂	40
41	.38323̂	34	.50338	45	.40341	33̂	.53018̂	44̂	41
42	.38357̂	34	.50383	44	.40374	33	.53063	44̂	42
43	.38391̂	34	.50427̂	45	.40407	33̂	.53107̂	44̂	43
44	.38425̂	34	.50472̂	45	.40440̂	33	.53152	44̂	44
45	9.38459̂	34	9.50517̂	45	9.40473̂	33	9.53196̂	44̂	45
46	.38493̂	34	.50562̂	45	.40506̂	33	.53240̂	44̂	46
47	.38527̂	34	.50607̂	44	.40540	33̂	.53285	44̂	47
48	.38561̂	34	.50652	45	.40573̂	33	.53329̂	44̂	48
49	.38595̂	34	.50697	45	.40606	33	.53374	44̂	49
50	9.38629̂	34	9.50742	45	9.40639	33	9.53418	44̂	50
51	.38663̂	34	.50787	45	.40672	33	.53462̂	44̂	51
52	.38697̂	33̂	.50831̂	44	.40705	33	.53507	44̂	52
53	.38731	34	.50876̂	45	.40738	33	.53551̂	44̂	53
54	.38765	34	.50921̂	44̂	.40771̂	33	.53595̂	44̂	54
55	9.38799	34	9.50966	44̂	9.40804	33	9.53640	44̂	55
56	.38833	33̂	.51011	45	.40837	33	.53684̂	44̂	56
57	.38866̂	33̂	.51055̂	44	.40870	33	.53728	44̂	57
58	.38900̂	33̂	.51100̂	45	.40903	33	.53773	44̂	58
59	.38934	34	.51145̂	44	.40936	33	.53817	44̂	59
60	9.38968		9.51190		9.40969		9.53861̂		60
′	Log. Vers.	D	Log. Exsec.	D	Log. Vers.	D	Log. Exsec.	D	′

P. P.

	45̂	45
6	4.5̂	4.5
7	5.3	5.2
8	6.0	6.0
9	6.8	6.7
10	7.6	7.5
20	15.1̂	15.0
30	22.7̂	22.5
40	30.3̂	30.0
50	37.9	37.5

	44̂	44
6	4.4	4.4
7	5.2	5.1̂
8	5.9	5.8
9	6.7	6.6
10	7.4	7.3̂
20	14.8	14.6
30	22.2	22.0
40	29.6̂	29.3̂
50	37.1	36.6̂

	35	34̂
6	3.5	3.4
7	4.1	4.0
8	4.6̂	4.6
9	5.2	5.2
10	5.8	5.7
20	11.6	11.5
30	17.5	17.2̂
40	23.3̂	23.0
50	29.1̂	28.7̂

	34	33̂
6	3.4	3.3
7	3.9	3.9
8	4.5	4.4
9	5.1	5.0
10	5.6̂	5.6
20	11.3	11.1̂
30	17.0	16.7̂
40	22.6̂	22.3̂
50	28.3	27.9

	33
6	3.3
7	3.8
8	4.4
9	4.9
10	5.5
20	11.0
30	16.5
40	22.0
50	27.5

TABLE VIII.—LOGARITHMIC VERSED SINES AND EXTERNAL SECANTS.

42°　　　43°

′	Log. Vers.	D	Log. Exsec.	D	Log. Vers.	D	Log. Exsec.	D	′
0	9.40969	3̂2	9.53861	4̂4	9.42918	32	9.56505	4̂3	0
1	.41001	33	.53906	44	.42950	32	.56549	44	1
2	.41034	33	.53950	4̂4	.42982	3̂2	.56593	44	2
3	.41067	33	.53994	44	.43014	3̂2	.56637	44	3
4	.41100	33	.54038	4̂4	.43046	32	.56680	4̂3	4
5	9.41133	3̂2	9.54083	44	9.43078	32	9.56724	44	5
6	.41166	33	.54127	4̂4	.43110	3̂1	.56768	44	6
7	.41199	33	.54171	44	.43142	32	.56812	4̂3	7
8	.41231	3̂2	.54215	44	.43174	32	.56856	44	8
9	.41264	33	.54259	44	.43206	3̂2	.56899	4̂3	9
10	9.41297	3̂2	9.54304	44	9.43238	32	9.56943	44	10
11	.41330	33	.54348	44	.43270	32	.56987	44	11
12	.41362	3̂2	.54392	44	.43302	32	.57031	4̂3	12
13	.41395	33	.54436	44	.43334	3̂1	.57075	4̂3	13
14	.41428	3̂2	.54480	44	.43365	32	.57118	44	14
15	9.41461	33	9.54525	4̂4	9.43397	32	9.57162	44	15
16	.41493	3̂2	.54569	44	.43429	32	.57206	4̂3	16
17	.41526	33	.54613	44	.43461	3̂1	.57250	44	17
18	.41559	3̂2	.54657	4̂4	.43493	32	.57293	4̂3	18
19	.41591	3̂2	.54701	44	.43525	3̂1	.57337	44	19
20	9.41624	33	9.54745	44	9.43557	32	9.57381	4̂3	20
21	.41657	3̂2	.54790	44	.43588	3̂1	.57424	4̂3	21
22	.41689	3̂2	.54834	44	.43620	32	.57468	44	22
23	.41722	3̂2	.54878	44	.43652	3̂1	.57512	4̂3	23
24	.41754	3̂2	.54922	4̂4	.43684	32	.57556	44	24
25	9.41787	3̂2	9.54966	44	9.43715	3̂1	9.57599	4̂3	25
26	.41819	3̂2	.55010	44	.43747	3̂1	.57643	44	26
27	.41852	33	.55054	44	.43779	3̂1	.57687	4̂3	27
28	.41885	3̂2	.55098	44	.43810	32	.57730	4̂3	28
29	.41917	3̂2	.55142	44	.43842	3̂1	.57774	44	29
30	9.41950	3̂2	9.55186	44	9.43874	32	9.57818	4̂3	30
31	.41982	32	.55230	4̂4	.43906	3̂1	.57861	4̂3	31
32	.42014	3̂2	.55275	44	.43937	3̂1	.57905	44	32
33	.42047	3̂2	.55319	44	.43969	3̂1	.57949	4̂3	33
34	.42079	3̂2	.55363	44	.44000	32	.57992	4̂3	34
35	9.42112	3̂2	9.55407	44	9.44032	3̂1	9.58036	4̂3	35
36	.42144	3̂2	.55451	44	.44064	3̂1	.58079	44	36
37	.42177	32	.55495	44	.44095	3̂1	.58123	4̂3	37
38	.42209	3̂2	.55539	44	.44127	3̂1	.58167	4̂3	38
39	.42241	3̂2	.55583	44	.44158	3̂1	.58210	4̂3	39
40	9.42274	3̂2	9.55627	44	9.44190	3̂1	9.58254	4̂3	40
41	.42306	32	.55671	44	.44221	3̂1	.58297	44	41
42	.42338	3̂2	.55715	44	.44253	3̂1	.58341	4̂3	42
43	.42371	32	.55759	44	.44284	3̂1	.58385	4̂3	43
44	.42403	3̂2	.55803	44	.44316	3̂1	.58428	4̂3	44
45	9.42435	32	9.55847	4̂3	9.44347	3̂1	9.58472	4̂3	45
46	.42467	3̂2	.55890	4̂3	.44379	3̂1	.58515	4̂3	46
47	.42500	32	.55934	44	.44410	3̂1	.58559	4̂3	47
48	.42532	3̂2	.55978	44	.44442	3̂1	.58602	4̂3	48
49	.42564	32	.56022	44	.44473	3̂1	.58646	4̂3	49
50	9.42596	32	9.56066	44	9.44504	31	9.58689	4̂3	50
51	.42629	32	.56110	44	.44536	3̂1	.58733	4̂3	51
52	.42661	32	.56154	4̂3	.44567	3̂1	.58776	44	52
53	.42693	3̂2	.56198	44	.44599	31	.58820	4̂3	53
54	.42725	32	.56242	44	.44630	3̂1	.58864	4̂3	54
55	9.42757	32	9.56286	44	9.44661	3̂1	9.58907	4̂3	55
56	.42789	3̂2	.56330	44	.44693	31	.58951	4̂3	56
57	.42822	32	.56374	4̂3	.44724	3̂1	.58994	4̂3	57
58	.42854	32	.56417	44	.44755	3̂1	.59037	4̂3	58
59	.42886	32	.56461	4̂3	.44787	31	.59081	4̂3	59
60	9.42918		9.56505		9.44818		9.59124		60
′	Log. Vers.	D	Log. Exsec.	D	Log. Vers.	D	Log. Exsec.	D	′

P. P.

	44̂	44
6	4.4	4.4
7	5.2	5.1
8	5.9	5.8
9	6.7	6.6
10	7.4	7.3
20	14.8	14.6
30	22.2	22.0
40	29.6	29.3
50	37.1	36.6

	4̂3	43
6	4.3	4.3
7	5.1	5.0
8	5.8	5.7
9	6.5	6.4
10	7.2	7.1
20	14.5	14.3
30	21.7	21.5
40	29.0	28.6
50	36.2	35.8

	33	3̂2
6	3.3	3.2
7	3.8	3.8
8	4.4	4.3
9	4.9	4.9
10	5.5	5.4
20	11.0	10.8
30	16.5	16.2
40	22.0	21.6
50	27.5	27.1

	32	3̂1
6	3.2	3.1
7	3.7	3.7
8	4.2	4.2
9	4.8	4.7
10	5.3	5.2
20	10.6	10.5
30	16.0	15.7
40	21.3	21.0
50	26.6	26.2

	31
6	3.1
7	3.6
8	4.1
9	4.6
10	5.1
20	10.3
30	15.5
40	20.6
50	25.8

TABLE VIII.—LOGARITHMIC VERSED SINES AND EXTERNAL SECANTS.

44°   45°

′	Log. Vers.	D	Log. Exsec.	D	Log. Vers.	D	Log. Exsec.	D	′	P. P.
0	9.44818	3î	9.59124̂	4³	9.46671	3ô	9.61722̂	43	0	
1	.44849̂	3I	.59168	4³	.4670î	3ô	.61765̂	43	1	
2	.44886̂	3î	.5921î	4³	.46732	3ô	.61808̂	43	2	
3	.44912	3I	.59255	4³	.46762̂	3ô	.61852	4³	3	
4	.44943	3î	.59298	4³	.46793	3ô	.61895	43	4	**43³** · **43**
5	9.44974̂	3I	9.59342	4³	9.46823̂	3ô	9.61938	43	5	6 · 4.3 · 4.3
6	.45005̂	3I	.59385̂	4³	.46853̂	30	.61981	43	6	7 · 5.1 · 5.0
7	.45036̂	3î	.59429	4³	.46884	3ô	.62024	43	7	8 · 5.8 · 5.7
8	.45068	3I	.59472̂	4³	.46914̂	3ô	.62067̂	43	8	9 · 6.5 · 6.4
9	.45099	3I	.5951̂5	43	.46945	3ô	.6211̂0	43	9	10 · 7.2 · 7.1
10	9.45130	3I	9.59559	4³	9.46975̂	3ô	9.62153̂	43	10	20 · 14.5 · 14.3
11	.45161	3î	.5960̂2	4³	.47005̂	30	.62196̂	43	11	30 · 21.7 · 21.5
12	.45192̂	3I	.59646	4³	.47036	3ô	.6223̂9	43	12	40 · 29.0 · 28.6
13	.45223̂	3I	.5968̂9	4³	.47066̂	3ô	.62282̂	4³	13	50 · 36.2 · 35.8
14	.45254̂	3I	.5973̂2	43	.47096	3ô	.62326	4³	14	
15	9.45285̂	3I	9.59776	4³	9.47127	3ô	9.62369	43	15	
16	.45316	3I	.5981̂9	4³	.47157̂	3ô	.62412	43	16	**4²**
17	.45348	3î	.59863	4³	.47187̂	30	.62455	43	17	6 · 4.2
18	.45379	3I	.59906	43	.47218	3ô	.62498	43	18	7 · 4.9
19	.45410	3I	.5994̂9	43	.47248	3ô	.62541	43	19	8 · 5.6
20	9.45441	3I	9.59993	43	9.47278̂	3ô	9.62584	43	20	9 · 6.4
21	.45472	3I	.60036	43	.47308	30	.62627	43	21	10 · 7.1
22	.45503	3I	.6007̂9	4³	.47339	3ô	.62670	43	22	20 · 14.1
23	.45534	3I	.60123	43	.47369	30	.62713	43	23	30 · 21.2
24	.45565	3ô	.60166	4³	.47399̂	3ô	.62756	43	24	40 · 28.3
25	9.45595̂	3I	9.6020̂9	4³	9.47429̂	30	9.62799	43	25	50 · 35.4
26	.45626̂	3I	.60253	4³	.47459̂	30	.62842	43	26	
27	.45657̂	3I	.60296	4³	.47490	3ô	.62885̂	43	27	**3î** · **31**
28	.45688̂	3I	.6033̂9	4³	.47520	30	.62928	43	28	6 · 3.1 · 3.1
29	.45719̂	3I	.60383	43	.47550	30	.62971	43	29	7 · 3.7 · 3.6
30	9.45750	3I	9.60426	4³	9.47580̂	3ô	9.63014	43	30	8 · 4.2 · 4.1
31	.45781	3ô	.6046̂9	4³	.4761̂0	30	.63057	43	31	9 · 4.7 · 4.6
32	.45812	3I	.6051̂2	4³	.47640̂	30	.63100	43	32	10 · 5.2 · 5.1
33	.45843	3ô	.60556	43	.47670̂	30	.63143	43	33	20 · 10.5 · 10.3
34	.45873̂	3I	.60599	43	.47700̂	3ô	.63186	43	34	30 · 15.7 · 15.5
35	9.45904̂	3I	9.6064̂2	4³	9.47731	3ô	9.63229	43	35	40 · 21.0 · 20.6
36	.45935̂	3ô	.6068̂5	4³	.47761	30	.63272	43	36	50 · 26.2 · 25.8
37	.45966	3I	.60729	43	.47791	30	.63315	43	37	
38	.45997	3ô	.60772	4³	.47821	30	.63358	43	38	
39	.46027	31	.6081̂5	43	.47851	30	.63401	43	39	**3ô** · **30**
40	9.46058	3ô	9.60858	4³	9.47881	30	9.63443̂	4²	40	6 · 3.0 · 3.0
41	.46089	3ô	.60902	4³	.47911	30	.63486̂	43	41	7 · 3.5 · 3.5
42	.46120	3I	.60945	43	.47941	30	.6352̂9	43	42	8 · 4.0 · 4.0
43	.46150̂	3ô	.6098̂8	4³	.47971	30	.6357̂2	43	43	9 · 4.6 · 4.5
44	.46181̂	3I	.6103̂1	43	.48001	30	.63615̂	43	44	10 · 5.1 · 5.0
45	9.46212	3ô	9.61075	4³	9.48031	30	9.63658	43	45	20 · 10.1 · 10.0
46	.46242̂	3ô	.61118	4³	.48061	30	.63701	4²	46	30 · 15.2 · 15.0
47	.46273̂	3I	.6116̂1	4³	.48090̂	2ô	.63744	43	47	40 · 20.3 · 20.0
48	.46304	3ô	.6120̂4	43	.48120̂	30	.63787	43	48	50 · 25.4 · 25.0
49	.46334̂	3ô	.61247	43	.48150̂	30	.63830	43	49	
50	9.46365̂	3I	9.61291	4³	9.48180̂	30	9.63873	43	50	
51	.46396	3ô	.61334	43	.48210̂	30	.6391̂5	4²	51	**2ô**
52	.46426̂	3ô	.61377	4³	.48240	2ô	.6395̂8	43	52	6 · 2.9
53	.46457	3ô	.6142̂0	43	.48270	30	.6400î	43	53	7 · 3.4
54	.46487̂	3ô	.6146̂3	43	.48300	30	.6404̂4	43	54	8 · 3.9
55	9.46518̂	3I	9.6150̂6	4³	9.48329̂	2ô	9.64087	4²	55	9 · 4.4
56	.46549	3ô	.61550	43	.48359̂	30	.64130	43	56	10 · 4.9
57	.46579̂	3ô	.61593	43	.48389	30	.64173	43	57	20 · 9.8
58	.46610	3ô	.61636	4³	.48419	2ô	.64216	43	58	30 · 14.7
59	.46640̂	3ô	.6167̂9	43	.48449	30	.64258̂	4²	59	40 · 19.6
60	9.46671		9.61722̂		9.48478̂	2ô	9.6430î	43	60	50 · 24.6
′	Log. Vers.	D	Log. Exsec.	D	Log. Vers.	D	Log. Exsec.	D	′	P. P.

TABLE VIII.—LOGARITHMIC VERSED SINES AND EXTERNAL SECANTS.

46°   47°

′	Log. Vers.	D	Log. Exsec.	D	Log. Vers.	D	Log. Exsec.	D	′	P. P.
0	9.48478	30	9.64301	43	9.50243	29	9.66864	42	0	
1	.48508	29	.64344	42	.50272	29	.66907	43	1	
2	.48538	30	.64387	43	.50301	29	.66950	42	2	
3	.48568	29	.64430	43	.50330	29	.66992	42	3	
4	.48597	30	.64473	42	.50359	29	.67035	42	4	
5	9.48627	29	9.64515	43	9.50388	29	9.67077	42	5	**43** **42**
6	.48657	29	.64558	43	.50417	29	.67120	42	6	6 4.3 4.2
7	.48686	30	.64601	42	.50446	29	.67162	43	7	7 5.0 4.9
8	.48716	29	.64644	43	.50475	29	.67205	42	8	8 5.7 5.6
9	.48746	29	.64687	42	.50504	29	.67248	42	9	9 6.4 6.4
10	9.48775	30	9.64729	43	9.50533	29	9.67290	42	10	10 7.1 7.1
11	.48805	29	.64772	43	.50562	29	.67333	42	11	20 14.3 14.1
12	.48835	29	.64815	43	.50591	28	.67375	42	12	30 21.5 21.2
13	.48864	29	.64858	43	.50619	29	.67418	42	13	40 28.6 28.3
14	.48894	29	.64901	42	.50648	29	.67460	42	14	50 35.8 35.4
15	9.48923	30	9.64943	43	9.50677	29	9.67503	43	15	
16	.48953	29	.64986	43	.50706	28	.67546	42	16	**42**
17	.48983	29	.65029	43	.50735	29	.67588	42	17	6 4.2
18	.49012	29	.65072	42	.50764	29	.67631	42	18	7 4.9
19	.49042	29	.65114	43	.50793	28	.67673	42	19	8 5.6
20	9.49071	29	9.65157	42	9.50821	29	9.67716	42	20	9 6.3
21	.49101	29	.65200	43	.50850	29	.67758	42	21	10 7.0
22	.49130	29	.65243	42	.50879	28	.67801	42	22	20 14.0
23	.49160	29	.65285	43	.50908	29	.67843	42	23	30 21.0
24	.49189	29	.65328	43	.50937	28	.67886	42	24	40 28.0
25	9.49219	29	9.65371	43	9.50965	29	9.67928	42	25	50 35.0
26	.49248	29	.65414	42	.50994	29	.67971	42	26	
27	.49278	29	.65456	43	.51023	29	.68013	42	27	**30** **29**
28	.49307	29	.65499	43	.51052	28	.68056	42	28	6 3.0 2.9
29	.49336	29	.65542	43	.51080	29	.68098	42	29	7 3.5 3.4
30	9.49366	29	9.65585	42	9.51109	29	9.68141	42	30	8 4.0 3.9
31	.49395	29	.65627	43	.51138	29	.68183	42	31	9 4.5 4.4
32	.49425	29	.65670	43	.51167	28	.68226	42	32	10 5.0 4.9
33	.49454	29	.65713	42	.51195	28	.68268	42	33	20 10.0 9.8
34	.49483	29	.65755	43	.51224	29	.68311	42	34	30 15.0 14.7
35	9.49513	29	9.65798	43	9.51253	28	9.68353	43	35	40 20.0 19.6
36	.49542	29	.65841	43	.51281	29	.68396	42	36	50 25.0 24.6
37	.49571	29	.65884	42	.51310	28	.68438	42	37	
38	.49601	29	.65926	43	.51338	29	.68481	42	38	**29** **28**
39	.49630	29	.65969	43	.51367	29	.68523	42	39	6 2.9 2.8
40	9.49659	29	9.66012	42	9.51396	28	9.68566	42	40	7 3.4 3.3
41	.49689	29	.66054	43	.51424	29	.68608	42	41	8 3.8 3.8
42	.49718	29	.66097	43	.51453	28	.68651	42	42	9 4.3 4.3
43	.49747	29	.66140	42	.51481	29	.68693	42	43	10 4.8 4.7
44	.49776	29	.66182	43	.51510	29	.68735	42	44	20 9.6 9.5
45	9.49806	29	9.66225	43	9.51539	28	9.68778	42	45	30 14.5 14.2
46	.49835	29	.66268	42	.51567	29	.68820	42	46	40 19.3 19.0
47	.49864	29	.66310	43	.51596	28	.68863	42	47	50 24.1 23.7
48	.49893	29	.66353	43	.51624	29	.68905	42	48	
49	.49922	29	.66396	42	.51653	28	.68948	42	49	
50	9.49952	29	9.66438	43	9.51681	29	9.68990	42	50	**28**
51	.49981	29	.66481	42	.51710	28	.69033	42	51	6 2.8
52	.50010	29	.66523	43	.51738	29	.69075	42	52	7 3.2
53	.50039	29	.66566	43	.51767	28	.69117	42	53	8 3.7
54	.50068	29	.66609	42	.51795	28	.69160	42	54	9 4.2
55	9.50097	29	9.66651	43	9.51823	29	9.69202	42	55	10 4.6
56	.50126	29	.66694	43	.51852	28	.69245	42	56	20 9.3
57	.50155	29	.66737	42	.51880	29	.69287	42	57	30 14.0
58	.50185	29	.66779	43	.51909	28	.69330	42	58	40 18.6
59	.50214	29	.66822	42	.51937	28	.69372	42	59	50 23.3
60	9.50243		9.66864		9.51965		9.69414		60	
	Log. Vers.	D	Log. Exsec.	D	Log. Vers.	D	Log. Exsec.	D	′	P. P.

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TABLE VIII.—LOGARITHMIC VERSED SINES AND EXTERNAL SECANTS.

48° **49°**

′	Log. Vers.	D	Log. Exsec.	D	Log. Vers.	D	Log. Exsec.	D	′	P. P.
0	9.51965	28	9.69414	42	9.53648	27	9.71954	42	0	
1	.51994	28	.69457	42	.53676	28	.71996	42	1	
2	.52022	28	.69499	42	.53704	27	.72038	42	2	
3	.52050	28	.69542	42	.53731	27	.72081	42	3	
4	.52079	28	.69584	42	.53759	28	.72123	42	4	
5	9.52107	28	9.69626	42	9.53787	27	9.72165	42	5	
6	.52135	28	.69669	42	.53814	27	.72207	42	6	
7	.52164	28	.69711	42	.53842	28	.72250	42	7	
8	.52192	28	.69753	42	.53870	27	.72292	42	8	
9	.52220	28	.69796	42	.53897	27	.72334	42	9	
10	9.52249	28	9.69838	42	9.53925	27	9.72376	42	10	
11	.52277	28	.69881	42	.53952	28	.72419	42	11	
12	.52305	28	.69923	42	.53980	27	.72461	42	12	
13	.52333	28	.69965	42	.54008	27	.72503	42	13	
14	.52362	28	.70008	42	.54035	27	.72545	42	14	**4̂2** **42**
15	9.52390	28	9.70050	42	9.54063	27	9.72587	42	15	6 4.2 4.2
16	.52418	28	.70092	42	.54090	27	.72630	42	16	7 4.9 4.9
17	.52446	28	.70135	42	.54118	27	.72672	42	17	8 5.6 5.6
18	.52474	28	.70177	42	.54145	27	.72714	42	18	9 6.4 6.3
19	.52503	28	.70220	42	.54173	27	.72756	42	19	10 7.1 7.0
20	9.52531	28	9.70262	42	9.54200	27	9.72799	42	20	20 14.1 14.0
21	.52559	28	.70304	42	.54228	27	.72841	42	21	30 21.2 21.0
22	.52587	28	.70347	42	.54255	27	.72883	42	22	40 28.3 28.0
23	.52615	28	.70389	42	.54283	27	.72925	42	23	50 35.4 35.0
24	.52643	28	.70431	42	.54310	27	.72967	42	24	
25	9.52671	28	9.70474	42	9.54338	27	9.73010	42	25	
26	.52699	28	.70516	42	.54365	27	.73052	42	26	**2̂8** **28**
27	.52727	28	.70558	42	.54393	27	.73094	42	27	6 2.8 2.8
28	.52756	28	.70601	42	.54420	27	.73136	42	28	7 3.3 3.2
29	.52784	28	.70643	42	.54448	27	.73178	42	29	8 3.8 3.7
30	9.52812	28	9.70685	42	9.54475	27	9.73221	42	30	9 4.3 4.2
31	.52840	28	.70728	42	.54502	27	.73263	42	31	10 4.7 4.6
32	.52868	28	.70770	42	.54530	27	.73305	42	32	20 9.5 9.3
33	.52896	28	.70812	42	.54557	27	.73347	42	33	30 14.2 14.0
34	.52924	28	.70854	42	.54585	27	.73389	42	34	40 19.0 18.6
35	9.52952	28	9.70897	42	9.54612	27	9.73431	42	35	50 23.7 23.3
36	.52980	28	.70939	42	.54639	27	.73474	42	36	
37	.53008	28	.70981	42	.54667	27	.73516	42	37	
38	.53036	28	.71024	42	.54694	27	.73558	42	38	
39	.53064	28	.71066	42	.54721	27	.73600	42	39	**2̂7** **27**
40	9.53092	28	9.71108	42	9.54748	27	9.73642	42	40	6 2.7 2.7
41	.53120	27	.71151	42	.54776	27	.73685	42	41	7 3.2 3.1
42	.53147	28	.71193	42	.54803	27	.73727	42	42	8 3.6 3.6
43	.53175	28	.71235	42	.54830	27	.73769	42	43	9 4.1 4.0
44	.53203	28	.71278	42	.54858	27	.73811	42	44	10 4.6 4.5
45	9.53231	27	9.71320	42	9.54885	27	9.73853	42	45	20 9.1 9.0
46	.53259	28	.71362	42	.54912	27	.73895	42	46	30 13.7 13.5
47	.53287	28	.71404	42	.54939	27	.73938	42	47	40 18.3 18.0
48	.53315	28	.71447	42	.54967	27	.73980	42	48	50 22.9 22.5
49	.53343	27	.71489	42	.54994	27	.74022	42	49	
50	9.53370	28	9.71531	42	9.55021	27	9.74064	42	50	
51	.53398	28	.71573	42	.55048	27	.74106	42	51	
52	.53426	28	.71616	42	.55075	27	.74148	42	52	
53	.53454	28	.71658	42	.55103	27	.74191	42	53	
54	.53482	27	.71700	42	.55130	27	.74233	42	54	
55	9.53509	28	9.71743	42	9.55157	27	9.74275	42	55	
56	.53537	28	.71785	42	.55184	27	.74317	42	56	
57	.53565	28	.71827	42	.55211	27	.74359	42	57	
58	.53593	27	.71869	42	.55238	27	.74401	42	58	
59	.53620	28	.71912	42	.55265	27	.74444	42	59	
60	9.53648		9.71954		9.55292		9.74486		60	
′	Log. Vers.	D	Log. Exsec.	D	Log. Vers.	D	Log. Exsec.	D	′	P. P.

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TABLE VIII.—LOGARITHMIC VERSED SINES AND EXTERNAL SECANTS.

50°   51°

′	Log. Vers.	D	Log. Exsec.	D	Log. Vers.	D	Log. Exsec.	D	′
0	9.55292̂	27	9.74486	42	9.56900	26̂	9.77012̂	42̂	0
1	.55319̂	27̂	.74528	42	.56926̂	26̂	.77055	42	1
2	.55347	27	.74570	42̂	.56953	26	.77097	42	2
3	.55374	27	.74612̂	42	.56979	26	.77139	42	3
4	.55401	27	.74654̂	42	.57005̂	26̂	.77181	42	4
5	9.55428	27	9.74696̂	42̂	9.57032	26̂	9.77223	42	5
6	.55455	27	.74739	42	.57058	26̂	.77265	42	6
7	.55482	27	.74781	42	.57085	26̂	.77307	42̂	7
8	.55509	27	.74823	42	.57111̂	26̂	.77349̂	42	8
9	.55536	27	.74865	42	.57138	26	.77391̂	42	9
10	9.55563	27	9.74907	42̂	9.57164	26̂	9.77433̂	42	10
11	.55590	27	.74949̂	42	.57190̂	26̂	.77475̂	42	11
12	.55617	27	.74991̂	42	.57217	26̂	.77517	42̂	12
13	.55644	27	.75033̂	42	.57243̂	26	.77560	42	13
14	.55671	27	.75076	42̂	.57269̂	26	.77602	42	14
15	9.55698	27	9.75118	42	9.57296	26	9.77644	42	15
16	.55725	26	.75160	42	.57322̂	26	.77686	42	16
17	.55751̂	27	.75202	42	.57348̂	26	.77728	42	17
18	.55778	27	.75244̂	42̂	.57375	26	.77770	42	18
19	.55805̂	27	.75286̂	42	.57401̂	26	.77812	42	19
20	9.55832	27	9.75328	42	9.57427̂	26	9.77854̂	42̂	20
21	.55859̂	26	.75376̂	42	.57454	26	.77896̂	42	21
22	.55886	27	.75413	42̂	.57480	26	.77938	42	22
23	.55913̂	27	.75455	42	.57506̂	26	.77980̂	42	23
24	.55940	26	.75497	42	.57532̂	26	.78022̂	42	24
25	9.55966̂	27	9.75539	42̂	9.57559	26̂	9.78064̂	42̂	25
26	.55993̂	27	.75581̂	42	.57585	26	.78107	42	26
27	.56020̂	26	.75623̂	42	.57611̂	26	.78149	42	27
28	.56047	27	.75665̂	42	.57637̂	26	.78191	42	28
29	.56074	27	.75707̂	42	.57664	26̂	.78233	42	29
30	9.56101	26	9.75750	42̂	9.57690	26	9.78275	42	30
31	.56127̂	27	.75792	42	.57716	26	.78317	42	31
32	.56154̂	27	.75834	42	.57742̂	26	.78359	42	32
33	.56181	27	.75876	42	.57768̂	26̂	.78401̂	42̂	33
34	.56208	26	.75918	42̂	.57794̂	26	.78443̂	42	34
35	9.56234̂	27	9.75966̂	42	9.57821	26	9.78485̂	42	35
36	.56261̂	26	.76002̂	42	.57847	26̂	.78527̂	42	36
37	.56288	27	.76044̂	42	.57873	26	.78569̂	42	37
38	.56315	26	.76086̂	42	.57899	26̂	.78611̂	42̂	38
39	.56341̂	27	.76128̂	42̂	.57925̂	26	.78653̂	42	39
40	9.56368	26	9.76171	42	9.57951̂	26	9.78696	42	40
41	.56395	26̂	.76213	42	.57977̂	26	.78738	42	41
42	.56421̂	26	.76255	42	.58003̂	26	.78780	42	42
43	.56448	27	.76297	42	.58029̂	26	.78822	42	43
44	.56475	26	.76339	42̂	.58055̂	26	.78864	42̂	44
45	9.56501̂	27	9.76381̂	42	9.58082	26̂	9.78906	42	45
46	.56528	26	.76423̂	42	.58108	26	.78948	42	46
47	.56554̂	27	.76465̂	42	.58134	26	.78990̂	42̂	47
48	.56581̂	26	.76507	42	.58160	26	.79032̂	42	48
49	.56608	26	.76549̂	42̂	.58186	26	.79074̂	42	49
50	9.56634̂	26	9.76592	42	9.58212	26	9.79116̂	42	50
51	.56661	26	.76634	42	.58238	26	.79158	42	51
52	.56687̂	26	.76676	42	.58264	26	.79200̂	42	52
53	.56714	27	.76718	42	.58290	26	.79242̂	42̂	53
54	.56741	26	.76760	42̂	.58316	26	.79285	42	54
55	9.56767̂	26	9.76802	42	9.58342	25	9.79327	42	55
56	.56794	26	.76844̂	42̂	.58367̂	26	.79369	42	56
57	.56820̂	26	.76886̂	42	.58393̂	26	.79411	42	57
58	.56847	26	.76928	42	.58419̂	26	.79453	42	58
59	.56873̂	26	.76970̂	42	.58445̂	26	.79495	42	59
60	9.56900		9.77012̂		9.58471̂		9.79537		60
′	Log. Vers.	D	Log. Exsec.	D	Log. Vers.	D	Log. Exsec.	D	′

P. P.

	42̂	42
6	4.2	4.2
7	4.9	4.9
8	5.6	5.6
9	6.4	6.3
10	7.1	7.0
20	14.1	14.0
30	21.2	21.0
40	28.3	28.0
50	35.4	35.0

	27̂	27
6	2.7	2.7
7	3.2	3.1
8	3.6	3.6
9	4.1	4.0
10	4.6	4.5
20	9.1	9.0
30	13.7	13.5
40	18.3	18.0
50	22.9	22.5

	26̂	26
6	2.6	2.6
7	3.1	3.0
8	3.5	3.4
9	4.0	3.9
10	4.4	4.3
20	8.8	8.6
30	13.2	13.0
40	17.6	17.3
50	22.1	21.6

	25
6	2.5
7	3.0
8	3.4
9	3.8
10	4.2
20	8.5
30	12.7
40	17.0
50	21.2

TABLE VIII.—LOGARITHMIC VERSED SINES AND EXTERNAL SECANTS.

52°   53°

′	Log. Vers.	D	Log. Exsec.	D	Log. Vers.	D	Log. Exsec.	D	′	P. P.
0	9.5847î	26	9.79537	4â2	9.60008	2ⁿ5	9.82062	4â2	**0**	
1	.58497	2ⁿ5	.7957ⁿ9	42	.60034	2ⁿ5	.8210â4	42	1	
2	.58523	26	.7962î	42	.60059	2ⁿ5	.8214â6	42	2	
3	.58549	26	.79663	42	.6008â4	2ⁿ5	.8218ⁿ8	42	3	
4	.58575	26	.7970ⁿ5	42	.60110	2ⁿ5	.8223ⁿ0	42	4	
5	9.58601	26	9.79747	42	9.60135	25	9.8227â2	42	**5**	
6	.5862ⁿ6	2ⁿ5	.7978ⁿ9	4â2	.6016ⁿ0	2ⁿ5	.82315	4â2	6	
7	.5865â2	26	.7983î	42	.6018ⁿ5	2ⁿ5	.82357	42	7	
8	.58678	2ⁿ5	.79874	4â2	.60211	25	.82399	42	8	
9	.58704	26	.79916	42	.60236	25	.82441	42	9	
10	9.58730	26	9.79958	42	9.6026î	2ⁿ5	9.8248â3	4â2	**10**	
11	.5875ⁿ6	2ⁿ5	.80000	42	.6028ⁿ6	25	.8252ⁿ5	42	11	
12	.5878î	26	.80042	42	.60312	25	.8256ⁿ7	42	12	
13	.58807	26	.80084	42	.60337	25	.8260ⁿ9	42	13	
14	.58833	2ⁿ5	.80126	42	.6036â2	2ⁿ5	.8265î	42	14	
15	9.58859	26	9.8016â8	4â2	9.6038ⁿ7	25	9.82694	4â2	**15**	
16	.5888â4	2ⁿ5	.8021ⁿ0	42	.6041â2	2ⁿ5	.82736	42	16	6 4.2̂ 4.2
17	.5891ⁿ0	26	.8025â2	42	.60438	2ⁿ5	.82778	42	17	7 4.9̂ 4.9
18	.58936	2ⁿ5	.8029â4	42	.60463	25	.82820	4â2	18	8 5.6̂ 5.6
19	.58962	26	.80336	42	.6048ⁿ8	25	.8286â2	42	19	9 6.4 6.3
20	9.58987	2ⁿ5	9.8037ⁿ8	42	9.6051â3	25	9.8290â4	42	**20**	10 7.1 7.0
21	.59013	26	.8042ⁿ0	42	.6053ⁿ8	25	.82946	42	21	20 14.1̂ 14.0
22	.59039	2ⁿ5	.80463	4â2	.6056â3	2ⁿ5	.82988	42	22	30 21.2̂ 21.0
23	.5906â4	26	.80505	42	.60589	2ⁿ5	.83031	4â2	23	40 28.3̂ 28.0
24	.5909ⁿ0	2ⁿ5	.80547	42	.60614	25	.83073	42	24	50 35.4 35.0
25	9.59116	2ⁿ5	9.80589	42	9.60639	25	9.83115	42	**25**	
26	.5914î	2ⁿ5	.80631	42	.60664	2ⁿ5	.83157	4â2	26	
27	.59167	2ⁿ5	.80673	42	.6068ⁿ9	2ⁿ5	.8319ⁿ9	42	27	
28	.59193	2ⁿ5	.8071ⁿ5	4â2	.6071â4	25	.8324î	42	28	26 25̂
29	.59218	2ⁿ5	.80757	42	.6073ⁿ9	25	.8328â3	42	29	6 2.6 2.5̂
30	9.59244	26	9.8079ⁿ9	42	9.60764	25	9.8332ⁿ5	42	**30**	7 3.0̂ 3.0
31	.59270	2ⁿ5	.8084î	42	.6078ⁿ9	25	.83368	4â2	31	8 3.4 3.4
32	.5929ⁿ5	2ⁿ5	.8088â3	42	.6081â4	25	.83410	42	32	9 3.9 3.8̂
33	.59321	2ⁿ5	.8092ⁿ5	4â2	.6083ⁿ9	25	.83452	42	33	10 4.3̂ 4.2
34	.5934â6	2ⁿ5	.80968	42	.6086â4	25	.83494	4â2	34	20 8.6 8.5
35	9.59372	2ⁿ5	9.81010	42	9.6088ⁿ9	25	9.8353â6	42	**35**	30 13.0 12.7
36	.59397	26	.81052	42	.6091â4	25	.83578	42	36	40 17.3̂ 17.0
37	.5942â3	2ⁿ5	.81094	42	.60939	25	.83620	42	37	50 21.6̂ 21.2̂
38	.59449	2ⁿ5	.81136	42	.6096â4	25	.83663	4â2	38	
39	.5947â4	2ⁿ5	.81178	42	.6098ⁿ9	25	.83705	42	39	
40	9.59500	2ⁿ5	9.8122ⁿ0	4â2	9.6101â4	25	9.83747	42	**40**	25 24̂
41	.5952ⁿ6	2ⁿ5	.8126â2	42	.6103ⁿ9	25	.8378ⁿ9	4â2	41	6 2.5 2.4̂
42	.59551	2ⁿ5	.8130â4	42	.6106â4	25	.8383î	42	42	7 2.9 2.8
43	.5957ⁿ6	2ⁿ5	.81346	42	.6108ⁿ9	25	.83873	42	43	8 3.3̂ 3.2
44	.59602	2ⁿ5	.8138â8	42	.6111â4	25	.83916	4â2	44	9 3.7 3.7
45	9.5962ⁿ7	2ⁿ5	9.8143ⁿ0	42	9.6113ⁿ9	25	9.83958	42	**45**	10 4.1̂ 4.1
46	.59653	2ⁿ5	.81473	4â2	.6116â4	25	.84000	42	46	20 8.3̂ 8.2̂
47	.5967ⁿ8	2ⁿ5	.81515	42	.6118ⁿ9	2ⁿ5	.8404â2	4â2	47	30 12.5 12.2̂
48	.59704	25	.81557	42	.61214	24	.8408â4	42	48	40 16.6̂ 16.3̂
49	.59729	2ⁿ5	.81599	42	.61239	25	.8412ⁿ6	42	49	50 20.8 20.4
50	9.5975â4	2ⁿ5	9.81641	42	9.61264	25	9.84168	42	**50**	
51	.59780	2ⁿ5	.8168â3	4â2	.61289	24	.84211	4â2	51	
52	.5980ⁿ5	2ⁿ5	.8172ⁿ5	42	.6131â3	25	.84253	42	52	
53	.59831	2ⁿ5	.8176â7	42	.61338	24	.8429ⁿ5	4â2	53	
54	.5985â6	2ⁿ5	.8180ⁿ9	42	.6136â3	25	.8433ⁿ7	42	54	
55	9.5988î	25	9.8185î	42	9.6138ⁿ8	25	9.8437ⁿ9	42	**55**	
56	.59907	2ⁿ5	.81894	4â2	.61413	24	.84422	4â2	56	
57	.5993â2	2ⁿ5	.81936	42	.61438	25	.84464	42	57	
58	.59958	2ⁿ5	.81978	42	.6146â2	24	.84506	42	58	
59	.59983	2ⁿ5	.82020	42	.6148ⁿ7	25	.84548	4â2	59	
60	9.6000ⁿ8		9.82062	42	9.6151â2	25	9.8459ⁿ0		**60**	
′	Log. Vers.	D	Log. Exsec.	D	Log. Vers.	D	Log. Exsec.	D	′	P. P.

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TABLE VIII.—LOGARITHMIC VERSED SINES AND EXTERNAL SECANTS.

54° 55°

'	Log. Vers.	D	Log. Exsec.	D	Log. Vers.	D	Log. Exsec.	D	'
0	9.61512	24̂	9.84590	42	9.62984	24̂	9.87125	42̂	0
1	.61537	25	.84632	42̂	.63008	24	.87167	42	1
2	.61562	24̂	.84675	42	.63032	24̂	.87209	42	2
3	.61586	25	.84717	42	.63057	24	.87252	42	3
4	.61611	24̂	.84759	42̂	.63081	24	.87294	42	4
5	9.61636	25	9.84801	42	9.63105	24̂	9.87336	42̂	5
6	.61661	24̂	.84843	42̂	.63129	24̂	.87379	42	6
7	.61685	25	.84886	42	.63154	24	.87421	42	7
8	.61710	24̂	.84928	42	.63178	24	.87463	42̂	8
9	.61735	25	.84970	42	.63202	24	.87506	42	9
10	9.61760	24̂	9.85012	42̂	9.63226	24	9.87548	42̂	10
11	.61784	25	.85054	42	.63250	24	.87590	42	11
12	.61809	24̂	.85097	42	.63274	24̂	.87633	42	12
13	.61834	25	.85139	42	.63299	24	.87675	42	13
14	.61858	24̂	.85181	42̂	.63323	24	.87717	42	14
15	9.61883	25	9.85223	42̂	9.63347	24	9.87760	42	15
16	.61908	24̂	.85265	42	.63371	24	.87802	42	16
17	.61932	24̂	.85308	42	.63395	24	.87844	42̂	17
18	.61957	25	.85350	42̂	.63419	24	.87887	42	18
19	.61982	24̂	.85392	42	.63443	24̂	.87929	42	19
20	9.62006	24̂	9.85434	42	9.63468	24	9.87971	42̂	20
21	.62031	24̂	.85476	42̂	.63492	24	.88014	42	21
22	.62055	25	.85519	42	.63516	24	.88056	42̂	22
23	.62080	24̂	.85561	42	.63540	24	.88099	42	23
24	.62105	24̂	.85603	42	.63564	24	.88141	42	24
25	9.62129	24̂	9.85645	42̂	9.63588	24	9.88183	42̂	25
26	.62154	24̂	.85688	42	.63612	24	.88226	42	26
27	.62178	24̂	.85730	42	.63636	24̂	.88268	42	27
28	.62203	24̂	.85772	42	.63660	24	.88310	42̂	28
29	.62227	24̂	.85814	42̂	.63684	24	.88353	42	29
30	9.62252	24̂	9.85857	42	9.63708	24	9.88395	42̂	30
31	.62276	24̂	.85899	42	.63732	24	.88438	42	31
32	.62301	24̂	.85941	42	.63756	24	.88480	42	32
33	.62325	24̂	.85983	42̂	.63780	24	.88522	42̂	33
34	.62350	24̂	.86026	42	.63804	24	.88565	42	34
35	9.62374	24̂	9.86068	42	9.63828	24	9.88607	42̂	35
36	.62399	24̂	.86110	42	.63852	24	.88650	42	36
37	.62423	24̂	.86152	42̂	.63876	24	.88692	42	37
38	.62448	24̂	.86195	42	.63900	24	.88734	42̂	38
39	.62472	24̂	.86237	42	.63924	24	.88777	42	39
40	9.62497	24̂	9.86279	42	9.63948	24	9.88819	42̂	40
41	.62521	24̂	.86321	42̂	.63972	24	.88862	42	41
42	.62546	24̂	.86364	42	.63996	23̂	.88904	42̂	42
43	.62570	24̂	.86406	42	.64019	24	.88947	42	43
44	.62594	24	.86448	42̂	.64043	24	.88989	42̂	44
45	9.62619	24̂	9.86490	42	9.64067	24	9.89031	42̂	45
46	.62643	24̂	.86533	42	.64091	24	.89074	42	46
47	.62668	24̂	.86575	42	.64115	23̂	.89116	42̂	47
48	.62692	24	.86617	42̂	.64139	24	.89159	42	48
49	.62716	24̂	.86659	42̂	.64163	24	.89201	42̂	49
50	9.62741	24̂	9.86702	42	9.64187	23̂	9.89244	42̂	50
51	.62765	24	.86744	42	.64210	24	.89286	42̂	51
52	.62789	24̂	.86786	42̂	.64234	24	.89329	42	52
53	.62814	24	.86829	42	.64258	23̂	.89371	42̂	53
54	.62838	24	.86871	42	.64282	24	.89414	42	54
55	9.62862	24̂	9.86913	42̂	9.64306	24	9.89456	42̂	55
56	.62887	24̂	.86956	42	.64330	23̂	.89499	42̂	56
57	.62911	24	.86998	42	.64353	24	.89541	42	57
58	.62935	24̂	.87040	42̂	.64377	24	.89583	42̂	58
59	.62960	24	.87082	42̂	.64401	24	.89626	42̂	59
60	9.62984		9.87125		9.64425		9.89668		60
'	Log. Vers.	D	Log. Exsec.	D	Log. Vers.	D	Log. Exsec.	D	'

P. P.

	42̂	42
6	4.2̂	4.2
7	4.9̂	4.9
8	5.6	5.6
9	6.4	6.3
10	7.1	7.0
20	14.1̂	14.0
30	21.2̂	21.0
40	28.3̂	28.0
50	35.4	35.0

	25	24̂
6	2.5	2.4̂
7	2.9	2.8̂
8	3.3	3.2
9	3.7	3.7
10	4.1̂	4.1
20	8.3̂	8.1
30	12.5	12.2̂
40	16.6̂	16.3̂
50	20.8	20.4

	24	23̂
6	2.4	2.3̂
7	2.8	2.7̂
8	3.2	3.1
9	3.6	3.5
10	4.0	3.9
20	8.0	7.8
30	12.0	11.7
40	16.0	15.6̂
50	20.0	19.6

TABLE VIII.—LOGARITHMIC VERSED SINES AND EXTERNAL SECANTS.

56° 57°

′	Log. Vers.	D	Log. Exsec.	D	Log. Vers.	D	Log. Exsec.	D	′
0	9.64425	23	9.89668	42	9.65835	23	9.92224	43	0
1	.64448	24	.89711	42	.65859	23	.92267	42	1
2	.64472	23	.89753	42	.65882	23	.92310	43	2
3	.64496	24	.89796	42	.65905	23	.92353	42	3
4	.64520	23	.89838	42	.65928	23	.92395	43	4
5	9.64543	24	9.89881	42	9.65952	23	9.92438	43	5
6	.64567	23	.89923	42	.65975	23	.92481	42	6
7	.64591	23	.89966	42	.65998	23	.92524	43	7
8	.64614	24	.90008	42	.66021	23	.92566	43	8
9	.64638	23	.90051	42	.66044	23	.92609	43	9
10	9.64662	23	9.90094	43	9.66068	23	9.92652	43	10
11	.64685	23	.90136	42	.66091	23	.92695	43	11
12	.64709	24	.90179	42	.66114	23	.92737	43	12
13	.64733	23	.90221	42	.66137	23	.92780	43	13
14	.64756	23	.90264	42	.66160	23	.92823	43	14
15	9.64780	24	9.90306	42	9.66183	23	9.92866	43	15
16	.64804	23	.90349	42	.66207	23	.92909	43	16
17	.64827	23	.90391	42	.66230	23	.92951	43	17
18	.64851	23	.90434	42	.66253	23	.92994	43	18
19	.64875	24	.90476	42	.66276	23	.93037	42	19
20	9.64898	23	9.90519	42	9.66299	23	9.93080	43	20
21	.64922	23	.90561	43	.66322	23	.93123	42	21
22	.64945	23	.90604	43	.66345	23	.93165	43	22
23	.64969	23	.90647	42	.66368	23	.93208	43	23
24	.64992	24	.90689	42	.66391	23	.93251	43	24
25	9.65016	24	9.90732	42	9.66415	23	9.93294	42	25
26	.65040	23	.90774	42	.66438	23	.93337	43	26
27	.65063	23	.90817	43	.66461	23	.93380	42	27
28	.65087	23	.90860	43	.66484	23	.93422	43	28
29	.65110	23	.90902	42	.66507	23	.93465	43	29
30	9.65134	23	9.90945	42	9.66530	23	9.93508	43	30
31	.65157	23	.90987	42	.66553	23	.93551	43	31
32	.65181	23	.91030	43	.66576	23	.93594	43	32
33	.65204	23	.91073	42	.66599	23	.93637	43	33
34	.65228	23	.91115	43	.66622	23	.93680	42	34
35	9.65251	23	9.91158	42	9.66645	23	9.93722	43	35
36	.65275	23	.91200	43	.66668	23	.93765	43	36
37	.65298	23	.91243	43	.66691	23	.93808	43	37
38	.65321	23	.91286	42	.66714	23	.93851	43	38
39	.65345	23	.91328	43	.66737	23	.93894	43	39
40	9.65368	23	9.91371	43	9.66760	23	9.93937	43	40
41	.65392	23	.91414	42	.66783	23	.93980	43	41
42	.65415	23	.91456	43	.66805	23	.94023	43	42
43	.65439	23	.91499	42	.66828	23	.94066	43	43
44	.65462	23	.91541	43	.66851	23	.94109	43	44
45	9.65485	23	9.91584	43	9.66874	23	9.94151	43	45
46	.65509	23	.91627	42	.66897	23	.94194	43	46
47	.65532	23	.91669	43	.66920	23	.94237	43	47
48	.65556	23	.91712	43	.66943	23	.94280	43	48
49	.65579	23	.91755	42	.66966	23	.94323	43	49
50	9.65602	23	9.91797	43	9.66989	23	9.94366	43	50
51	.65626	23	.91840	43	.67012	22	.94409	43	51
52	.65649	23	.91883	43	.67034	23	.94452	43	52
53	.65672	23	.91926	42	.67057	23	.94495	43	53
54	.65696	23	.91968	43	.67080	23	.94538	43	54
55	9.65719	23	9.92011	43	9.67103	22	9.94581	43	55
56	.65742	23	.92054	42	.67126	23	.94624	43	56
57	.65765	23	.92096	43	.67149	22	.94667	43	57
58	.65789	23	.92139	43	.67171	23	.94710	43	58
59	.65812	23	.92182	42	.67194	22	.94753	43	59
60	9.65835		9.92224		9.67217		9.94796		60
′	Log. Vers.	D	Log. Exsec.	D	Log. Vers.	D	Log. Exsec.	D	′

P. P.

	43	42
6	4.3	4.2
7	5.0	4.9
8	5.7	5.6
9	6.4	6.4
10	7.1	7.1
20	14.3	14.1
30	21.5	21.2
40	28.6	28.3
50	35.8	35.4

	24	23
6	2.4	2.3
7	2.8	2.7
8	3.2	3.1
9	3.6	3.5
10	4.0	3.9
20	8.0	7.8
30	12.0	11.7
40	16.0	15.6
50	20.0	19.6

	23	22
6	2.3	2.2
7	2.7	2.6
8	3.0	3.0
9	3.4	3.4
10	3.8	3.7
20	7.6	7.5
30	11.5	11.2
40	15.3	15.0
50	19.1	18.7

TABLE VIII.—LOGARITHMIC VERSED SINES AND EXTERNAL SECANTS.

58°   59°

′	Log. Vers.	D	Log. Exsec.	D	Log. Vers.	D	Log. Exsec.	D	′	P. P.
0	9.67217	23	9.94796	43	9.68571	22	9.97387	43	0	
1	.67240	23	.94839	43	.68593	22	.97430	43	1	
2	.67263	22	.94882	43	.68615	22	.97473	43	2	
3	.67285	23	.94925	43	.68637	22	.97517	43	3	
4	.67308	22	.94968	43	.68660	22	.97560	43	4	
5	9.67331	23	9.95011	43	9.68682	22	9.97603	43	5	
6	.67354	22	.95054	43	.68704	22	.97647	43	6	
7	.67376	23	.95097	43	.68727	22	.97690	43	7	
8	.67399	22	.95140	43	.68749	22	.97734	43	8	
9	.67422	23	.95183	43	.68771	22	.97777	43	9	6 44 43 / 4.4 4.3
10	9.67445	22	9.95226	43	9.68793	22	9.97820	43	10	7 5.1 5.1
11	.67467	23	.95269	43	.68816	22	.97864	43	11	8 5.8 5.8
12	.67490	23	.95313	43	.68838	22	.97907	43	12	9 6.6 6.5
13	.67513	22	.95356	43	.68860	22	.97951	43	13	10 7.3 7.2
14	.67535	23	.95399	43	.68882	22	.97994	43	14	20 14.6 14.5
15	9.67558	23	9.95442	43	9.68905	22	9.98038	43	15	30 22.0 21.7
16	.67581	22	.95485	43	.68927	22	.98081	43	16	40 29.3 29.0
17	.67603	23	.95528	43	.68949	22	.98125	43	17	50 36.6 36.2
18	.67626	23	.95571	43	.68971	22	.98168	43	18	
19	.67649	22	.95614	43	.68993	22	.98211	43	19	
20	9.67671	23	9.95657	43	9.69016	22	9.98255	43	20	
21	.67694	23	.95700	43	.69038	22	.98298	43	21	
22	.67717	22	.95744	43	.69060	22	.98342	43	22	43
23	.67739	23	.95787	43	.69082	22	.98385	43	23	6 4.3
24	.67762	22	.95830	43	.69104	22	.98429	43	24	7 5.0
25	9.67784	23	9.95873	43	9.69126	22	9.98472	43	25	8 5.7
26	.67807	23	.95916	43	.69149	22	.98516	43	26	9 6.4
27	.67830	22	.95959	43	.69171	22	.98559	44	27	10 7.1
28	.67852	23	.96002	43	.69193	22	.98603	43	28	20 14.3
29	.67875	22	.96046	43	.69215	22	.98647	43	29	30 21.5
30	9.67897	23	9.96089	43	9.69237	22	9.98690	43	30	40 28.6
31	.67920	22	.96132	43	.69259	22	.98734	43	31	50 35.8
32	.67942	23	.96175	43	.69281	22	.98777	43	32	
33	.67965	22	.96218	43	.69303	22	.98821	43	33	
34	.67987	23	.96261	43	.69325	22	.98864	43	34	23 22
35	9.68010	22	9.96305	43	9.69347	22	9.98908	44	35	6 2.3 2.2
36	.68032	23	.96348	43	.69369	22	.98952	43	36	7 2.7 2.6
37	.68055	22	.96391	43	.69392	22	.98995	43	37	8 3.0 3.0
38	.68077	23	.96434	43	.69414	22	.99039	43	38	9 3.4 3.4
39	.68100	22	.96478	43	.69436	22	.99082	43	39	10 3.8 3.7
40	9.68122	23	9.96521	43	9.69458	22	9.99126	43	40	20 7.6 7.5
41	.68145	22	.96564	43	.69480	22	.99170	44	41	30 11.5 11.2
42	.68167	23	.96607	43	.69502	22	.99213	43	42	40 15.3 15.0
43	.68190	22	.96650	43	.69524	22	.99257	43	43	50 19.1 18.7
44	.68212	23	.96694	43	.69546	22	.99300	43	44	
45	9.68235	22	9.96737	43	9.69568	22	9.99344	44	45	
46	.68257	23	.96780	43	.69590	22	.99388	43	46	22 21
47	.68280	22	.96824	43	.69612	22	.99431	44	47	6 2.2 2.1
48	.68302	23	.96867	43	.69634	22	.99475	43	48	7 2.5 2.5
49	.68324	22	.96910	43	.69656	22	.99519	43	49	8 2.9 2.8
50	9.68347	23	9.96953	43	9.69678	22	9.99562	44	50	9 3.3 3.2
51	.68369	22	.96997	43	.69700	21	.99606	44	51	10 3.6 3.6
52	.68392	23	.97040	43	.69721	22	.99650	44	52	20 7.3 7.1
53	.68414	22	.97083	43	.69743	22	.99694	43	53	30 11.0 10.7
54	.68436	23	.97127	43	.69765	22	.99737	44	54	40 14.6 14.3
55	9.68459	22	9.97170	43	9.69787	22	9.99781	44	55	50 18.3 17.9
56	.68481	22	.97213	43	.69809	22	.99825	43	56	
57	.68503	23	.97257	43	.69831	21	.99868	44	57	
58	.68526	22	.97300	43	.69853	22	.99912	44	58	
59	.68548	23	.97343	43	.69875	22	9.99956	44	59	
60	9.68571		9.97387		9.69897		10.00000		60	
′	Log. Vers.	D	Log. Exsec.	D	Log. Vers.	D	Log. Exsec.	D	′	P. P.

TABLE VIII.—LOGARITHMIC VERSED SINES AND EXTERNAL SECANTS.

60°   61°

′	Log. Vers.	D	Log. Exsec.	D	Log. Vers.	D	Log. Exsec.	D	′
0	9.69897	22	10.00000	44	9.71197	2î	10.02639	44â	0
1	.69919	2î	.00044	43	.71218	2î	.02684	44	1
2	.69940	22	.00087	44	.71239	2î	.02728	44â	2
3	.69962	22	.00131	43	.71261	2î	.02772	44	3
4	.69984	22	.00175	44	.71282	2î	.02816	44	4
5	9.70006	2î	10.00219	43	9.71304	2î	10.02861	44	5
6	.70028	22	.00262	44	.71325	21	.02905	44	6
7	.70050	22	.00306	44	.71346	2î	.02949	44	7
8	.70072	2î	.00350	43	.71368	2î	.02994	44	8
9	.70093	2î	.00394	44	.71389	2î	.03038	44	9
10	9.70115	2î	10.00438	44	9.71411	2î	10.03082	44â	10
11	.70137	22	.00482	43	.71432	21	.03127	44â	11
12	.70159	22	.00525	44	.71453	21	.03171	44â	12
13	.70181	2î	.00569	44	.71475	2î	.03215	44â	13
14	.70202	22	.00613	44	.71496	21	.03260	44â	14
15	9.70224	2î	10.00657	43	9.71517	2î	10.03304	44â	15
16	.70246	22	.00701	44	.71539	2î	.03348	44â	16
17	.70268	2î	.00745	44	.71560	21	.03393	44â	17
18	.70289	22	.00789	44	.71581	21	.03437	44â	18
19	.70311	2î	.00833	43	.71603	21	.03481	44â	19
20	9.70333	22	10.00876	44	9.71624	21	10.03526	44â	20
21	.70355	2î	.00920	44	.71645	2î	.03570	44â	21
22	.70376	22	.00964	44	.71667	21	.03615	44â	22
23	.70398	2î	.01008	44	.71688	21	.03659	44â	23
24	.70420	2î	.01052	44	.71709	21	.03704	44	24
25	9.70441	22	10.01096	44	9.71730	21	10.03748	44â	25
26	.70463	2î	.01140	44	.71752	21	.03793	44â	26
27	.70485	22	.01184	44	.71773	21	.03837	44â	27
28	.70507	2î	.01228	44	.71794	21	.03882	44â	28
29	.70528	2î	.01272	44	.71815	21	.03926	44â	29
30	9.70550	22	10.01316	44	9.71837	21	10.03970	44â	30
31	.70572	2î	.01360	44	.71858	21	.04015	44â	31
32	.70593	2î	.01404	44	.71879	21	.04059	44â	32
33	.70615	2î	.01448	44	.71900	2î	.04104	45	33
34	.70636	22	.01492	44	.71922	21	.04149	44â	34
35	9.70658	2î	10.01536	44	9.71943	21	10.04193	44â	35
36	.70680	2î	.01580	44	.71964	21	.04238	44â	36
37	.70701	2î	.01624	44	.71985	21	.04282	44â	37
38	.70723	22	.01668	44	.72006	21	.04327	44â	38
39	.70745	2î	.01712	44	.72028	21	.04371	44â	39
40	9.70766	2î	10.01756	44	9.72049	21	10.04416	45	40
41	.70788	2î	.01800	44	.72070	2î	.04461	44â	41
42	.70809	2î	.01844	44â	.72091	21	.04505	44â	42
43	.70831	2î	.01889	44	.72112	21	.04550	44â	43
44	.70852	22	.01933	44	.72133	21	.04594	45	44
45	9.70874	2î	10.01977	44	9.72154	21	10.04639	44â	45
46	.70896	2î	.02021	44â	.72176	2î	.04684	44â	46
47	.70917	2î	.02065	44	.72197	21	.04728	44â	47
48	.70939	2î	.02109	44	.72218	21	.04773	45	48
49	.70960	2î	.02153	44	.72239	21	.04818	44â	49
50	9.70982	2î	10.02197	44	9.72260	2î	10.04862	45	50
51	.71003	2î	.02242	44â	.72281	21	.04907	44â	51
52	.71025	2î	.02286	44	.72302	21	.04952	45	52
53	.71046	2î	.02330	44â	.72323	21	.04996	45	53
54	.71068	2î	.02374	44	.72344	21	.05041	44â	54
55	9.71089	2î	10.02418	44	9.72365	21	10.05086	45	55
56	.71111	2î	.02463	44â	.72386	2î	.05131	44â	56
57	.71132	2î	.02507	44	.72408	21	.05175	45	57
58	.71154	2î	.02551	44â	.72429	21	.05220	45	58
59	.71175	2î	.02595	44	.72450	21	.05265	45	59
60	9.71197		10.02639		9.72471		10.05310		60
′	Log. Vers.	D	Log. Exsec.	D	Log. Vers.	D	Log. Exsec.	D	′

P. P.

	45	44â
6	4.5	4.4
7	5.2	5.2
8	6.0	5.9
9	6.7	6.7
10	7.5	7.4
20	15.0	14.8
30	22.5	22.2
40	30.0	29.6
50	37.5	37.1

	44	43â
6	4.4	4.3
7	5.1	5.1
8	5.8	5.8
9	6.6	6.5
10	7.3	7.2
20	14.6	14.5
30	22.0	21.7
40	29.3	29.0
50	36.6	36.2

	22	2î
6	2.2	2.1
7	2.5	2.5
8	2.9	2.8
9	3.3	3.2
10	3.6	3.6
20	7.3	7.1
30	11.0	10.7
40	14.6	14.3
50	18.3	17.9

	21
6	2.1
7	2.4
8	2.8
9	3.1
10	3.5
20	7.0
30	10.5
40	14.0
50	17.5

TABLE VIII.—LOGARITHMIC VERSED SINES AND EXTERNAL SECANTS.

62°   63°

′	Log. Vers.	D	Log. Exsec.	D	Log. Vers.	D	Log. Exsec.	D	′
0	9.72471	21	10.05310	44	9.73720	20	10.08015	45	0
1	.72492	21	.05354	45	.73740	20	.08061	45	1
2	.72513	21	.05399	45	.73761	21	.08106	45	2
3	.72534	21	.05444	44	.73782	20	.08151	45	3
4	.72555	21	.05489	45	.73802	20	.08197	45	4
5	9.72576	21	10.05534	45	9.73823	20	10.08242	45	5
6	.72597	21	.05579	44	.73843	20	.08288	45	6
7	.72618	21	.05623	45	.73864	20	.08333	45	7
8	.72639	21	.05668	45	.73884	21	.08379	45	8
9	.72660	21	.05713	45	.73905	21	.08424	45	9
10	9.72681	21	10.05758	44	9.73926	20	10.08470	45	10
11	.72701	20	.05803	45	.73946	21	.08515	45	11
12	.72722	21	.05848	45	.73967	20	.08561	45	12
13	.72743	21	.05893	45	.73987	21	.08606	45	13
14	.72764	21	.05938	45	.74008	20	.08652	45	14
15	9.72785	21	10.05983	45	9.74028	21	10.08697	46	15
16	.72806	21	.06028	44	.74049	20	.08743	45	16
17	.72827	20	.06072	45	.74069	21	.08789	45	17
18	.72848	21	.06117	45	.74090	20	.08834	45	18
19	.72869	21	.06162	45	.74110	21	.08880	46	19
20	9.72890	21	10.06207	45	9.74131	20	10.08926	45	20
21	.72911	20	.06252	45	.74151	21	.08971	46	21
22	.72931	21	.06297	45	.74172	20	.09017	45	22
23	.72952	21	.06342	45	.74192	21	.09062	46	23
24	.72973	21	.06387	45	.74213	20	.09108	46	24
25	9.72994	21	10.06432	45	9.74233	21	10.09154	46	25
26	.73015	21	.06477	45	.74254	20	.09200	45	26
27	.73036	21	.06522	46	.74274	20	.09245	46	27
28	.73057	20	.06568	45	.74294	21	.09291	46	28
29	.73077	21	.06613	45	.74315	20	.09337	45	29
30	9.73098	21	10.06658	45	9.74335	20	10.09382	46	30
31	.73119	20	.06703	45	.74356	20	.09428	46	31
32	.73140	21	.06748	45	.75376	20	.09474	46	32
33	.73161	20	.06793	45	.74396	20	.09520	46	33
34	.73181	21	.06838	45	.74417	20	.09566	45	34
35	9.73202	21	10.06883	45	9.74437	20	10.09611	46	35
36	.73223	20	.06928	46	.74458	20	.09657	46	36
37	.73244	21	.06974	45	.74478	20	.09703	46	37
38	.73265	20	.07019	45	.74498	21	.09749	46	38
39	.73285	21	.07064	45	.74519	20	.09795	46	39
40	9.73306	21	10.07109	45	9.74539	20	10.09841	45	40
41	.73327	21	.07154	46	.74559	21	.09886	46	41
42	.73348	20	.07200	45	.74580	20	.09932	46	42
43	.73368	21	.07245	45	.74600	20	.09978	46	43
44	.73389	21	.07290	45	.74620	21	.10024	46	44
45	9.73410	20	10.07335	45	9.74641	20	10.10070	46	45
46	.73430	21	.07380	46	.74661	20	.10116	46	46
47	.73451	21	.07426	45	.74681	21	.10162	46	47
48	.73472	21	.07471	45	.74702	20	.10208	46	48
49	.73493	20	.07516	46	.74722	20	.10254	46	49
50	9.73513	21	10.07562	45	9.74742	20	10.10300	46	50
51	.73534	21	.07607	45	.74762	21	.10346	46	51
52	.73555	20	.07652	45	.74783	20	.10392	46	52
53	.73575	20	.07697	46	.74803	20	.10438	46	53
54	.73596	21	.07743	45	.74823	21	.10484	46	54
55	9.73617	20	10.07788	46	9.74844	20	10.10530	46	55
56	.73637	21	.07834	45	.74864	20	.10576	46	56
57	.73658	21	.07879	45	.74884	20	.10622	46	57
58	.73679	20	.07924	46	.74904	20	.10668	46	58
59	.73699	20	.07970	45	.74924	20	.10714	46	59
60	9.73720		10.08015		9.74945		10.10760		60
′	Log. Vers.	D	Log. Exsec.	D	Log. Vers.	D	Log. Exsec.	D	′

P. P.

46 / 46
6	4.6	4.6
7	5.4	5.3
8	6.2	6.1
9	7.0	6.9
10	7.7	7.6
20	15.5	15.3
30	23.2	23.0
40	31.0	30.6
50	38.7	38.3

45 / 45
6	4.5	4.5
7	5.3	5.2
8	6.0	6.0
9	6.8	6.7
10	7.6	7.5
20	15.1	15.0
30	22.7	22.5
40	30.3	30.0
50	37.9	37.5

44
6	4.4
7	5.2
8	5.9
9	6.7
10	7.4
20	14.8
30	22.2
40	29.6
50	37.1

21 / 20
6	2.1	2.0
7	2.4	2.4
8	2.8	2.7
9	3.1	3.1
10	3.5	3.4
20	7.0	6.8
30	10.5	10.2
40	14.0	13.6
50	17.5	17.1

20
6	2.0
7	2.3
8	2.6
9	3.0
10	3.3
20	6.6
30	10.0
40	13.3
50	16.6

′	Log. Vers.	D	Log. Exsec.	D	Log. Vers.	D	Log. Exsec.	D	′
0	9.74945	20	10.10766̂	46	9.76146̂	19̂	10.13551̂	47	**0**
1	.74965	20̂	.10807	46	.76166	20	.13598̂	47	1
2	.74985̂	20	.10853	46	.76186	20	.13645̂	47	2
3	.75005̂	20̂	.10899	46	.76206	19̂	.13692̂	47	3
4	.75026	20	.10945̂	46	.76225̂	20	.13739̂	47	4
5	9.75046	20	10.10991̂	46	9.76245̂	19̂	10.13786̂	47	**5**
6	.75066	20̂	.11037̂	46	.76265	20	.13833̂	47	6
7	.75086̂	20	.11084	46	.76285	19̂	.13880̂	47	7
8	.75106̂	20	.11130	46	.76304̂	19̂	.13927	47	8
9	.75126̂	20̂	.11176̂	46	.76324̂	20	.13974̂	47	9
10	9.75147	20	10.11222̂	46	9.76344̂	20	10.14021̂	47	**10**
11	.75167	20	.11269	46	.76364	19̂	.14068̂	47	11
12	.75187	20	.11315	46	.76384	20	.14115̂	47	12
13	.75207	20	.11361̂	46	.76403̂	19̂	.14162̂	47̂	13
14	.75227̂	20	.11407̂	46	.76423̂	20	.14210	47	14
15	9.75247̂	20	10.11454	46	9.76443̂	19̂	10.14257̂	47	**15**
16	.75267̂	20	.11500̂	46	.76463	20	.14304	47	16
17	.75287̂	20̂	.11546̂	46	.76482̂	19̂	.14351	47̂	17
18	.75308	20	.11593	46	.76502	19̂	.14398	47	18
19	.75328	20	.11639̂	46	.76522	20	.14445̂	47̂	19
20	9.75348	20	10.11685̂	46	9.76541̂	19̂	10.14493	47	**20**
21	.75368	20	.11732	46	.76561̂	20	.14540	47	21
22	.75388	20	.11778̂	46	.76581	19̂	.14587	47̂	22
23	.75408	20	.11825	46	.76600̂	19̂	.14634̂	47̂	23
24	.75428̂	20̂	.11871̂	46	.76620̂	20	.14682	47	24
25	9.75448̂	20	10.11917̂	46	9.76640	19̂	10.14729	47	**25**
26	.75468̂	20	.11964	46	.76659̂	19̂	.14776̂	47̂	26
27	.75488̂	20	.12010̂	46	.76679̂	20	.14823̂	47	27
28	.75508̂	20	.12057	46	.76699	19̂	.14871	47̂	28
29	.75528̂	20	.12103̂	46	.76718̂	19̂	.14918̂	47̂	29
30	9.75548̂	20	10.12150	46	9.76738̂	20	10.14965	47	**30**
31	.75568̂	20	.12196̂	46	.76758	19̂	.15013̂	47̂	31
32	.75588̂	20	.12243	46	.76777̂	19̂	.15060̂	47̂	32
33	.75608̂	20	.12289̂	46	.76797	19̂	.15108	47	33
34	.75628̂	20	.12336	46	.76817	20	.15155	47̂	34
35	9.75648̂	20	10.12383	47	9.76836̂	19̂	10.15202̂	47̂	**35**
36	.75668̂	20	.12429̂	46	.76856	20	.15250	47̂	36
37	.75688̂	20	.12476	46	.76875̂	19̂	.15297	47̂	37
38	.75708̂	20	.12522̂	46	.76895	20	.15345	47̂	38
39	.75728̂	20	.12569	46	.76915	19̂	.15392̂	47̂	39
40	9.75748̂	20	10.12616	46	9.76934̂	19̂	10.15440	47̂	**40**
41	.75768̂	20	.12662̂	46	.76954	19̂	.15487̂	47̂	41
42	.75788̂	20	.12709	47	.76973̂	19̂	.15535	47̂	42
43	.75808̂	20	.12756	46	.76993	19̂	.15582̂	47̂	43
44	.75828	19̂	.12802̂	47	.77012̂	19̂	.15630	47̂	44
45	9.75848̂	20	10.12849	46	9.77032̂	20	10.15678	48	**45**
46	.75868̂	20	.12896	47	.77052	19̂	.15725̂	47̂	46
47	.75888̂	20	.12942̂	46	.77071̂	19̂	.15773	47̂	47
48	.75908	20	.12989̂	47	.77091	19̂	.15820̂	48	48
49	.75928	20	.13036	46	.77110̂	19̂	.15868	48	49
50	9.75947̂	19̂	10.13083	47	9.77130	19̂	10.15916	47̂	**50**
51	.75967̂	20	.13130	47	.77149̂	19̂	.15963̂	48	51
52	.75987̂	20	.13176̂	46	.77169	19̂	.16011̂	48	52
53	.76007̂	20	.13223̂	47	.77188̂	19̂	.16059	47̂	53
54	.76027	19̂	.13270	47	.77208	19̂	.16106̂	48	54
55	9.76047̂	20	10.13317	47	9.77227̂	19̂	10.16154̂	48	**55**
56	.76067̂	20	.13364	47	.77247	19̂	.16202	47̂	56
57	.76087̂	20	.13411	46	.77266̂	19̂	.16250	48	57
58	.76106̂	19̂	.13457̂	47	.77286	19̂	.16298	47̂	58
59	.76126̂	20	.13504̂	47	.77305̂	19̂	.16345̂	48	59
60	9.76146̂		10.13551̂		9.77325		10.16393̂		**60**
′	Log. Vers.	D	Log. Exsec.	D	Log. Vers.	D	Log. Exsec.	D	′

P. P.

	48	47̂
6	4.8	4.7̂
7	5.6	5.5
8	6.4	6.3
9	7.2	7.1
10	8.0	7.9
20	16.0	15.8̂
30	24.0	23.7̂
40	32.0	31.6
50	40.0	39.6

	47	46̂
6	4.7	4.6̂
7	5.5	5.4
8	6.2	6.2
9	7.0̂	7.0
10	7.8̂	7.7̂
20	15.6	15.5̂
30	23.5̂	23.2
40	31.3̂	31.0
50	39.1	38.7̂

	46
6	4.6
7	5.3̂
8	6.1̂
9	6.9
10	7.6̂
20	15.3
30	23.0
40	30.6̂
50	38.3

	2ô	20
6	2.0̂	2.0
7	2.4̂	2.3̂
8	2.7̂	2.6
9	3.1	3.0
10	3.4̂	3.3̂
20	6.8̂	6.6
30	10.2̂	10.0
40	13.6̂	13.3̂
50	17.1	16.6

	19̂
6	1.9̂
7	2.3
8	2.6
9	2.9
10	3.2̂
20	6.5
30	9.7̂
40	13.0
50	16.2

TABLE VIII.—LOGARITHMIC VERSED SINES AND EXTERNAL SECANTS.

66°   67°

′	Log. Vers.	D	Log. Exsec.	D	Log. Vers.	D	Log. Exsec.	D	′
0	9.77325	19	10.16393	48	9.78481	19	10.19293	49	0
1	.77344	19	.16441	47	.78500	19	.19342	49	1
2	.77363	19	.16489	48	.78519	19	.19391	48	2
3	.77383	19	.16537	48	.78538	19	.19439	49	3
4	.77402	19	.16585	48	.78557	19	.19488	49	4
5	9.77422	19	10.16633	47	9.78576	19	10.19537	49	5
6	.77441	19	.16680	48	.78595	19	.19586	49	6
7	.77461	19	.16728	48	.78614	19	.19635	49	7
8	.77480	19	.16776	48	.78633	19	.19684	49	8
9	.77499	19	.16824	48	.78652	19	.19733	49	9
10	9.77519	19	10.16872	48	9.78671	19	10.19782	49	10
11	.77538	19	.16920	48	.78690	19	.19831	49	11
12	.77557	19	.16968	48	.78709	19	.19880	49	12
13	.77577	19	.17016	48	.78728	19	.19929	49	13
14	.77596	19	.17064	48	.78747	19	.19979	49	14
15	9.77616	19	10.17112	48	9.78766	19	10.20028	49	15
16	.77635	19	.17160	48	.78785	19	.20077	49	16
17	.77654	19	.17209	48	.78804	19	.20126	49	17
18	.77674	19	.17257	48	.78823	19	.20175	49	18
19	.77693	19	.17305	48	.78842	19	.20224	49	19
20	9.77712	19	10.17353	48	9.78861	19	10.20273	49	20
21	.77732	19	.17401	48	.78880	19	.20323	49	21
22	.77751	19	.17449	48	.78899	19	.20372	49	22
23	.77770	19	.17498	48	.78918	18	.20421	49	23
24	.77790	19	.17546	48	.78937	19	.20470	49	24
25	9.77809	19	10.17594	48	9.78956	19	10.20520	49	25
26	.77828	19	.17642	48	.78975	19	.20569	49	26
27	.77847	19	.17690	48	.78994	19	.20618	49	27
28	.77867	19	.17739	48	.79013	19	.20668	49	28
29	.77886	19	.17787	48	.79032	19	.20717	49	29
30	9.77905	19	10.17835	48	9.79051	18	10.20767	49	30
31	.77925	19	.17884	48	.79069	19	.20816	49	31
32	.77944	19	.17932	48	.79088	19	.20865	49	32
33	.77963	19	.17980	48	.79107	19	.20915	49	33
34	.77982	19	.18029	48	.79126	19	.20964	49	34
35	9.78002	19	10.18077	48	9.79145	19	10.21014	49	35
36	.78021	19	.18126	48	.79164	18	.21063	49	36
37	.78040	19	.18174	48	.79183	19	.21113	49	37
38	.78059	19	.18222	48	.79202	18	.21162	50	38
39	.78078	19	.18271	48	.79220	19	.21212	49	39
40	9.78098	19	10.18319	48	9.79239	19	10.21262	49	40
41	.78117	19	.18368	48	.79258	19	.21311	49	41
42	.78136	19	.18416	49	.79277	18	.21361	49	42
43	.78155	19	.18465	48	.79296	19	.21410	50	43
44	.78174	19	.18514	48	.79315	19	.21460	50	44
45	9.78194	19	10.18562	48	9.79333	18	10.21510	50	45
46	.78213	19	.18611	48	.79352	19	.21560	49	46
47	.78232	19	.18659	48	.79371	19	.21609	50	47
48	.78251	19	.18708	48	.79390	18	.21659	50	48
49	.78270	19	.18757	49	.79409	19	.21709	49	49
50	9.78289	19	10.18805	48	9.79427	18	10.21759	50	50
51	.78309	19	.18854	49	.79446	19	.21808	49	51
52	.78328	19	.18903	48	.79465	19	.21858	50	52
53	.78347	19	.18951	49	.79484	18	.21908	50	53
54	.78366	19	.19000	49	.79503	19	.21958	49	54
55	9.78385	19	10.19049	48	9.79521	18	10.22008	50	55
56	.78404	19	.19098	49	.79540	18	.22058	50	56
57	.78423	19	.19146	48	.79559	19	.22108	50	57
58	.78442	19	.19195	49	.79578	18	.22158	50	58
59	.78462	19	.19244	49	.79596	19	.22208	50	59
60	9.78481		10.19293		9.79615		10.22258		60

P. P.

	50	49
6	5.0	4.9
7	5.8	5.8
8	6.6	6.6
9	7.5	7.4
10	8.3	8.2
20	16.6	16.5
30	25.0	24.7
40	33.3	33.0
50	41.6	41.2

	49	48
6	4.9	4.8
7	5.7	5.6
8	6.5	6.4
9	7.3	7.3
10	8.1	8.1
20	16.3	16.1
30	24.5	24.2
40	32.6	32.3
50	40.8	40.4

	48	47
6	4.8	4.7
7	5.6	5.5
8	6.4	6.3
9	7.2	7.1
10	8.0	7.9
20	16.0	15.8
30	24.0	23.7
40	32.0	31.6
50	40.0	39.6

	19	19
6	1.9	1.9
7	2.3	2.2
8	2.6	2.5
9	2.9	2.8
10	3.2	3.1
20	6.5	6.3
30	9.7	9.5
40	13.0	12.6
50	16.2	15.8

	18
6	1.8
7	2.1
8	2.4
9	2.8
10	3.1
20	6.1
30	9.2
40	12.3
50	15.4

TABLE VIII.—LOGARITHMIC VERSED SINES AND EXTERNAL SECANTS.

68°   69°

′	Log. Vers.	D	Log. Exsec.	D	Log. Vers.	D	Log. Exsec.	D	′
0	9.79615	18	10.22258	50	9.80728	18	10.25295	51	0
1	.79634	19	.22308	50	.80747	18	.25347	51	1
2	.79653	18	.22358	50	.80765	18	.25398	51	2
3	.79671	18	.22408	50	.80783	18	.25449	51	3
4	.79690	19	.22458	50	.80802	18	.25501	51	4
5	9.79709	18	10.22508	50	9.80820	18	10.25552	51	5
6	.79727	19	.22558	50	.80839	18	.25604	51	6
7	.79746	18	.22608	50	.80857	18	.25655	51	7
8	.79765	18	.22658	50	.80875	18	.25707	51	8
9	.79783	19	.22708	50	.80894	18	.25758	51	9
10	9.79802	18	10.22759	50	9.80912	18	10.25810	51	10
11	.79821	18	.22809	50	.80930	18	.25861	51	11
12	.79839	19	.22859	50	.80949	18	.25913	51	12
13	.79858	18	.22909	50	.80967	18	.25964	51	13
14	.79877	18	.22960	50	.80985	18	.26016	51	14
15	9.79895	18	10.23010	50	9.81003	18	10.26065	51	15
16	.79914	19	.23060	50	.81022	18	.26119	51	16
17	.79933	18	.23110	50	.81040	18	.26171	51	17
18	.79951	18	.23161	50	.81058	18	.26222	52	18
19	.79970	18	.23211	50	.81077	18	.26274	51	19
20	9.79988	18	10.23262	50	9.81095	18	10.26326	52	20
21	.80007	19	.23312	50	.81113	18	.26378	51	21
22	.80026	18	.23362	50	.81131	18	.26429	52	22
23	.80044	18	.23413	50	.81150	18	.26481	52	23
24	.80063	18	.23463	50	.81168	18	.26533	52	24
25	9.80081	19	10.23514	50	9.81186	18	10.26585	51	25
26	.80100	18	.23564	50	.81204	18	.26637	52	26
27	.80119	18	.23615	50	.81223	18	.26689	52	27
28	.80137	18	.23666	50	.81241	18	.26741	52	28
29	.80156	18	.23716	50	.81259	18	.26793	52	29
30	9.80174	18	10.23767	50	9.81277	18	10.26845	52	30
31	.80193	18	.23817	51	.81295	18	.26897	52	31
32	.80211	18	.23868	50	.81314	18	.26949	52	32
33	.80230	18	.23919	50	.81332	18	.27001	52	33
34	.80248	18	.23969	51	.81350	18	.27053	52	34
35	9.80267	19	10.24020	51	9.81368	18	10.27105	52	35
36	.80286	18	.24071	50	.81386	18	.27157	52	36
37	.80304	18	.24122	51	.81405	18	.27209	52	37
38	.80323	18	.24172	51	.81423	18	.27261	52	38
39	.80341	18	.24223	51	.81441	18	.27314	52	39
40	9.80360	18	10.24274	51	9.81459	18	10.27366	52	40
41	.80378	18	.24325	50	.81477	18	.27418	52	41
42	.80397	18	.24376	51	.81495	18	.27470	52	42
43	.80415	18	.24427	51	.81513	18	.27523	52	43
44	.80434	18	.24478	51	.81532	18	.27575	52	44
45	9.80452	18	10.24529	51	9.81550	18	10.27627	52	45
46	.80470	18	.24580	51	.81568	18	.27680	52	46
47	.80489	18	.24631	51	.81586	18	.27732	52	47
48	.80507	18	.24682	51	.81604	18	.27785	52	48
49	.80526	18	.24733	51	.81622	18	.27837	52	49
50	9.80544	18	10.24784	51	9.81646	18	10.27890	52	50
51	.80563	18	.24835	51	.81658	18	.27942	52	51
52	.80587	18	.24886	51	.81676	18	.27995	52	52
53	.80600	18	.24937	51	.81695	18	.28047	52	53
54	.80618	18	.24988	51	.81713	18	.28100	52	54
55	9.80636	18	10.25039	51	9.81731	18	10.28152	52	55
56	.80655	18	.25090	51	.81749	18	.28205	53	56
57	.80673	18	.25142	51	.81767	18	.28258	52	57
58	.80692	18	.25193	51	.81785	18	.28310	53	58
59	.80710	18	.25244	51	.81803	18	.28363	52	59
60	9.80728		10.25295		9.81821		10.28416		60
′	Log. Vers.	D	Log. Exsec.	D	Log. Vers.	D	Log. Exsec.	D	′

P. P.

	53	52
6	5.3	5.2
7	6.2	6.1
8	7.0	7.0
9	7.9	7.9
10	8.8	8.7
20	17.6	17.5
30	26.5	26.2
40	35.3	35.0
50	44.1	43.7

	52	51
6	5.2	5.1
7	6.0	6.0
8	6.9	6.8
9	7.8	7.7
10	8.6	8.6
20	17.3	17.1
30	26.0	25.7
40	34.6	34.3
50	43.3	42.9

	51	50
6	5.1	5.0
7	5.9	5.9
8	6.8	6.7
9	7.6	7.6
10	8.5	8.4
20	17.0	16.8
30	25.5	25.2
40	34.0	33.6
50	42.5	42.1

	50
6	5.0
7	5.8
8	6.6
9	7.5
10	8.3
20	16.6
30	25.0
40	33.3
50	41.6

	19	18
6	1.9	1.8
7	2.2	2.1
8	2.5	2.4
9	2.8	2.8
10	3.1	3.1
20	6.3	6.1
30	9.5	9.2
40	12.6	12.3
50	15.8	15.4

	18
6	1.8
7	2.1
8	2.4
9	2.7
10	3.0
20	6.0
30	9.0
40	12.0
50	15.0

TABLE VIII.—LOGARITHMIC VERSED SINES AND EXTERNAL SECANTS.

70°   71°

′	Log. Vers.	D	Log. Exsec.	D	Log. Vers.	D	Log. Exsec.	D	′	P. P.	
0	9.81821	18	10.28416	53	9.82894	17	10.31629	54	0		
1	.81839	18	.28469	52	.82911	17	.31684	54	1		
2	.81857	18	.28521	53	.82929	18	.31738	54	2		
3	.81875	18	.28574	53	.82947	17	.31793	54	3	56	56
4	.81893	18	.28627	52	.82964	18	.31847	54	4	6 5.6	5.6
5	9.81911	18	10.28680	53	9.82982	17	10.31902	54	5	7 6.6 / 8 7.5	6.5 / 7.4
6	.81929	18	.28733	53	.83000	17	.31956	54	6	9 8.5	8.4
7	.81947	18	.28786	53	.83017	18	.32011	54	7	10 9.4	9.3
8	.81965	18	.28839	53	.83035	18	.32066	55	8	20 18.8	18.6
9	.81983	18	.28892	53	.83053	17	.32120	54	9	30 28.2 / 40 37.6	28.0 / 37.3
10	9.82001	18	10.28945	53	9.83070	18	10.32175	54	10	50 47.1	46.6
11	.82019	18	.28998	53	.83088	18	.32230	55	11		
12	.82037	18	.29051	53	.83106	17	.32284	54	12		
13	.82055	18	.29104	53	.83123	17	.32339	55	13	55	55
14	.82073	18	.29157	53	.83141	17	.32394	54	14	6 5.5	5.5
15	9.82091	18	10.29210	53	9.83159	18	10.32449	55	15	7 6.4 / 8 7.4	6.4 / 7.3
16	.82109	17	.29263	53	.83176	17	.32504	55	16	9 8.3	8.2
17	.82127	18	.29316	53	.83194	17	.32558	54	17	10 9.2	9.1
18	.82145	18	.29370	53	.83211	17	.32613	55	18	20 18.5	18.3
19	.82163	18	.29423	53	.83229	18	.32668	55	19	30 27.7 / 40 37.0	27.5 / 36.6
20	9.82181	18	10.29476	53	9.83247	17	10.32723	55	20	50 46.2	45.8
21	.82199	18	.29529	53	.83264	17	.32778	55	21		
22	.82217	18	.29583	53	.83282	17	.32833	55	22		
23	.82235	18	.29636	53	.83299	18	.32888	55	23	54	54
24	.82252	17	.29689	53	.83317	18	.32944	55	24	6 5.4	5.4
25	9.82270	18	10.29743	53	9.83335	17	10.32999	55	25	7 6.3 / 8 7.2	6.3 / 7.2
26	.82288	18	.29796	53	.83352	17	.33054	55	26	9 8.2	8.1
27	.82306	18	.29850	53	.83370	17	.33109	55	27	10 9.1	9.0
28	.82324	18	.29903	53	.83387	17	.33164	55	28	20 18.1	18.0
29	.82342	17	.29957	53	.83405	17	.33220	55	29	30 27.2 / 40 36.3	27.0 / 36.0
30	9.82360	18	10.30010	53	9.83422	17	10.33275	55	30	50 45.4	45.0
31	.82378	18	.30064	53	.83440	18	.33330	55	31		
32	.82396	17	.30117	54	.83458	17	.33385	55	32		
33	.82413	18	.30171	54	.83475	17	.33441	55	33	53	53
34	.82431	18	.30225	53	.83493	17	.33496	55	34	6 5.3	5.3
35	9.82449	18	10.30278	53	9.83510	17	10.33552	55	35	7 6.2 / 8 7.1	6.2 / 7.0
36	.82467	17	.30332	54	.83528	17	.33607	55	36	9 8.0	7.9
37	.82485	18	.30386	53	.83545	17	.33663	55	37	10 8.9	8.8
38	.82503	18	.30440	54	.83563	17	.33718	55	38	20 17.8	17.6
39	.82520	17	.30493	53	.83580	17	.33774	55	39	30 26.7 / 40 35.6	26.5 / 35.2
40	9.82538	18	10.30547	54	9.83598	17	10.33829	55	40	50 44.6	44.1
41	.82556	18	.30601	53	.83615	17	.33885	56	41		
42	.82574	17	.30655	54	.83633	17	.33941	55	42		
43	.82592	17	.30709	54	.83650	17	.33996	55	43		52
44	.82609	18	.30763	54	.83668	17	.34052	56	44	6	5.2
45	9.82627	18	10.30817	54	9.83685	17	10.34108	55	45	7 / 8	6.1 / 7.0
46	.82645	18	.30871	54	.83703	17	.34164	56	46	9 / 10	7.9 / 8.7
47	.82663	17	.30925	54	.83720	17	.34220	55	47	20	17.5
48	.82681	18	.30979	54	.83737	17	.34275	56	48	30	26.2
49	.82698	17	.31033	54	.83755	17	.34331	56	49	40	35.0
50	9.82716	17	10.31087	54	9.83772	17	10.34387	56	50	50	43.7
51	.82734	18	.31141	54	.83790	17	.34443	56	51	18 17	17
52	.82752	17	.31195	54	.83807	17	.34499	56	52	6 1.8 1.7	1.7
53	.82769	17	.31249	54	.83825	17	.34555	56	53	7 2.1 2.0	2.0
54	.82787	18	.31303	54	.83842	17	.34611	56	54	8 2.4 2.3	2.2
55	9.82805	17	10.31358	54	9.83859	17	10.34667	56	55	9 2.7 2.6	2.5
56	.82823	18	.31412	54	.83877	17	.34723	56	56	10 3.0 2.9	2.8
57	.82840	17	.31466	54	.83894	17	.34780	56	57	20 6.0 5.8	5.6
58	.82858	18	.31521	54	.83912	17	.34836	56	58	30 9.0 8.7	8.5
59	.82876	17	.31575	54	.83929	17	.34892	56	59	40 12.0 11.6	11.2
60	9.82894	18	10.31629	54	9.83946	17	10.34948	56	60	50 15.0 14.6	14.1
′	Log. Vers.	D	Log. Exsec.	D	Log. Vers.	D	Log. Exsec.	D	′	P. P.	

TABLE VIII.—LOGARITHMIC VERSED SINES AND EXTERNAL SECANTS.

72° **73°**

′	Log. Vers.	D	Log. Exsec.	D	Log. Vers.	D	Log. Exsec.	D	′
0	9.83946	17	10.34948	56	9.84986	17	10.38387	58	0
1	.83964	17	.35005	56	.84997	17	.38445	58	1
2	.83981	17	.35061	56	.85014	17	.38504	58	2
3	.83999	17	.35117	56	.85031	17	.38562	58	3
4	.84016	17	.35174	56	.85049	17	.38621	58	4
5	9.84033	17	10.35230	56	9.85066	17	10.38679	58	5
6	.84051	17	.35286	56	.85083	17	.38738	58	6
7	.84068	17	.35343	56	.85100	17	.38796	59	7
8	.84085	17	.35399	57	.85117	17	.38855	58	8
9	.84103	17	.35456	57	.85134	17	.38914	59	9
10	9.84120	17	10.35513	56	9.85151	17	10.38973	59	10
11	.84137	17	.35569	56	.85168	17	.39031	58	11
12	.84155	17	.35626	56	.85185	17	.39090	59	12
13	.84172	17	.35683	57	.85202	17	.39149	59	13
14	.84189	17	.35739	56	.85219	17	.39208	58	14
15	9.84207	17	10.35796	57	9.85236	17	10.39267	59	15
16	.84224	17	.35853	56	.85253	17	.39326	59	16
17	.84241	17	.35910	57	.85270	17	.39385	59	17
18	.84259	17	.35967	56	.85287	17	.39444	59	18
19	.84276	17	.36023	57	.85304	17	.39503	59	19
20	9.84293	17	10.36080	57	9.85321	17	10.39562	59	20
21	.84310	17	.36137	57	.85338	17	.39621	59	21
22	.84328	17	.36194	57	.85355	17	.39681	59	22
23	.84345	17	.36251	57	.85372	17	.39740	59	23
24	.84362	17	.36308	57	.85389	16	.39799	59	24
25	9.84380	17	10.36366	57	9.85405	17	10.39859	59	25
26	.84397	17	.36423	57	.85422	17	.39918	59	26
27	.84414	17	.36480	57	.85439	17	.39977	59	27
28	.84431	17	.36537	57	.85456	17	.40037	59	28
29	.84449	17	.36594	57	.85473	17	.40096	59	29
30	9.84466	17	10.36652	57	9.85490	17	10.40156	59	30
31	.84483	17	.36709	57	.85507	17	.40216	60	31
32	.84500	17	.36766	57	.85524	16	.40275	59	32
33	.84517	17	.36824	57	.85541	17	.40335	60	33
34	.84535	17	.36881	57	.85558	17	.40395	60	34
35	9.84552	17	10.36938	57	9.85575	17	10.40454	59	35
36	.84569	17	.36996	58	.85592	16	.40514	60	36
37	.84586	17	.37054	57	.85608	17	.40574	59	37
38	.84603	17	.37111	57	.85625	17	.40634	60	38
39	.84620	17	.37169	57	.85642	17	.40694	60	39
40	9.84638	17	10.37226	57	9.85659	17	10.40754	60	40
41	.84655	17	.37284	58	.85676	16	.40814	60	41
42	.84672	17	.37342	57	.85693	17	.40874	60	42
43	.84689	17	.37399	58	.85710	16	.40934	60	43
44	.84706	17	.37457	58	.85726	17	.40994	60	44
45	9.84724	17	10.37515	57	9.85743	17	10.41054	60	45
46	.84741	17	.37573	58	.85760	16	.41114	60	46
47	.84758	17	.37631	58	.85777	17	.41174	60	47
48	.84775	17	.37689	58	.85794	17	.41235	60	48
49	.84792	17	.37747	58	.85811	16	.41295	60	49
50	9.84809	17	10.37805	58	9.85827	17	10.41355	60	50
51	.84826	17	.37863	58	.85844	17	.41416	60	51
52	.84844	17	.37921	58	.85861	16	.41476	60	52
53	.84861	17	.37979	58	.85878	17	.41537	60	53
54	.84878	17	.38037	58	.85895	16	.41597	60	54
55	9.84895	17	10.38095	58	9.85911	17	10.41658	61	55
56	.84912	17	.38153	58	.85928	17	.41719	60	56
57	.84929	17	.38212	58	.85945	16	.41779	61	57
58	.84946	17	.38270	58	.85962	16	.41840	61	58
59	.84963	17	.38328	58	.85979	16	.41901	61	59
60	9.84986		10.38387		9.85995		10.41962		60
′	Log. Vers.	D	Log. Exsec.	D	Log. Vers.	D	Log. Exsec.	D	′

P. P.

	61	66
6	6.1	6.6
7	7.1	7.6
8	8.1	8.8
9	9.1	9.1
10	10.1	10.1
20	20.3	20.1
30	30.5	30.2
40	40.6	40.3
50	50.8	50.4

	60	59
6	6.0	5.9
7	7.0	6.9
8	8.0	7.9
9	9.0	8.9
10	10.0	9.9
20	20.0	19.8
30	30.0	29.7
40	40.0	39.6
50	50.0	49.6

	59	58
6	5.9	5.8
7	6.9	6.8
8	7.8	7.8
9	8.8	8.8
10	9.8	9.7
20	19.6	19.5
30	29.5	29.2
40	39.3	39.0
50	49.1	48.7

	58	57
6	5.8	5.7
7	6.7	6.7
8	7.7	7.6
9	8.7	8.6
10	9.6	9.6
20	19.3	19.1
30	29.0	28.7
40	38.6	38.3
50	48.3	47.9

	57	56
6	5.7	5.6
7	6.6	6.6
8	7.6	7.5
9	8.5	8.5
10	9.5	9.4
20	19.0	18.8
30	28.5	28.2
40	38.0	37.6
50	47.5	47.1

	17	17	16
6	1.7	1.7	1.6
7	2.0	2.0	1.9
8	2.3	2.2	2.2
9	2.6	2.6	2.5
10	2.9	2.8	2.7
20	5.8	5.6	5.5
30	8.7	8.7	8.2
40	11.6	11.3	11.0
50	14.6	14.1	13.7

P. P.

TABLE VIII.—LOGARITHMIC VERSED SINES AND EXTERNAL SECANTS.

74°   75°

′	Log. Vers.	D	Log. Exsec.	D	Log. Vers.	D	Log. Exsec.	D	′
0	9.85995	17	10.41962	66	9.86992	16	10.45693	63	0
1	.86012	16	.42022	61	.87009	16	.45756	63	1
2	.86029	17	.42083	61	.87025	16	.45820	64	2
3	.86046	16	.42144	61	.87042	16	.45884	63	3
4	.86062	17	.42205	61	.87058	16	.45947	64	4
5	9.86079	16	10.42266	61	9.87074	16	10.46011	64	5
6	.86096	17	.42327	61	.87091	16	.46075	64	6
7	.86113	16	.42388	61	.87107	16	.46139	64	7
8	.86129	17	.42450	61	.87124	16	.46203	64	8
9	.86146	16	.42511	61	.87140	16	.46267	64	9
10	9.86163	16	10.42572	61	9.87157	16	10.46331	64	10
11	.86179	17	.42633	61	.87173	16	.46395	64	11
12	.86196	17	.42695	61	.87189	16	.46460	64	12
13	.86213	17	.42756	61	.87206	16	.46524	64	13
14	.86230	16	.42817	61	.87222	16	.46588	64	14
15	9.86246	16	10.42879	61	9.87239	16	10.46652	64	15
16	.86263	17	.42940	61	.87255	16	.46717	64	16
17	.86280	16	.43002	61	.87271	16	.46781	64	17
18	.86296	16	.43063	62	.87288	16	.46846	64	18
19	.86313	17	.43125	61	.87304	16	.46910	64	19
20	9.86330	16	10.43187	62	9.87320	16	10.46975	65	20
21	.86346	16	.43249	61	.87337	16	.47040	64	21
22	.86363	17	.43310	62	.87353	16	.47104	65	22
23	.86380	16	.43372	62	.87370	16	.47169	65	23
24	.86396	16	.43434	62	.87386	16	.47234	65	24
25	9.86413	17	10.43496	62	9.87402	16	10.47299	65	25
26	.86430	16	.43558	62	.87419	16	.47364	65	26
27	.86446	16	.43620	62	.87435	16	.47429	65	27
28	.86463	16	.43682	62	.87451	16	.47494	65	28
29	.86479	17	.43744	62	.87468	16	.47559	65	29
30	9.86496	16	10.43806	62	9.87484	16	10.47624	65	30
31	.86513	16	.43868	63	.87500	16	.47689	65	31
32	.86529	16	.43931	62	.87516	16	.47754	65	32
33	.86546	16	.43993	62	.87533	16	.47820	65	33
34	.86562	17	.44055	62	.87549	16	.47885	65	34
35	9.86579	16	10.44118	62	9.87565	16	10.47950	65	35
36	.86596	16	.44180	62	.87582	16	.48016	65	36
37	.86612	16	.44242	62	.87598	16	.48081	65	37
38	.86629	16	.44305	63	.87614	16	.48147	66	38
39	.86645	16	.44368	62	.87631	16	.48213	65	39
40	9.86662	16	10.44430	63	9.87647	16	10.48278	65	40
41	.86678	17	.44493	63	.87663	16	.48344	66	41
42	.86695	16	.44556	62	.87679	16	.48410	66	42
43	.86712	16	.44618	63	.87696	16	.48476	66	43
44	.86728	16	.44681	63	.87712	16	.48542	65	44
45	9.86745	16	10.44744	63	9.87728	16	10.48607	66	45
46	.86761	16	.44807	63	.87744	16	.48674	66	46
47	.86778	16	.44870	63	.87761	16	.48740	66	47
48	.86794	16	.44933	63	.87777	16	.48806	66	48
49	.86811	16	.44996	63	.87793	16	.48872	66	49
50	9.86827	16	10.45059	63	9.87809	16	10.48938	66	50
51	.86844	16	.45122	63	.87825	16	.49004	66	51
52	.86860	16	.45185	63	.87842	16	.49071	66	52
53	.86877	16	.45248	63	.87858	16	.49137	66	53
54	.86893	16	.45312	63	.87874	16	.49204	66	54
55	9.86910	16	10.45375	63	9.87890	16	10.49270	66	55
56	.86926	16	.45439	63	.87906	16	.49337	66	56
57	.86943	16	.45502	63	.87923	16	.49403	67	57
58	.86959	16	.45565	63	.87939	16	.49470	66	58
59	.86976	16	.45629	64	.87955	16	.49537	67	59
60	9.86992		10.45693		9.87971		10.49604		60

| ′ | Log. Vers. | D | Log. Exsec. | D | Log. Vers. | D | Log. Exsec. | D | ′ |

P. P.

	67	66	66
6	6.7	6.6	6.6
7	7.8	7.7	7.7
8	8.9	8.8	8.8
9	10.0	10.0	9.9
10	11.1	11.1	11.0
20	22.3	22.1	22.0
30	33.5	33.2	33.0
40	44.6	44.3	44.0
50	55.8	55.4	55.0

	65	65	64
6	6.5	6.5	6.4
7	7.6	7.6	7.5
8	8.7	8.6	8.6
9	9.8	9.7	9.7
10	10.9	10.8	10.7
20	21.8	21.6	21.5
30	32.7	32.5	32.2
40	43.6	43.3	43.0
50	54.6	54.1	53.7

	64	63	63
6	6.4	6.3	6.3
7	7.4	7.4	7.3
8	8.5	8.4	8.4
9	9.6	9.5	9.4
10	10.6	10.6	10.5
20	21.3	21.1	21.0
30	31.7	31.7	31.5
40	42.6	42.3	42.0
50	53.3	52.9	52.5

	62	62	61
6	6.2	6.2	6.1
7	7.3	7.2	7.2
8	8.3	8.2	8.2
9	9.4	9.3	9.2
10	10.4	10.3	10.2
20	20.8	20.6	20.5
30	31.2	31.0	30.7
40	41.6	41.3	41.0
50	52.1	51.6	51.2

	61	60
6	6.1	6.0
7	7.1	7.0
8	8.1	8.0
9	9.1	9.1
10	10.1	10.1
20	20.3	20.1
30	30.5	30.2
40	40.6	40.3
50	50.8	50.4

	17	16	16
6	1.7	1.6	1.6
7	2.0	1.9	1.8
8	2.2	2.2	2.1
9	2.5	2.5	2.4
10	2.8	2.7	2.6
20	5.6	5.5	5.3
30	8.5	8.2	8.0
40	11.3	11.0	10.6
50	14.1	13.7	13.3

TABLE VIII.—LOGARITHMIC VERSED SINES AND EXTERNAL SECANTS.

76°   77°

'	Log. Vers.	D	Log. Exsec.	D	Log. Vers.	D	Log. Exsec.	D	'
0	9.87971	16	10.49604	66	9.88933	16	10.53724	70	0
1	.87987	16	.49670	67	.88949	15	.53794	71	1
2	.88003	16	.49737	67	.88964	16	.53865	70	2
3	.88020	16	.49804	67	.88980	16	.53936	71	3
4	.88036	16	.49871	67	.88996	16	.54007	71	4
5	9.88052	16	10.49939	67	9.89012	15	10.54078	71	5
6	.88068	16	.50006	67	.89028	16	.54149	71	6
7	.88084	16	.50073	67	.89044	16	.54220	71	7
8	.88100	16	.50140	67	.89060	16	.54291	71	8
9	.88116	16	.50208	67	.89075	15	.54362	71	9
10	9.88133	16	10.50275	67	9.89091	16	10.54433	71	10
11	.88149	16	.50342	67	.89107	16	.54505	71	11
12	.88165	16	.50410	67	.89123	16	.54576	71	12
13	.88181	16	.50477	68	.89139	16	.54647	71	13
14	.88197	16	.50545	67	.89155	15	.54719	72	14
15	9.88213	16	10.50613	68	9.89170	16	10.54791	71	15
16	.88229	16	.50681	67	.89186	15	.54862	71	16
17	.88245	16	.50748	68	.89202	16	.54934	72	17
18	.88261	16	.50816	68	.89218	16	.55006	71	18
19	.88277	16	.50884	68	.89234	15	.55078	72	19
20	9.88294	16	10.50952	68	9.89249	16	10.55150	72	20
21	.88310	16	.51020	68	.89265	15	.55222	72	21
22	.88326	16	.51088	68	.89281	16	.55294	72	22
23	.88342	16	.51157	68	.89297	15	.55366	72	23
24	.88358	16	.51225	68	.89312	16	.55438	72	24
25	9.88374	16	10.51293	68	9.89328	15	10.55511	72	25
26	.88390	16	.51361	68	.89344	16	.55583	72	26
27	.88406	16	.51430	68	.89360	16	.55655	72	27
28	.88422	16	.51498	68	.89376	15	.55728	73	28
29	.88438	16	.51567	69	.89391	15	.55801	72	29
30	9.88454	16	10.51636	68	9.89407	15	10.55873	72	30
31	.88470	16	.51704	68	.89423	15	.55946	73	31
32	.88486	16	.51773	69	.89438	16	.56019	72	32
33	.88502	16	.51842	69	.89454	16	.56092	73	33
34	.88518	16	.51911	69	.89470	15	.56165	73	34
35	9.88534	16	10.51980	69	9.89486	16	10.56238	73	35
36	.88550	16	.52049	69	.89501	15	.56311	73	36
37	.88566	16	.52118	69	.89517	16	.56384	73	37
38	.88582	16	.52187	69	.89533	15	.56457	73	38
39	.88598	16	.52256	69	.89548	16	.56531	73	39
40	9.88614	16	10.52325	69	9.89564	15	10.56604	73	40
41	.88630	16	.52394	69	.89580	16	.56678	73	41
42	.88646	16	.52464	69	.89596	15	.56751	73	42
43	.88662	15	.52533	69	.89611	16	.56825	74	43
44	.88678	16	.52603	69	.89627	16	.56899	73	44
45	9.88694	16	10.52672	70	9.89643	15	10.56973	74	45
46	.88710	16	.52742	69	.89658	16	.57047	74	46
47	.88726	16	.52812	69	.89674	16	.57120	73	47
48	.88742	16	.52881	70	.89690	15	.57195	74	48
49	.88758	16	.52951	70	.89705	16	.57269	74	49
50	9.88774	16	10.53021	70	9.89721	15	10.57343	74	50
51	.88790	15	.53091	70	.89737	15	.57417	74	51
52	.88805	16	.53161	70	.89752	16	.57491	74	52
53	.88821	16	.53231	70	.89768	15	.57566	74	53
54	.88837	16	.53301	70	.89783	16	.57640	74	54
55	9.88853	16	10.53372	70	9.89799	16	10.57715	75	55
56	.88869	15	.53442	70	.89815	15	.57790	74	56
57	.88885	16	.53512	70	.89830	15	.57864	74	57
58	.88901	16	.53583	70	.89846	16	.57939	75	58
59	.88917	16	.53653	70	.89862	15	.58014	75	59
60	9.88933		10.53724		9.89877		10.58089		60
'	Log. Vers.	D	Log. Exsec.	D	Log. Vers.	D	Log. Exsec.	D	'

P. P.

```
      75    74    73
6    7.5   7.4   7.3
7    8.7   8.6   8.5
8   10.0   9.8   9.7
9   11.2  11.1  10.9
10  12.5  12.3  12.1
20  25.0  24.6  24.3
30  37.5  37.0  36.5
40  50.0  49.3  48.6
50  62.5  61.6  60.8

      72    71    70
6    7.2   7.1   7.0
7    8.4   8.3   8.2
8    9.6   9.4   9.4
9   10.8  10.6  10.6
10  12.0  11.8  11.7
20  24.0  23.6  23.3
30  36.0  35.5  35.2
40  48.0  47.3  47.0
50  60.0  59.1  58.7

      69    68    67
6    6.9   6.8   6.7
7    8.0   7.9   7.8
8    9.2   9.0   8.9
9   10.3  10.2  10.0
10  11.5  11.3  11.1
20  23.0  22.6  22.3
30  34.5  34.0  33.5
40  46.0  45.3  44.6
50  57.5  56.6  55.8

      66     0
6    6.6   0.0
7    7.7   0.0
8    8.8   0.0
9    9.9   0.1
10  11.0   0.1
20  22.0   0.1
30  33.0   0.2
40  44.0   0.3
50  55.0   0.4

      18    16    15
6    1.6   1.6   1.5
7    1.9   1.8   1.8
8    2.2   2.1   2.0
9    2.5   2.4   2.3
10   2.7   2.6   2.6
20   5.5   5.3   5.1
30   8.2   8.0   7.7
40  11.0  10.6  10.3
50  13.7  13.3  12.9
```

TABLE VIII.—LOGARITHMIC VERSED SINES AND EXTERNAL SECANTS.

78°   79°

′	Log. Vers.	D	Log. Exsec.	D	Log. Vers.	D	Log. Exsec.	D	′	P. P.
0	9.89877	1⅝	10.58089	75	9.90805	1⅝	10.62745	86	0	
1	.89893	1⅝	.58164	75	.90820	15	.62825	86	1	
2	.89908	1⅝	.58239	7⅝	.90835	1⅝	.62906	86	2	
3	.89924	15	.58315	7⅝	.90851	1⅝	.62986	81	3	**86** **85** **84**
4	.89939	16	.58390	75	.90866	15	.63067	86	4	6 8.6 8.5 8.4
5	9.89955	1⅝	10.58465	75	9.90881	1⅝	10.63148	81	5	7 10.0 9.9 9.8
6	.89971	1⅝	.58541	7⅝	.90897	1⅝	.63229	81	6	8 11.4 11.3 11.2
7	.89986	1⅝	.58616	76	.90912	15	.63310	81	7	9 12.9 12.7 12.6
8	.90002	1⅝	.58692	75	.90927	1⅝	.63391	81	8	10 14.3 14.1 14.0
9	.90017	1⅝	.58768	76	.90943	1⅝	.63472	81	9	20 28.6 28.3 28.0
10	9.90033	1⅝	10.58844	76	9.90958	15	10.63553	81	10	30 43.0 42.5 42.0
11	.90048	1⅝	.58920	75	.90973	1⅝	.63634	81	11	40 57.3 56.6 56.0
12	.90064	16	.58995	7⅝	.90988	1⅝	.63716	81	12	50 71.6 70.8 70.0
13	.90080	15	.59072	76	.91004	1⅝	.63797	81	13	**83** **82** **81**
14	.90095	1⅝	.59148	76	.91019	15	.63879	82	14	6 8.3 8.2 8.1
15	9.90111	1⅝	10.59224	76	9.91034	1⅝	10.63961	82	15	7 9.3 9.5 9.4
16	.90126	1⅝	.59300	76	.91049	15	.64043	82	16	8 11.0 10.9 10.8
17	.90142	1⅝	.59377	76	.91065	15	.64125	82	17	9 12.4 12.3 12.1
18	.90157	1⅝	.59453	76	.91080	1⅝	.64207	82	18	10 13.8 13.6 13.5
19	.90173	1⅝	.59530	76	.91095	1⅝	.64289	82	19	20 27.6 27.3 27.0
20	9.90188	1⅝	10.59606	76	9.91110	15	10.64371	8⅝	20	30 41.5 41.0 40.5
21	.90204	1⅝	.59683	77	.91126	15	.64453	82	21	40 55.3 54.6 54.0
22	.90219	1⅝	.59760	76	.91141	15	.64536	8⅝	22	50 69.1 68.3 67.5
23	.90235	1⅝	.59837	77	.91156	1⅝	.64618	82	23	**80** **79** **78**
24	.90250	15	.59914	77	.91171	1⅝	.64701	83	24	6 8.0 7.9 7.8
25	9.90266	1⅝	10.59991	77	9.91187	15	10.64784	8⅝	25	7 9.3 9.2 9.1
26	.90281	1⅝	.60068	77	.91202	15	.64867	83	26	8 10.6 10.5 10.4
27	.90297	1⅝	.60145	77	.91217	15	.64950	83	27	9 12.0 11.8 11.7
28	.90312	1⅝	.60223	77	.91232	15	.65033	83	28	10 13.3 13.1 13.0
29	.90328	1⅝	.60300	77	.91247	15	.65116	83	29	20 26.6 26.3 26.0
30	9.90343	1⅝	10.60378	77	9.91263	1⅝	10.65199	8⅝	30	30 40.0 39.5 39.0
31	.90359	15	.60455	77	.91278	15	.65283	8⅝	31	40 53.3 52.6 52.0
32	.90374	15	.60533	78	.91293	1⅝	.65366	8⅝	32	50 66.6 65.8 65.0
33	.90389	1⅝	.60611	77	.91308	15	.65450	84	33	**77** **76** **75**
34	.90405	15	.60688	78	.91323	15	.65534	83	34	6 7.7 7.6 7.5
35	9.90420	1⅝	10.60766	78	9.91338	1⅝	10.65617	84	35	7 9.0 8.8 8.7
36	.90436	1⅝	.60844	78	.91354	15	.65701	84	36	8 10.2 10.1 10.0
37	.90451	1⅝	.60923	78	.91369	15	.65785	8⅝	37	9 11.5 11.4 11.2
38	.90467	1⅝	.61001	78	.91384	1⅝	.65870	84	38	10 12.8 12.6 12.5
39	.90482	15	.61079	78	.91399	15	.65954	8⅝	39	20 25.6 25.3 25.0
40	9.90497	1⅝	10.61158	7⅝	9.91414	15	10.66038	8⅝	40	30 38.5 38.0 37.5
41	.90513	1⅝	.61236	78	.91429	1⅝	.66123	8⅝	41	40 51.3 50.6 50.0
42	.90528	1⅝	.61315	78	.91445	15	.66207	8⅝	42	50 64.1 63.3 62.5
43	.90544	1⅝	.61393	78	.91460	15	.66292	8⅝	43	**δ**
44	.90559	15	.61472	79	.91475	15	.66377	85	44	6 0.6
45	9.90574	1⅝	10.61551	7⅝	9.91490	1⅝	10.66462	85	45	7 0.6
46	.90590	1⅝	.61630	79	.91505	15	.66547	85	46	8 0.6
47	.90605	1⅝	.61709	79	.91520	1⅝	.66632	8⅝	47	9 0 1
48	.90621	1⅝	.61788	7⅝	.91535	1⅝	.66717	8⅝	48	10 0.1
49	.90636	15	.61867	79	.91550	15	.66803	8⅝	49	20 0.1
50	9.90651	15	10.61947	7⅝	9.91565	1⅝	10.66888	8⅝	50	30 0.2
51	.90667	1⅝	.62026	79	.91581	15	.66974	8⅝	51	40 0.3
52	.90682	1⅝	.62105	7⅝	.91596	15	.67059	86	52	50 0.4
53	.90697	15	.62185	80	.91611	15	.67145	86	53	**16** **1⅝** **15**
54	.90713	1⅝	.62265	79	.91626	15	.67231	86	54	6 1.6 1.5 1.5
55	9.90728	1⅝	10.62345	80	9.91641	15	10.67317	86	55	7 1.8 1.8 1.7
56	.90744	1⅝	.62424	7⅝	.91656	1⅝	.67403	8⅝	56	8 2.1 2.0 2.0
57	.90759	15	.62504	80	.91671	15	.67490	8⅝	57	9 2.4 2.3 2.2
58	.90774	1⅝	.62585	80	.91686	15	.67576	8⅝	58	10 2.6 2.6 2.5
59	.90790	15	.62665	80	.91701	15	.67663	8⅝	59	20 5.3 5.1 5.0
60	9.90805		10.62745		9.91716		10.67749		60	30 8.0 7.7 7.5
										40 10.6 10.3 10.0
										50 13.3 12.9 12.5
′	Log. Vers.	D	Log. Exsec.	D	Log. Vers.	D	Log. Exsec.	D	′	P. P.

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TABLE VIII.—LOGARITHMIC VERSED SINES AND EXTERNAL SECANTS.

80°   81°

'	Log. Vers.	D	Log. Exsec.	D	Log. Vers.	D	Log. Exsec.	D	'
0	9.91716	15	10.67749	86	9.92612	14	10.73178	95	0
1	.91731	15	.67836	87	.92626	15	.73273	94	1
2	.91746	15	.67923	87	.92641	14	.73368	95	2
3	.91761	15	.68010	87	.92656	15	.73463	95	3
4	.91776	15	.68097	87	.92671	15	.73558	95	4
5	9.91791	15	10.68184	87	9.92686	14	10.73653	95	5
6	.91807	15	.68272	87	.92700	15	.73748	95	6
7	.91822	15	.68359	87	.92715	14	.73844	96	7
8	.91837	15	.68447	87	.92730	15	.73940	95	8
9	.91852	15	.68534	88	.92745	14	.74035	96	9
10	9.91867	15	10.68622	88	9.92759	15	10.74131	96	10
11	.91882	15	.68710	88	.92774	14	.74227	96	11
12	.91897	15	.68798	88	.92789	15	.74324	96	12
13	.91912	15	.68886	88	.92804	14	.74420	96	13
14	.91927	15	.68975	88	.92818	15	.74517	96	14
15	9.91942	15	10.69063	88	9.92833	15	10.74613	97	15
16	.91957	15	.69152	88	.92848	14	.74710	97	16
17	.91972	15	.69240	89	.92862	15	.74807	97	17
18	.91987	15	.69329	89	.92877	15	.74905	97	18
19	.92002	15	.69418	89	.92892	14	.75002	97	19
20	9.92016	14	10.69507	89	9.92907	15	10.75099	98	20
21	.92031	15	.69596	89	.92921	14	.75197	97	21
22	.92046	15	.69686	89	.92936	15	.75295	98	22
23	.92061	15	.69775	89	.92951	14	.75393	98	23
24	.92076	15	.69865	90	.92965	15	.75491	98	24
25	9.92091	15	10.69955	89	9.92980	15	10.75589	98	25
26	.92106	15	.70044	90	.92995	14	.75688	98	26
27	.92121	15	.70134	90	.93009	15	.75786	99	27
28	.92136	15	.70224	90	.93024	15	.75885	99	28
29	.92151	14	.70315	90	.93039	14	.75984	99	29
30	9.92166	15	10.70405	90	9.93053	15	10.76083	99	30
31	.92181	15	.70495	90	.93068	14	.76182	99	31
32	.92196	15	.70586	91	.93083	14	.76282	99	32
33	.92211	15	.70677	91	.93097	15	.76382	100	33
34	.92226	14	.70768	91	.93112	14	.76481	99	34
35	9.92240	15	10.70859	91	9.93127	14	10.76581	100	35
36	.92255	15	.70950	91	.93141	14	.76681	100	36
37	.92270	15	.71041	91	.93156	15	.76782	100	37
38	.92285	15	.71133	91	.93171	14	.76882	100	38
39	.92300	14	.71224	91	.93185	14	.76983	100	39
40	9.92315	15	10.71316	92	9.93200	14	10.77083	101	40
41	.92330	15	.71408	92	.93214	15	.77184	101	41
42	.92345	15	.71500	92	.93229	14	.77286	101	42
43	.92360	15	.71592	92	.93244	14	.77387	101	43
44	.92374	14	.71684	92	.93258	14	.77488	101	44
45	9.92389	15	10.71776	92	9.93273	14	10.77590	102	45
46	.92404	14	.71869	92	.93287	15	.77692	102	46
47	.92419	15	.71961	93	.93302	14	.77794	102	47
48	.92434	15	.72054	92	.93317	14	.77896	102	48
49	.92449	14	.72147	93	.93331	14	.77998	102	49
50	9.92463	15	10.72240	93	9.93346	14	10.78101	102	50
51	.92478	15	.72333	93	.93360	15	.78203	102	51
52	.92493	15	.72427	93	.93375	14	.78306	103	52
53	.92508	15	.72520	93	.93389	14	.78409	103	53
54	.92523	15	.72614	93	.93404	15	.78513	103	54
55	9.92538	15	10.72707	93	9.93419	14	10.78616	103	55
56	.92552	14	.72801	94	.93433	14	.78720	104	56
57	.92567	15	.72895	94	.93448	14	.78823	103	57
58	.92582	15	.72990	94	.93462	14	.78927	104	58
59	.92597	15	.73084	94	.93477	14	.79031	104	59
60	9.92612		10.73178		9.93491		10.79136		60
'	Log. Vers.	D	Log. Exsec.	D	Log. Vers.	D	Log. Exsec.	D	'

P. P.

	90	80
6	9.0	8.0
7	10.5	9.3
8	12.0	10.6
9	13.5	12.0
10	15.0	13.3
20	30.0	26.6
30	45.0	40.0
40	60.0	53.3
50	75.0	66.6

	9	8
6	0.9	0.8
7	1.0	0.9
8	1.2	1.0
9	1.3	1.2
10	1.5	1.3
20	3.0	2.6
30	4.5	4.0
40	6.0	5.3
50	7.5	6.6

	7	6
6	0.7	0.6
7	0.8	0.7
8	0.9	0.8
9	1.0	0.9
10	1.1	1.0
20	2.3	2.0
30	3.5	3.0
40	4.6	4.0
50	5.8	5.0

	5	4
6	0.5	0.4
7	0.6	0.4
8	0.6	0.5
9	0.7	0.6
10	0.8	0.6
20	1.6	1.3
30	2.5	2.0
40	3.3	2.6
50	4.1	3.3

	1.5	1.5
6	1.5	1.5
7	1.8	1.7
8	2.0	2.0
9	2.3	2.2
10	2.6	2.5
20	5.1	5.0
30	7.7	7.5
40	10.3	10.0
50	12.9	12.5

	1.4
6	1.4
7	1.7
8	1.9
9	2.2
10	2.4
20	4.8
30	7.2
40	9.6
50	12.1

TABLE VIII.—LOGARITHMIC VERSED SINES AND EXTERNAL SECANTS.

82°   83°

′	Log. Vers.	D	Log. Exsec.	D	Log. Vers.	D	Log. Exsec.	D	′
0	9.9349î	1¼	10.79136	104	9.94356	14	10.8576ô	117	0
1	.93506	1¼	.7924ô	105	.94370	1¼	.85884	117	1
2	.93526	1¼	.79345	104	.94384	14	.8600î	117	2
3	.93535	1¼	.79450	105	.94398	1¼	.86119	118	3
4	.93549	1¼	.79555	105	.94413	1¼	.86237	118	4
5	9.93564	1¼	10.7966ô	105	9.94427	14	10.86355	118	5
6	.93578	1¼	.79766	105	.94441	1¼	.86474	118	6
7	.93593	1¼	.7987î	106	.94456	14	.86592	119	7
8	.93607	1¼	.79977	106	.94470	14	.8671î	119	8
9	.93622	1¼	.80083	106	.94484	14	.86831	119	9
10	9.93636	1¼	10.80189	106	9.94498	14	10.8695ô	120	10
11	.93651	1¼	.80296	106	.94512	14	.8707ô	120	11
12	.93665	1¼	.80402	107	.94527	14	.87190	120	12
13	.93680	1¼	.80509	107	.94541	14	.8731ô	120	13
14	.93694	1¼	.80616	107	.94555	14	.87431	121	14
15	9.93709	1¼	10.80723	107	9.9456ô	14	10.87552	121	15
16	.93723	1¼	.80831	107	.94584	14	.87673	12î	16
17	.93738	1¼	.80938	108	.94598	1¼	.87794	12î	17
18	.93752	1¼	.81046	108	.94612	14	.87916	122	18
19	.93767	1¼	.8115î	108	.94626	1¼	.88038	122	19
20	9.9378î	1¼	10.81262	108	9.9464ô	14	10.88160	122	20
21	.93796	1¼	.81371	108	.94655	14	.88282	122	21
22	.9381ô	1¼	.81479	109	.94669	14	.88405	123	22
23	.93824	1¼	.81588	109	.94683	1¼	.88528	123	23
24	.93839	1¼	.81697	109	.94697	14	.8865î	124	24
25	9.93853	1¼	10.81806	109	9.9471î	14	10.88775	123	25
26	.93868	1¼	.81916	109	.94726	14	.88898	124	26
27	.93882	1¼	.82025	110	.94740	14	.89022	124	27
28	.93897	14	.82135	110	.94754	1¼	.89147	124	28
29	.93911	1¼	.82245	110	.94768	14	.8927î	125	29
30	9.93925	1¼	10.82356	110	9.94782	14	10.89396	125	30
31	.93940	1¼	.82466	110	.94796	14	.89521	125	31
32	.93954	1¼	.82577	111	.9481ô	1¼	.89647	126	32
33	.93969	14	.82688	111	.94825	14	.89773	126	33
34	.93983	1¼	.82799	111	.94839	14	.89899	126	34
35	9.93997	1¼	10.82910	111	9.94853	14	10.90025	126	35
36	.94012	1¼	.83022	111	.94867	1¼	.90152	127	36
37	.94026	1¼	.83133	112	.9488î	14	.90279	127	37
38	.94041	14	.83245	112	.94895	14	.90406	127	38
39	.94055	1¼	.83358	112	.9490ô	14	.90533	128	39
40	9.94069	1¼	10.83470	112	9.94923	1¼	10.9066î	127	40
41	.94084	14	.83583	113	.94938	14	.90789	128	41
42	.94098	1¼	.83695	113	.94952	14	.90917	129	42
43	.94112	1¼	.83809	113	.94966	14	.91046	129	43
44	.94127	1¼	.83922	113	.94980	1¼	.91175	129	44
45	9.94141	1¼	10.84035	114	9.94994	14	10.91304	130	45
46	.94155	1¼	.84149	114	.95008	14	.91434	129	46
47	.94170	1¼	.84263	114	.95022	14	.91564	130	47
48	.94184	1¼	.84377	114	.95036	14	.91694	130	48
49	.94198	1¼	.84492	115	.9505ô	14	.91825	131	49
50	9.94213	14	10.84607	114	9.95064	14	10.91956	131	50
51	.94227	1¼	.84721	115	.95078	1¼	.92087	13î	51
52	.94241	1¼	.84837	115	.95093	14	.92218	13î	52
53	.94256	14	.84952	116	.95107	14	.92350	132	53
54	.94270	1¼	.85068	115	.95121	14	.92482	132	54
55	9.94284	1¼	10.85183	116	9.95135	14	10.92614	133	55
56	.94299	14	.85299	116	.95149	14	.92747	133	56
57	.94313	1¼	.85416	116	.95163	14	.9288ô	133	57
58	.94327	14	.85532	117	.95177	14	.93014	133	58
59	.9434î	1¼	.8564ô	117	.95191	14	.93147	134	59
60	9.94356		10.85766		9.95205		10.9328î		60
′	Log. Vers.	D	Log. Exsec.	D	Log. Vers.	D	Log. Exsec.	D	′

P. P.

	130	120
6	13.0	12.0
7	15.î	14.0
8	17.3	16.0
9	19.5	18.0
10	21.6	20.0
20	43.3	40.0
30	65.0	60.0
40	86.6	80.0
50	108.3	100.0

	110	100
6	11.0	10.0
7	12.8	11.6
8	14.6	13.3
9	16.5	15.0
10	18.3	16.6
20	36.6	33.3
30	55.0	50.0
40	73.3	66.6
50	91.6	83.3

	3	2
6	0.3	0.2
7	0.3	0.2
8	0.4	0.3
9	0.4	0.3
10	0.5	0.3
20	1.0	0.6
30	1.5	1.0
40	2.0	1.3
50	2.5	1.6

	1	ô
6	0.1	0.6
7	0.1	0.6
8	0.1	0.6
9	0.1	0.1
10	0.1	0.1
20	0.3	0.1
30	0.5	0.2
40	0.6	0.3
50	0.8	0.4

	1¼	14
6	1.4	1.4
7	1.7	1.6
8	1.9	1.8
9	2.2	2.1
10	2.4	2.3
20	4.8	4.6
30	7.2	7.0
40	9.6	9.3
50	12.1	11.6

TABLE VIII.—LOGARITHMIC VERSED SINES AND EXTERNAL SECANTS.
84°   **85°**

'	Log. Vers.	D	Log. Exsec.	D	Log. Vers.	D	Log. Exsec.	D	'
0	9.95205	14	10.93281	134	9.96039	14	11.02010	158	0
1	.95219	14	.93416	135	.96053	13	.02168	159	1
2	.95233	14	.93551	135	.96067	13	.02327	159	2
3	.95247	14	.93686	135	.96081	14	.02487	159	3
4	.95261	14	.93821	135	.96095	13	.02646	160	4
5	9.95275	14	10.93957	136	9.96108	14	11.02807	161	5
6	.95289	14	.94093	136	.96122	14	.02968	161	6
7	.95303	14	.94229	137	.96136	14	.03129	161	7
8	.95317	14	.94366	137	.96150	13	.03291	162	8
9	.95331	14	.94503	137	.96163	14	.03453	163	9
10	9.95345	14	10.94641	137	9.96177	14	11.03616	163	10
11	.95359	14	.94778	13	.96191	13	.03780	163	11
12	.95373	13	.94917	138	.96205	14	.03944	164	12
13	.95387	14	.95055	138	.96218	14	.04108	164	13
14	.95401	14	.95194	139	.96232	14	.04273	165	14
15	9.95415	14	10.95333	139	9.96246	13	11.04438	165	15
16	.95429	14	.95473	139	.96259	14	.04604	166	16
17	.95443	14	.95613	140	.96273	14	.04771	167	17
18	.95457	14	.95753	140	.96287	14	.04938	167	18
19	.95471	14	.95894	141	.96301	13	.05106	167	19
20	9.95485	14	10.96035	141	9.96314	14	11.05274	168	20
21	.95499	14	.96176	142	.96328	14	.05443	169	21
22	.95513	14	.96318	143	.96342	13	.05612	169	22
23	.95527	13	.96461	142	.96355	14	.05782	170	23
24	.95540	14	.96603	143	.96369	14	.05952	171	24
25	9.95554	14	10.96746	143	9.96383	14	11.06123	171	25
26	.95568	14	.96889	144	.96397	13	.06295	172	26
27	.95582	14	.97033	144	.96410	14	.06467	173	27
28	.95596	14	.97177	144	.96424	14	.06640	173	28
29	.95610	14	.97322	145	.96438	13	.06813	174	29
30	9.95624	14	10.97467	145	9.96451	14	11.06987	174	30
31	.95638	14	.97612	145	.96465	14	.07161	175	31
32	.95652	14	.97758	146	.96479	13	.07336	176	32
33	.95666	14	.97904	146	.96492	14	.07512	176	33
34	.95680	13	.98050	147	.96506	13	.07688	177	34
35	9.95693	14	10.98197	147	9.96519	14	11.07865	177	35
36	.95707	14	.98345	147	.96533	14	.08043	178	36
37	.95721	14	.98492	148	.96547	13	.08221	179	37
38	.95735	14	.98640	149	.96560	14	.08400	179	38
39	.95749	13	.98789	149	.96574	14	.08579	180	39
40	9.95763	14	10.98938	149	9.96588	13	11.08759	180	40
41	.95777	14	.99087	150	.96601	14	.08940	181	41
42	.95791	13	.99237	150	.96615	14	.09121	182	42
43	.95804	14	.99387	151	.96629	13	.09303	182	43
44	.95818	14	.99538	151	.96642	14	.09486	183	44
45	9.95832	14	10.99689	151	9.96656	13	11.09669	184	45
46	.95846	14	.99841	152	.96669	14	.09853	185	46
47	.95860	14	10.99993	152	.96683	14	.10038	185	47
48	.95874	14	11.00145	152	.96697	13	.10223	186	48
49	.95888	13	.00298	153	.96710	14	.10409	186	49
50	9.95901	14	11.00451	153	9.96724	13	11.10595	187	50
51	.95915	14	.00605	154	.96737	14	.10783	188	51
52	.95929	14	.00759	154	.96751	13	.10971	189	52
53	.95943	14	.00914	155	.96764	14	.11160	189	53
54	.95957	13	.01069	155	.96778	14	.11349	190	54
55	9.95970	14	11.01225	156	9.96792	13	11.11539	191	55
56	.95984	14	.01381	156	.96805	14	.11730	191	56
57	.95998	14	.01537	157	.96819	13	.11922	192	57
58	.96012	14	.01694	157	.96832	14	.12114	193	58
59	.96026	13	.01852	158	.96846	13	.12307	193	59
60	9.96039		11.02010		9.96859		11.12501		60
'	Log. Vers.	D	Log. Exsec.	D	Log. Vers.	D	Log. Exsec.	D	'

P. P.

	190	180
6	19.0	18.0
7	22.1	21.0
8	25.3	24.0
9	28.5	27.0
10	31.6	30.0
20	63.3	60.0
30	95.0	90.0
40	126.6	120.0
50	158.3	150.0

	170	160
6	17.0	16.0
7	19.8	18.6
8	22.6	21.3
9	25.5	24.0
10	28.3	26.6
20	56.6	53.3
30	85.0	80.0
40	113.3	106.6
50	141.6	133.3

	150	140
6	15.0	14.0
7	17.5	16.3
8	20.0	18.6
9	22.5	21.0
10	25.0	23.3
20	50.0	46.6
30	75.0	70.0
40	100.0	93.3
50	125.0	116.6

	130	9	8
6	13.0	0.9	0.8
7	15.1	1.0	0.9
8	17.3	1.2	1.0
9	19.5	1.3	1.2
10	21.6	1.5	1.3
20	43.3	3.0	2.6
30	65.0	4.5	4.0
40	86.6	6.0	5.3
50	108.3	7.5	6.6

	7	6	5
6	0.7	0.6	0.5
7	0.8	0.7	0.6
8	0.9	0.8	0.6
9	1.0	0.9	0.7
10	1.1	1.0	0.8
20	2.3	2.0	1.6
30	3.5	3.0	2.5
40	4.6	4.0	3.3
50	5.8	5.0	4.1

	1⅔	1⅓	1⅓
6	1.4	1.4	1.3
7	1.7	1.6	1.6
8	1.9	1.8	1.8
9	2.2	2.1	2.0
10	2.4	2.3	2.2
20	4.8	4.6	4.5
30	7.2	7.0	6.7
40	9.6	9.3	9.0
50	12.1	11.6	11.2

TABLE VIII.—LOGARITHMIC VERSED SINES AND EXTERNAL SECANTS.

86°   87°

'	Log. Vers.	D	Log. Exsec.	D	Log. Vers.	D	Log. Exsec.	D	'
0	9.96859	13	11.12501	195	9.97665	13	11.25785	255	0
1	.96873	14	.12696	195	.97679	13	.26040	256	1
2	.96887	13	.12891	196	.97692	13	.26297	257	2
3	.96900	13	.13087	196	.97705	13	.26554	259	3
4	.96914	13	.13284	198	.97718	13	.26814	260	4
5	9.96927	13	11.13482	198	9.97732	13	11.27074	262	5
6	.96941	13	.13680	199	.97745	13	.27336	263	6
7	.96954	13	.13879	200	.97758	13	.27599	265	7
8	.96968	13	.14079	201	.97772	13	.27864	266	8
9	.96981	13	.14280	201	.97785	13	.28131	267	9
10	9.96995	13	11.14482	202	9.97798	13	11.28398	269	10
11	.97008	13	.14684	203	.97811	13	.28668	270	11
12	.97022	13	.14887	204	.97825	13	.28938	272	12
13	.97035	13	.15092	205	.97838	13	.29211	274	13
14	.97049	13	.15297	205	.97851	13	.29485	275	14
15	9.97062	13	11.15502	206	9.97864	13	11.29760	277	15
16	.97076	13	.15709	208	.97878	13	.30037	278	16
17	.97089	13	.15917	208	.97891	13	.30316	279	17
18	.97103	13	.16125	209	.97904	13	.30596	282	18
19	.97116	13	.16334	210	.97917	13	.30878	283	19
20	9.97130	13	11.16544	211	9.97931	13	11.31162	285	20
21	.97143	13	.16755	212	.97944	13	.31447	287	21
22	.97157	13	.16967	213	.97957	13	.31734	288	22
23	.97170	13	.17180	214	.97970	13	.32023	290	23
24	.97183	13	.17394	214	.97984	13	.32313	292	24
25	9.97197	13	11.17609	215	9.97997	13	11.32606	294	25
26	.97210	13	.17824	216	.98010	13	.32900	296	26
27	.97224	13	.18041	218	.98023	13	.33196	298	27
28	.97237	13	.18259	218	.98036	13	.33494	299	28
29	.97251	13	.18477	219	.98050	13	.33793	301	29
30	9.97264	13	11.18697	220	9.98063	13	11.34095	303	30
31	.97277	13	.18917	221	.98076	13	.34398	305	31
32	.97291	13	.19138	222	.98089	13	.34704	307	32
33	.97304	13	.19361	223	.98102	13	.35011	309	33
34	.97318	13	.19584	224	.98116	13	.35321	311	34
35	9.97331	13	11.19809	224	9.98129	13	11.35632	313	35
36	.97345	13	.20034	225	.98142	13	.35946	315	36
37	.97358	13	.20261	227	.98155	13	.36261	318	37
38	.97371	13	.20489	227	.98168	13	.36579	320	38
39	.97385	13	.20717	228	.98181	13	.36899	322	39
40	9.97398	13	11.20947	230	9.98195	13	11.37221	322	40
41	.97412	13	.21178	230	.98208	13	.37546	324	41
42	.97425	13	.21410	232	.98221	13	.37872	326	42
43	.97438	13	.21643	233	.98234	13	.38201	328	43
44	.97452	13	.21877	234	.98247	13	.38532	331	44
45	9.97465	13	11.22112	235	9.98260	13	11.38866	333	45
46	.97478	13	.22349	236	.98273	13	.39201	335	46
47	.97492	13	.22586	237	.98287	13	.39540	338	47
48	.97505	13	.22825	239	.98300	13	.39880	340	48
49	.97519	13	.23065	239	.98313	13	.40224	343	49
50	9.97532	13	11.23306	241	9.98326	13	11.40569	345	50
51	.97545	13	.23548	242	.98339	13	.40918	348	51
52	.97559	13	.23792	243	.98352	13	.41269	351	52
53	.97572	13	.24037	245	.98365	13	.41622	353	53
54	.97585	13	.24283	246	.98378	13	.41979	356	54
55	9.97599	13	11.24530	247	9.98392	13	11.42338	359	55
56	.97612	13	.24778	248	.98405	13	.42699	361	56
57	.97625	13	.25028	250	.98418	13	.43064	364	57
58	.97639	13	.25279	251	.98431	13	.43431	367	58
59	.97652	13	.25531	252	.98444	13	.43802	370	59
60	9.97665	13	11.25785	254	9.98457	13	11.44175	373	60
'	Log. Vers.	D	Log. Exsec.	D	Log. Vers.	D	Log. Exsec.	D	'

P. P.

	250	240
6	25.0	24.0
7	29.1	28.0
8	33.3	32.0
9	37.5	36.0
10	41.6	40.0
20	83.3	80.0
30	125.0	120.0
40	166.6	160.0
50	208.3	200.0

	230	220
6	23.0	22.0
7	26.8	25.6
8	30.6	29.3
9	34.5	33.0
10	38.3	36.6
20	76.6	73.3
30	115.0	110.0
40	153.3	146.6
50	191.6	183.3

	210	200
6	21.0	20.0
7	24.5	23.3
8	28	26.6
9	31.5	30.0
10	35.0	33.3
20	70.0	66.6
30	105.0	100.0
40	140.0	133.3
50	175.0	166.6

	190	4	3
6	19.0	0.4	0.3
7	22.1	0.4	0.3
8	25.3	0.5	0.4
9	28.5	0.6	0.4
10	31.6	0.6	0.5
20	63.3	1.3	1.0
30	95.0	2.0	1.5
40	126.6	2.6	2.0
50	158.3	3.3	2.5

	2	1	6
6	0.2	0.1	0.6
7	0.2	0.1	0.6
8	0.2	0.1	0.6
9	0.3	0.1	0.1
10	0.6	0.1	0.1
20	0.6	0.3	0.1
30	1.0	0.5	0.2
40	1.3	0.6	0.3
50	1.6	0.8	0.4

	14	13	13
6	1.4	1.3	1.3
7	1.6	1.6	1.5
8	1.8	1.8	1.7
9	2.1	2.0	1.9
10	2.3	2.2	2.1
20	4.6	4.5	4.3
30	7.0	6.7	6.5
40	9.3	9.0	8.6
50	11.6	11.2	10.8

TABLE VIII.—LOGARITHMIC VERSED SINES AND EXTERNAL SECANTS.

88° 89°

′	Log. Vers.	D	Log. Exsec.	D	Log. Vers.	D	Log. Exsec.	D	′	P. P.
0	9.98457	13	11.44175	376	9.99235̂	12̂	11.75050	742̂	0	
I	.98470̂	13	.44551̂	376̂	.99248	13	.75792	755	I	
2	.98483̂	13	.44931	382̂	.99261	13	.76547̂	768	2	
3	.98496	13	.45313̂	386	.99274	13	.77316	781̂	3	
4	.98509̂	13	.45699̂	389	.99287	13	.78097̂	795	4	
5	9.98522̂	13	11.46088	392	9.99299̂	12̂	11.78892̂	809	5	
6	.98535̂	13	.46480	395	.99312̂	13	.79702	825	6	
7	.98548̂	13	.46876	399	.99325̂	12̂	.80527	840	7	
8	.98562̂	13	.47275̂	402	.99338̂	13	.81367	856	8	
9	.98575̂	13	.47677̂	406	.99351	13	.82223	872̂	9	
10	9.98588	13	11.48083̂	409	9.99363̂	12̂	11.83095̂	890	10	
II	.98601	13	.48493	413	.99376̂	13	.83986	908	II	
12	.98614	13	.48906	417	.99389̂	13	.84894	927	12	
13	.98627	13	.49323	420̂	.99402	12̂	.85821	947	13	
14	.98640	13	.49743̂	425	.99415	13	.86768	967	14	
15	9.98653	13	11.50168	428	9.99428	13	11.87735	989	15	
16	.98666	13	.50597	432̂	.99440̂	12̂	.88724	1009	16	
17	.98679	13	.51029̂	436	.99453̂	13	.89735	1034̂	17	
18	.98692	13	.51466	440̂	.99466̂	12̂	.90769̂	1059̂	18	
19	.98705	13	.51906̂	445	.99479	13	.91829	1085	19	
20	9.98718	13	11.52351̂	449̂	9.9949î	12̂	11.92914	1112̂	20	
21	.98731	13	.52801	454	.99504̂	13	.94026	1146̂	21	
22	.98744	13	.53255	458	.99517̂	12̂	.95167	1171	22	
23	.98757	13	.53713	463	.99530̂	13	.96338	1203	23	
24	.98770	13	.54176	467	.99543	12̂	.97541	1236	24	
25	9.98783	13	11.54643	472̂	9.99555̂	12̂	11.98777	1271̂	25	
26	.98796	13	.55116	477	.99568̂	13̂	12.00048	1309̂	26	
27	.98809	13	.55593̂	482̂	.99581	13	.01358	1349	27	
28	.98822	13	.56076	487	.99594	12̂	.02707	1391̂	28	
29	.98835	13	.56563̂	492̂	.99606̂	13̂	.04098	1436	29	
30	9.98848	13	11.57056	498	9.99619̂	12̂	12.05535	1485	30	
31	.98861	13	.57554	504	.99632	13	.07020	1537	31	
32	.98874	13	.58058	509	.99645	12̂	.08557	1592	32	
33	.98887	13	.58567	515	.99657̂	13	.10149	1652	33	
34	.98900	13	.59082	520̂	.99670̂	12̂	.11801	1716	34	
35	9.98913	13 / 12	11.59602	527	9.99683̂	12̂	12.13517	1785̂	35	
36	.98925̂	13	.60129̂	533	.99695̂	12̂	.15302̂	1861	36	
37	.98938̂	13	.60662̂	539̂	.99708̂	13	.17163̂	1943	37	
38	.98951̂	13	.61202	545	.99721	12̂	.19106̂	2033	38	
39	.98964̂	13	.61747̂	552̂	.99734	12̂	.21139	2131̂	39	
40	9.98977̂	13	11.62300	559	9.99746̂	13	12.23271	2240̂	40	
41	.98990̂	13	.62859	566	.99759̂	12̂	.25511̂	2361	41	
42	.99003̂	13	.63425	573	.99772	12̂	.27872̂	2495	42	
43	.99016̂	13 / 12	.63998	581	.99784̂	12̂	.30367	2645̂	43	
44	.99029	13	.64579	588	.99797̂	13	.33013	2815̂	44	
45	9.99042	13	11.65167	595	9.99810	12̂	12.35828	3009	45	
46	.99055	13	.65762̂	604	.99823̂	13	.38837	3231	46	
47	.99068	13	.66366̂	611̂	.99835̂	12̂	.42068	3489	47	
48	.99081	13 / 12	.66977̂	620̂	.99848̂	13	.45557̂	3791̂	48	
49	.99093̂	13	.67598̂	628̂	.99861	12̂	.49349	4152	49	
50	9.99106̂	13	11.68227	638	9.99873̂	12̂	12.53501	4588̂	50	
51	.99119̂	13	.68865	646	.99886	13	.58089̂	5127̂	51	
52	.99132̂	13 / 12	.69511̂	656̂	.99899	12̂	.63217	5812	52	
53	.99145̂	13 / 12	.70168	666	.99911̂	12̂	.69029	6707̂	53	
54	.99158	13	.70834	675̂	.99924	13	.75736	7931̂	54	
55	9.99171	13	11.71509	686̂	9.99937	12̂	12.83667	9704	55	
56	.99184	13	.72196	696̂	.99949̂	12̂	.93371	12506̂	56	
57	.99197	13 / 12	.72892̂	707̂	.99962	13	13.05877̂	17621̂	57	
58	.99209̂	13	.73600	719̂	.99974̂	12̂	.23499	30116	58	
59	.99222̂	13	.74319̂	730̂	.99987̂	12̂	.53615		59	
60	9.99235̂		11.75050		10.00000		Infinity		60	
′	Log. Vers.	D	Log. Exsec.	D	Log. Vers.	D	Log. Exsec.	D	′	P. P.

P. P.

	13̂	13
6	1.3̂	1.3
7	1.6	1.5
8	1.8	1.7
9	2.0	1.9̂
10	2.2̂	2.1
20	4.5	4.3̂
30	6.7̂	6.5
40	9.0	8.6̂
50	11.2̂	10.8

	12̂
6	1.2̂
7	1.4̂
8	1.6
9	1.9
10	2.1
20	4.1̂
30	6.2̂
40	8.3̂
50	10.4

TABLE IX.—NATURAL SINES, COSINES, TANGENTS, AND COTANGENTS.

0°–10°

° ′	Sin.	d.	Tan.	d.	Cot.	d.	Cos.	d.	P. P.
0 0	0.0000		0.0000		∞		1.0000		0 90
10	0.0029	29	0.0029	29	343.773		1.0000	0	50
20	0.0058	29	0.0058	29	171.885		1.0000	0	40
30	0.0087	29	0.0087	29	114.588		0.9999	0	30
40	0.0116	29	0.0116	29	85.9398		0.9999	0	20
50	0.0145	29	0.0145	29	68.7501		0.9999	0	10
1 0	0.0174	29	0.0174	29	57.2899	8.1866	0.9998	0	0 89
10	0.0203	29	0.0203	29	49.1039	6.1398	0.9998	0	50
20	0.0232	29	0.0233	29	42.9641	4.7756	0.9997	1	40
30	0.0262	29	0.0262	29	38.1885	3.8217	0.9996	0	30
40	0.0291	29	0.0291	29	34.3677	3.1261	0.9996	1	20
50	0.0320	29	0.0320	29	31.2416	2.6053	0.9995	1	10
2 0	0.0349	29	0.0349	29	28.6362	2.2046	0.9994	1	0 88
10	0.0378	29	0.0378	29	26.4316	1.8898	0.9993	1	50
20	0.0407	29	0.0407	29	24.5417	1.6380	0.9991	1	40
30	0.0436	29	0.0436	29	22.9037	1.4333	0.9990	1	30
40	0.0465	29	0.0466	29	21.4704	1.2648	0.9989	1	20
50	0.0494	29	0.0495	29	20.2055	1.1244	0.9988	1	10
3 0	0.0523	29	0.0524	29	19.0811	1.0061	0.9986	2	0 87
10	0.0552	29	0.0553	29	18.0750	9056	0.9984	1	50
20	0.0581	29	0.0582	29	17.1693	8195	0.9983	1	40
30	0.0610	29	0.0611	29	16.3498	7456	0.9981	2	30
40	0.0639	29	0.0641	29	15.6048	6804	0.9979	2	20
50	0.0668	29	0.0670	29	14.9244	6237	0.9977	2	10
4 0	0.0697	29	0.0699	29	14.3006	5739	0.9975	2	0 86
10	0.0726	29	0.0728	29	13.7267	5298	0.9973	2	50
20	0.0755	29	0.0758	29	13.1969	4907	0.9971	2	40
30	0.0784	29	0.0787	29	12.7062	4557	0.9969	2	30
40	0.0813	29	0.0816	29	12.2505	4243	0.9967	2	20
50	0.0842	29	0.0845	29	11.8261	3961	0.9964	2	10
5 0	0.0871	29	0.0875	29	11.4306	3706	0.9962	2	0 85
10	0.0900	29	0.0904	29	11.0594	3475	0.9959	2	50
20	0.0929	29	0.0933	29	10.7119	3265	0.9956	2	40
30	0.0958	29	0.0963	29	10.3854	3073	0.9954	3	30
40	0.0987	29	0.0992	29	10.0780	2899	0.9951	3	20
50	0.1016	29	0.1021	29	9.7881	2738	0.9948	3	10
6 0	0.1045	28	0.1051	29	9.5143	2590	0.9945	3	0 84
10	0.1074	29	0.1080	29	9.2553	2454	0.9942	3	50
20	0.1103	29	0.1110	29	9.0098	2329	0.9939	3	40
30	0.1132	29	0.1139	29	8.7769	2213	0.9935	3	30
40	0.1161	29	0.1169	29	8.5555	2106	0.9932	3	20
50	0.1190	28	0.1198	29	8.3449	2006	0.9929	3	10
7 0	0.1218	29	0.1228	29	8.1443	1913	0.9925	3	0 83
10	0.1247	29	0.1257	29	7.9530	1826	0.9922	4	50
20	0.1276	29	0.1287	29	7.7703	1746	0.9918	3	40
30	0.1305	28	0.1316	29	7.5957	1676	0.9914	4	30
40	0.1334	29	0.1346	30	7.4287	1599	0.9910	4	20
50	0.1363	28	0.1376	29	7.2687	1534	0.9906	4	10
8 0	0.1391	29	0.1405	29	7.1153	1471	0.9902	4	0 82
10	0.1420	29	0.1435	30	6.9682	1413	0.9898	4	50
20	0.1449	28	0.1465	29	6.8269	1358	0.9894	4	40
30	0.1478	29	0.1494	30	6.6911	1306	0.9890	4	30
40	0.1507	28	0.1524	30	6.5605	1257	0.9886	4	20
50	0.1535	29	0.1554	30	6.4348	1211	0.9881	4	10
9 0	0.1564	28	0.1584	29	6.3137	1167	0.9877	4	0 81
10	0.1593	29	0.1613	30	6.1970	1126	0.9872	5	50
20	0.1622	28	0.1643	30	6.0844	1087	0.9867	4	40
30	0.1650	28	0.1673	30	5.9757	1049	0.9863	5	30
40	0.1679	29	0.1703	30	5.8708	1014	0.9858	5	20
50	0.1708	28	0.1733	30	5.7693	980	0.9853	5	10
10 0	0.1736		0.1763		5.6713		0.9848		0 80
	Cos.	d.	Cot.	d.	Tan.	d.	Sin.	d.	′ °	P. P.

P. P.

30	2Ɋ	29
1	3.0	2.9	2.9
2	6.0	5.9	5.8
3	9.0	8.8	8.7
4	12.0	11.8	11.6
5	15.0	14.7	14.5
6	18.0	17.7	17.4
7	21.0	20.6	20.3
8	24.0	23.6	23.2
9	27.0	26.5	26.1

28	5	4̂	4
1	2.8	0.5	0.4	0.4
2	5.7	1.0	0.9	0.8
3	8.5	1.5	1.3	1.2
4	11.4	2.0	1.8	1.6
5	14.2	2.5	2.2	2.0
6	17.1	3.0	2.7	2.4
7	19.6	3.5	3.1	2.8
8	22.8	4.0	3.6	3.2
9	25.6	4.5	4.0	3.6

3	3	2̂	2
1	0.3	0.3	0.2	0.2
2	0.7	0.6	0.5	0.4
3	1.0	0.9	0.7	0.6
4	1.4	1.2	1.0	0.8
5	1.7	1.5	1.2	1.0
6	2.1	1.8	1.5	1.2
7	2.4	2.1	1.7	1.4
8	2.8	2.4	2.0	1.6
9	3.1	2.7	2.2	1.8

1̂	1	0̂
1	0.1	0.1	0.0
2	0.3	0.2	0.1
3	0.4	0.3	0.1
4	0.6	0.4	0.2
5	0.7	0.5	0.2
6	0.9	0.6	0.3
7	1.0	0.7	0.3
8	1.2	0.8	0.4
9	1.3	0.9	0.4

TABLE IX.—NATURAL SINES, COSINES, TANGENTS, AND COTANGENTS.
10°–20°

° ′	Sin.	d.	Tan.	d.	Cot.	d.	Cos.	d.	′ °
10 0	0.1736	28	0.1763	30	5.6713	949	0.9848	5	**0 80**
10	0.1765	28	0.1793	30	5.5764	919	0.9843	5	50
20	0.1793	29	0.1823	30	5.4845	890	0.9838	6	40
30	0.1822	29	0.1853	30	5.3955	862	0.9832	5	30
40	0.1851	28	0.1883	30	5.3093	836	0.9827	5	20
50	0.1879	29	0.1913	31	5.2256	811	0.9822	6	10
11 0	0.1908	28	0.1944	30	5.1445	787	0.9816	6	**0 79**
10	0.1936	29	0.1974	30	5.0658	764	0.9810	5	50
20	0.1965	28	0.2004	30	4.9894	742	0.9805	6	40
30	0.1993	29	0.2034	31	4.9152	721	0.9799	6	30
40	0.2022	28	0.2065	30	4.8430	701	0.9793	6	20
50	0.2050	29	0.2095	30	4.7728	682	0.9787	6	10
12 0	0.2079	28	0.2125	31	4.7046	664	0.9781	6	**0 78**
10	0.2107	29	0.2156	30	4.6382	646	0.9775	6	50
20	0.2136	28	0.2186	31	4.5736	629	0.9769	6	40
30	0.2164	29	0.2217	30	4.5107	613	0.9763	6	30
40	0.2193	28	0.2247	31	4.4494	597	0.9756	7	20
50	0.2221	29	0.2278	30	4.3897	582	0.9750	6	10
13 0	0.2250	28	0.2308	31	4.3315	568	0.9743	7	**0 77**
10	0.2278	28	0.2339	31	4.2747	553	0.9737	6	50
20	0.2306	28	0.2370	31	4.2193	540	0.9730	7	40
30	0.2334	29	0.2401	31	4.1653	527	0.9723	7	30
40	0.2363	28	0.2432	30	4.1126	515	0.9717	6	20
50	0.2391	28	0.2462	31	4.0611	502	0.9710	7	10
14 0	0.2419	28	0.2493	31	4.0108	491	0.9703	7	**0 76**
10	0.2447	28	0.2524	31	3.9617	480	0.9696	7	50
20	0.2475	29	0.2555	31	3.9136	469	0.9689	8	40
30	0.2504	28	0.2586	31	3.8667	458	0.9681	7	30
40	0.2532	28	0.2617	31	3.8208	449	0.9674	7	20
50	0.2560	28	0.2648	31	3.7760	439	0.9667	8	10
15 0	0.2588	28	0.2679	32	3.7321	429	0.9659	7	**0 75**
10	0.2616	28	0.2711	31	3.6891	420	0.9652	8	50
20	0.2644	28	0.2742	31	3.6470	411	0.9644	8	40
30	0.2672	28	0.2773	32	3.6059	403	0.9636	8	30
40	0.2700	28	0.2805	31	3.5656	394	0.9628	7	20
50	0.2728	28	0.2836	31	3.5261	387	0.9621	8	10
16 0	0.2756	28	0.2867	32	3.4874	379	0.9613	9	**0 74**
10	0.2784	28	0.2899	32	3.4495	371	0.9604	8	50
20	0.2812	28	0.2931	31	3.4124	364	0.9596	8	40
30	0.2840	28	0.2962	32	3.3759	357	0.9588	8	30
40	0.2868	28	0.2994	32	3.3402	350	0.9580	9	20
50	0.2896	28	0.3026	31	3.3052	343	0.9571	8	10
17 0	0.2924	28	0.3057	32	3.2709	337	0.9563	9	**0 73**
10	0.2952	27	0.3089	32	3.2371	331	0.9554	8	50
20	0.2979	28	0.3121	32	3.2041	324	0.9546	9	40
30	0.3007	28	0.3153	32	3.1716	319	0.9537	9	30
40	0.3035	27	0.3185	32	3.1397	313	0.9528	8	20
50	0.3062	28	0.3217	32	3.1084	307	0.9520	9	10
18 0	0.3090	28	0.3249	32	3.0777	302	0.9511	9	**0 72**
10	0.3118	27	0.3281	33	3.0475	296	0.9502	10	50
20	0.3145	28	0.3314	32	3.0178	291	0.9492	9	40
30	0.3173	28	0.3346	32	2.9887	286	0.9483	9	30
40	0.3201	27	0.3378	33	2.9600	281	0.9474	9	20
50	0.3228	28	0.3411	32	2.9319	277	0.9465	10	10
19 0	0.3256	27	0.3443	33	2.9042	272	0.9455	9	**0 71**
10	0.3283	28	0.3476	32	2.8770	267	0.9446	10	50
20	0.3311	27	0.3508	33	2.8502	263	0.9436	10	40
30	0.3338	27	0.3541	33	2.8239	259	0.9426	9	30
40	0.3365	28	0.3574	33	2.7980	254	0.9417	10	20
50	0.3393	27	0.3607	33	2.7725	250	0.9407	10	10
20 0	0.3420		0.3640		2.7475		0.9397		**0 70**
	Cos.	d.	Cot.	d.	Tan.	d.	Sin.	d.	′ °

P. P.

	33	32	31
1	3.3	3.2	3.1
2	6.6	6.4	6.2
3	9.9	9.6	9.3
4	13.2	12.8	12.4
5	16.5	16.0	15.5
6	19.8	19.2	18.6
7	23.1	22.4	21.7
8	26.4	25.6	24.8
9	29.7	28.8	27.9

	30	30	29
1	3.0	3.0	2.9
2	6.1	6.0	5.8
3	9.1	9.0	8.7
4	12.2	12.0	11.6
5	15.2	15.0	14.5
6	18.3	18.0	17.4
7	21.3	21.0	20.3
8	24.4	24.0	23.2
9	27.4	27.0	26.1

	28	28	27
1	2.8	2.8	2.7
2	5.7	5.6	5.4
3	8.5	8.4	8.1
4	11.4	11.2	10.8
5	14.2	14.0	13.5
6	17.1	16.8	16.2
7	19.9	19.6	18.9
8	22.8	22.4	21.6
9	25.6	25.2	24.3

	10	9	8
1	1.0	0.9	0.8
2	2.0	1.8	1.6
3	3.0	2.7	2.4
4	4.0	3.6	3.2
5	5.0	4.5	4.0
6	6.0	5.4	4.8
7	7.0	6.3	5.6
8	8.0	7.2	6.4
9	9.0	8.1	7.2

	7	7	6	5
1	0.7	0.7	0.6	0.5
2	1.5	1.4	1.2	1.0
3	2.2	2.1	1.8	1.5
4	3.0	2.8	2.4	2.0
5	3.7	3.5	3.0	2.5
6	4.5	4.2	3.6	3.0
7	5.2	4.9	4.2	3.5
8	6.0	5.6	4.8	4.0
9	6.7	6.3	5.4	4.5

TABLE IX.—NATURAL SINES, COSINES, TANGENTS, AND COTANGENTS.
20°–30°

° ′	Sin.	d.	Tan.	d.	Cot.	d.	Cos.	d.		
20 0	0.3420	27	0.3639	33	2.7475	247	0.9397	10	0	70
10	0.3447	27	0.3672	33	2.7228	243	0.9387	10	50	
20	0.3475	27	0.3705	33	2.6985	239	0.9377	10	40	
30	0.3502	27	0.3739	33	2.6746	235	0.9366	10	30	
40	0.3529	27	0.3772	33	2.6511	232	0.9356	10	20	
50	0.3556	27	0.3805	33	2.6279	228	0.9346	10	10	
21 0	0.3583	27	0.3838	33	2.6051	225	0.9336	10	0	69
10	0.3611	27	0.3872	33	2.5826	222	0.9325	10	50	
20	0.3638	27	0.3905	33	2.5604	218	0.9315	11	40	
30	0.3665	27	0.3939	33	2.5386	215	0.9304	10	30	
40	0.3692	27	0.3972	34	2.5171	212	0.9293	11	20	
50	0.3719	27	0.4006	34	2.4959	208	0.9282	10	10	
22 0	0.3746	27	0.4040	33	2.4751	206	0.9272	11	0	68
10	0.3773	27	0.4074	34	2.4545	203	0.9261	11	50	
20	0.3800	27	0.4108	34	2.4342	200	0.9250	11	40	
30	0.3827	26	0.4142	34	2.4142	197	0.9239	11	30	
40	0.3853	27	0.4176	34	2.3945	194	0.9228	11	20	
50	0.3880	27	0.4210	34	2.3750	192	0.9216	11	10	
23 0	0.3907	26	0.4244	34	2.3558	189	0.9205	11	0	67
10	0.3934	27	0.4279	34	2.3369	187	0.9194	11	50	
20	0.3961	26	0.4313	34	2.3182	184	0.9182	11	40	
30	0.3987	26	0.4348	35	2.2998	182	0.9170	11	30	
40	0.4014	26	0.4383	34	2.2817	179	0.9159	12	20	
50	0.4041	26	0.4417	35	2.2637	177	0.9147	11	10	
24 0	0.4067	26	0.4452	34	2.2460	175	0.9135	11	0	66
10	0.4094	26	0.4487	35	2.2285	172	0.9123	12	50	
20	0.4120	26	0.4522	35	2.2113	170	0.9111	12	40	
30	0.4147	26	0.4557	35	2.1943	168	0.9099	12	30	
40	0.4173	26	0.4592	35	2.1775	166	0.9087	12	20	
50	0.4200	26	0.4627	35	2.1609	164	0.9075	12	10	
25 0	0.4226	26	0.4663	35	2.1445	162	0.9063	12	0	65
10	0.4252	26	0.4698	35	2.1283	159	0.9051	12	50	
20	0.4279	26	0.4734	36	2.1123	158	0.9038	12	40	
30	0.4305	26	0.4770	35	2.0965	156	0.9026	13	30	
40	0.4331	26	0.4805	36	2.0809	154	0.9013	13	20	
50	0.4357	26	0.4841	36	2.0655	152	0.9001	13	10	
26 0	0.4383	26	0.4877	36	2.0503	150	0.8988	13	0	64
10	0.4410	26	0.4913	36	2.0353	148	0.8975	13	50	
20	0.4436	26	0.4949	36	2.0204	147	0.8962	13	40	
30	0.4462	26	0.4986	36	2.0057	145	0.8949	13	30	
40	0.4488	26	0.5022	36	1.9911	143	0.8936	13	20	
50	0.4514	26	0.5058	37	1.9768	142	0.8923	13	10	
27 0	0.4540	26	0.5095	37	1.9626	140	0.8910	13	0	63
10	0.4566	25	0.5132	37	1.9486	139	0.8897	13	50	
20	0.4591	26	0.5169	36	1.9347	137	0.8884	13	40	
30	0.4617	26	0.5205	37	1.9210	136	0.8870	13	30	
40	0.4643	25	0.5242	38	1.9074	134	0.8857	13	20	
50	0.4669	25	0.5280	37	1.8940	132	0.8843	13	10	
28 0	0.4694	26	0.5317	37	1.8807	131	0.8829	13	0	62
10	0.4720	25	0.5354	38	1.8676	130	0.8816	14	50	
20	0.4746	25	0.5392	37	1.8546	128	0.8802	14	40	
30	0.4771	25	0.5429	38	1.8417	127	0.8788	13	30	
40	0.4797	25	0.5467	38	1.8290	125	0.8774	14	20	
50	0.4822	25	0.5505	38	1.8165	124	0.8760	14	10	
29 0	0.4848	25	0.5543	38	1.8040	123	0.8746	14	0	61
10	0.4873	25	0.5581	38	1.7917	122	0.8732	14	50	
20	0.4899	25	0.5619	38	1.7796	120	0.8718	14	40	
30	0.4924	25	0.5657	38	1.7675	119	0.8704	14	30	
40	0.4946	25	0.5696	39	1.7556	118	0.8689	14	20	
50	0.4975	25	0.5735	38	1.7437	117	0.8675	14	10	
30 0	0.5000		0.5773		1.7320		0.8660		0	60
	Cos.	d.	Cot.	d.	Tan.	d.	Sin.	d.	′	°

P. P.

	39	38	37	36
1	3.9	3.8	3.7	3.6
2	7.8	7.6	7.4	7.2
3	11.7	11.4	11.1	10.8
4	15.6	15.2	14.8	14.4
5	19.5	19.0	18.5	18.0
6	23.4	22.8	22.2	21.6
7	27.3	26.6	25.9	25.2
8	31.2	30.4	29.6	28.8
9	35.1	34.2	33.3	32.4

	35.8	35	34	33
1	3.5	3.5	3.4	3.3
2	7.1	7.0	6.8	6.6
3	10.6	10.5	10.2	9.9
4	14.2	14.0	13.6	13.2
5	17.7	17.5	17.0	16.5
6	21.3	21.0	20.4	19.8
7	24.8	24.5	23.8	23.1
8	28.4	28.0	27.2	26.4
9	31.9	31.5	30.6	29.7

	27	27	26	25
1	2.7	2.7	2.6	2.5
2	5.5	5.4	5.2	5.0
3	8.2	8.1	7.8	7.5
4	11.0	10.8	10.4	10.0
5	13.7	13.5	13.0	12.5
6	16.5	16.2	15.6	15.0
7	19.2	18.9	18.2	17.5
8	22.0	21.6	20.8	20.0
9	24.7	24.3	23.4	22.5

	14	14	13	12
1	1.4	1.4	1.3	1.2
2	2.9	2.8	2.6	2.4
3	4.3	4.2	3.9	3.6
4	5.8	5.6	5.2	4.8
5	7.2	7.0	6.5	6.0
6	8.7	8.4	7.8	7.2
7	10.1	9.8	9.1	8.4
8	11.6	11.2	10.4	9.6
9	13.0	12.6	11.7	10.8

	11	11	10
1	1.1	1.1	1.0
2	2.3	2.2	2.0
3	3.4	3.3	3.0
4	4.6	4.4	4.0
5	5.7	5.5	5.0
6	6.9	6.6	6.0
7	8.0	7.7	7.0
8	9.2	8.8	8.0
9	10.3	9.9	9.0

TABLE IX.—NATURAL SINES, COSINES, TANGENTS, AND COTANGENTS.
30°–40°

o '	Sin.	d.	Tan.	d.	Cot.	d.	Cos.	d.		
30 0	0.5000		0.5773		1.7320		0.8660			**0 60**
10	0.5025	25	0.5812	39	1.7204	116	0.8645	15		50
20	0.5050	25	0.5851	39	1.7090	114	0.8631	14		40
30	0.5075	25	0.5890	39	1.6976	113	0.8616	14		30
40	0.5100	25	0.5929	39	1.6864	112	0.8601	15		20
50	0.5125	25	0.5969	39	1.6753	111	0.8586	15		10
31 0	0.5150	25	0.6008	39	1.6643	110	0.8571	15		**0 59**
10	0.5175	25	0.6048	40	1.6533	109	0.8556	15		50
20	0.5200	24	0.6088	39	1.6425	108	0.8541	15		40
30	0.5225	25	0.6128	40	1.6318	107	0.8526	15		30
40	0.5250	25	0.6168	40	1.6212	106	0.8511	15		20
50	0.5274	24	0.6208	40	1.6107	105	0.8496	15		10
32 0	0.5299	25	0.6248	40	1.6003	104	0.8480	15		**0 58**
10	0.5324	24	0.6289	41	1.5900	103	0.8465	15		50
20	0.5348	24	0.6330	40	1.5798	102	0.8449	15		40
30	0.5373	24	0.6370	41	1.5697	101	0.8434	16		30
40	0.5397	24	0.6411	41	1.5596	100	0.8418	16		20
50	0.5422	24	0.6453	41	1.5497	99	0.8402	15		10
33 0	0.5446	24	0.6494	41	1.5398	98	0.8386	16		**0 57**
10	0.5471	24	0.6535	41	1.5301	97	0.8371	15		50
20	0.5495	24	0.6577	41	1.5204	96	0.8355	16		40
30	0.5519	24	0.6619	42	1.5108	96	0.8339	16		30
40	0.5543	24	0.6661	42	1.5013	95	0.8323	16		20
50	0.5568	24	0.6703	42	1.4919	94	0.8306	16		10
34 0	0.5592	24	0.6745	42	1.4825	93	0.8290	16		**0 56**
10	0.5616	24	0.6787	42	1.4733	92	0.8274	16		50
20	0.5640	24	0.6830	42	1.4641	92	0.8257	16		40
30	0.5664	24	0.6873	43	1.4550	91	0.8241	16		30
40	0.5688	24	0.6915	42	1.4460	90	0.8225	16		20
50	0.5712	24	0.6959	43	1.4376	89	0.8208	17		10
35 0	0.5736	24	0.7002	43	1.4281	89	0.8191	16		**0 55**
10	0.5759	24	0.7045	43	1.4193	88	0.8175	17		50
20	0.5783	23	0.7089	43	1.4106	87	0.8158	17		40
30	0.5807	23	0.7133	44	1.4019	86	0.8141	17		30
40	0.5830	23	0.7177	44	1.3933	86	0.8124	17		20
50	0.5854	24	0.7221	44	1.3848	85	0.8107	17		10
36 0	0.5878	23	0.7265	44	1.3764	84	0.8090	17		**0 54**
10	0.5901	23	0.7310	44	1.3680	84	0.8073	17		50
20	0.5925	23	0.7354	45	1.3597	83	0.8056	17		40
30	0.5948	23	0.7399	45	1.3514	83	0.8038	17		30
40	0.5971	23	0.7444	45	1.3432	81	0.8021	17		20
50	0.5995	23	0.7490	45	1.3351	81	0.8004	17		10
37 0	0.6018	23	0.7535	45	1.3270	80	0.7986	17		**0 53**
10	0.6041	23	0.7581	46	1.3190	79	0.7969	18		50
20	0.6064	23	0.7627	46	1.3111	78	0.7951	17		40
30	0.6087	23	0.7673	46	1.3032	78	0.7933	17		30
40	0.6110	23	0.7719	46	1.2954	77	0.7916	18		20
50	0.6133	23	0.7766	47	1.2876	77	0.7898	18		10
38 0	0.6156	23	0.7813	47	1.2799	76	0.7880	18		**0 52**
10	0.6179	23	0.7860	47	1.2723	76	0.7862	18		50
20	0.6202	22	0.7907	47	1.2647	75	0.7844	18		40
30	0.6225	23	0.7954	47	1.2571	74	0.7826	18		30
40	0.6248	23	0.8002	48	1.2497	74	0.7808	18		20
50	0.6270	22	0.8050	48	1.2423	73	0.7790	18		10
39 0	0.6293	23	0.8098	48	1.2349	73	0.7771	18		**0 51**
10	0.6316	22	0.8146	48	1.2276	73	0.7753	18		50
20	0.6338	22	0.8194	49	1.2203	72	0.7734	18		40
30	0.6361	22	0.8243	49	1.2131	71	0.7716	18		30
40	0.6383	22	0.8292	49	1.2059	71	0.7697	18		20
50	0.6405	22	0.8341	49	1.1988	71	0.7679	18		10
40 0	0.6428		0.8391	46	1.1917	70	0.7660			**0 50**
	Cos.	d.	Cot.	d.	Tan.	d.	Sin.	d.	' o	

P. P.

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   49   49   48   47   46
1  4.9  4.9  4.8  4.7  4.6
2  9.9  9.8  9.6  9.4  9.2
3 14.8 14.7 14.4 14.1 13.8
4 19.8 19.6 19.2 18.8 18.4
5 24.7 24.5 24.0 23.5 23.0
6 29.7 29.4 28.8 28.2 27.6
7 34.6 34.3 33.6 32.9 32.2
8 39.6 39.2 38.4 37.6 36.8
9 44.5 44.1 43.2 42.3 41.4

   45   45   44   43   42
1  4.5  4.5  4.4  4.3  4.2
2  9.1  9.0  8.8  8.6  8.4
3 13.6 13.5 13.2 12.9 12.6
4 18.2 18.0 17.6 17.2 16.8
5 22.7 22.5 22.0 21.5 21.0
6 27.3 27.0 26.4 25.8 25.2
7 31.8 31.5 30.8 30.1 29.4
8 36.4 36.0 35.2 34.4 33.6
9 40.9 40.5 39.6 38.7 37.8

   41   41   40   39
1  4.1  4.1  4.0  3.9
2  8.3  8.2  8.0  7.8
3 12.4 12.3 12.0 11.7
4 16.6 16.4 16.0 15.6
5 20.7 20.5 20.0 19.5
6 24.9 24.6 24.0 23.4
7 29.0 28.7 28.0 27.3
8 33.2 32.8 32.0 31.2
9 37.3 36.9 36.0 35.1

   25   25   24   23
1  2.5  2.5  2.4  2.3
2  5.1  5.0  4.8  4.6
3  7.6  7.5  7.2  6.9
4 10.2 10.0  9.6  9.2
5 12.7 12.5 12.0 11.5
6 15.3 15.0 14.4 13.8
7 17.8 17.5 16.8 16.1
8 20.4 20.0 19.2 18.4
9 22.9 22.5 21.6 20.7

   22   22   18   18
1  2.2  2.2  1.8  1.8
2  4.5  4.4  3.7  3.6
3  6.7  6.6  5.5  5.4
4  9.0  8.8  7.4  7.2
5 11.2 11.0  9.2  9.0
6 13.5 13.2 11.1 10.8
7 15.7 15.4 12.9 12.6
8 18.0 17.6 14.8 14.4
9 20.2 19.8 16.6 16.2

   17   17   16   15   14
1  1.7  1.7  1.6  1.5  1.4
2  3.5  3.4  3.2  3.0  2.9
3  5.2  5.1  4.8  4.5  4.3
4  7.0  6.8  6.4  6.0  6.8
5  8.7  8.5  8.0  7.5  7.2
6 10.5 10.2  9.6  9.0  8.7
7 12.2 11.9 11.2 10.5 10.1
8 14.0 13.6 12.8 12.0 11.6
9 15.7 15.3 14.4 13.5 13.0
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TABLE IX.—NATURAL SINES, COSINES, TANGENTS, AND COTANGENTS.
40°–45°

° ′	Sin.	d.	Tan.	d.	Cot.	d.	Cos.	d.			P. P.			
40 0	0.6428	22	0.8391	49	1.1917	70	0.7660	19	**0 50**					
10	0.6450	22	0.8446	50	1.1847	70	0.7641	18	50					
20	0.6472	22	0.8490	56	1.1779	69	0.7623	19	40	**70**	**22**	**22**	**21**	**21**
30	0.6494	22	0.8541	50	1.1708	68	0.7604	19	30	1	7.0	2.2	2.2	2.1	2.1
40	0.6516	22	0.8591	51	1.1640	68	0.7585	19	20	2	14.0	4.5	4.4	4.3	4.2
50	0.6538	22	0.8642	51	1.1571	68	0.7566	19	10	3	21.0	6.7	6.6	6.4	6.3
41 0	0.6560	22	0.8693	51	1.1503	67	0.7547	19	**0 49**	4	28.0	9.0	8.8	8.6	8.4
10	0.6582	22	0.8744	51	1.1436	67	0.7528	19	50	5	35.0	11.2	11.0	10.7	10.5
20	0.6604	21	0.8795	52	1.1369	66	0.7509	19	40	6	42.0	13.5	13.2	12.9	12.6
30	0.6626	22	0.8847	51	1.1303	66	0.7489	19	30	7	49.0	15.7	15.4	15.0	14.7
40	0.6648	21	0.8899	52	1.1237	65	0.7470	19	20	8	56.0	18.0	17.6	17.2	16.8
50	0.6669	22	0.8951	52	1.1171	65	0.7451	19	10	9	63.0	20.2	19.8	19.3	18.9
42 0	0.6691	21	0.9004	53	1.1106	64	0.7431	19	**0 48**					
10	0.6713	21	0.9057	53	1.1041	64	0.7412	19	50		**69**	**20**	**20**	**19**	**19**
20	0.6734	21	0.9110	53	1.0977	64	0.7392	19	40	1	6.9	2.0	2.0	1.9	1.9
30	0.6756	21	0.9163	53	1.0913	63	0.7373	20	30	2	13.8	4.1	4.0	3.9	3.8
40	0.6777	21	0.9217	54	1.0849	63	0.7353	19	20	3	20.7	6.1	6.0	5.8	5.7
50	0.6798	21	0.9271	54	1.0786	63	0.7333	20	10	4	27.6	8.2	8.0	7.8	7.6
43 0	0.6820	21	0.9325	54	1.0723	62	0.7313	20	**0 47**	5	34.5	10.2	10.0	9.7	9.5
10	0.6841	21	0.9379	55	1.0661	62	0.7293	20	50	6	41.4	12.3	12.0	11.7	11.4
20	0.6862	21	0.9434	55	1.0599	61	0.7273	20	40	7	48.3	14.3	14.0	13.6	13.3
30	0.6883	21	0.9489	55	1.0538	61	0.7253	20	30	8	55.2	16.4	16.0	15.6	15.2
40	0.6904	21	0.9545	56	1.0476	60	0.7233	20	20	9	62.1	18.4	18.0	17.5	17.1
50	0.6925	21	0.9601	56	1.0416	60	0.7213	20	10					
44 0	0.6946	21	0.9657	56	1.0355	60	0.7193	20	**0 46**		**68**	**68**	**67**	**66**	**18**
10	0.6967	21	0.9713	56	1.0295	59	0.7173	20	50	1	6.8	6.8	6.7	6.6	1.8
20	0.6988	21	0.9770	57	1.0235	59	0.7153	20	40	2	13.7	13.6	13.4	13.2	3.7
30	0.7009	21	0.9827	57	1.0176	59	0.7132	20	30	3	20.5	20.4	20.1	19.8	5.5
40	0.7030	20	0.9884	57	1.0117	59	0.7112	20	20	4	27.4	27.2	26.8	26.4	7.4
50	0.7050	20	0.9942	57	1.0058	58	0.7091	20	10	5	34.2	34.0	33.5	33.0	9.2
		20		58		58		20		6	41.1	40.8	40.2	39.6	11.1
45 0	0.7071		1.0000		1.0000		0.7071		**0 45**	7	47.9	47.6	46.9	46.2	12.9
										8	54.8	54.4	53.6	52.8	14.8
	Cos.	d.	Cot.	d.	Tan.	d.	Sin.	d.	′ °	9	61.6	61.2	60.3	59.4	16.6

	65	64	64	63	62	61	60	59	59	58	58	57	57	56	56	55	54	54	53	53	52	52
1	6.5	6.4	6.4	6.3	6.2	6.1	6.0	5.9	5.9	5.8	5.8	5.7	5.7	5.6	5.6	5.5	5.4	5.4	5.3	5.3	5.2	5.2
2	13.1	12.9	12.8	12.6	12.4	12.3	12.1	11.9	11.8	11.7	11.6	11.5	11.4	11.3	11.2	11.0	10.9	10.8	10.7	10.6	10.5	10.4
3	19.6	19.3	19.2	18.9	18.6	18.4	18.1	17.8	17.7	17.5	17.4	17.2	17.1	16.9	16.8	16.5	16.3	16.2	16.0	15.9	15.7	15.6
4	26.2	25.8	25.6	25.2	24.8	24.6	24.2	23.8	23.6	23.4	23.2	23.0	22.8	22.6	22.4	22.0	21.8	21.6	21.4	21.2	21.0	20.8
5	32.7	32.2	32.0	31.5	31.0	30.7	30.2	29.7	29.5	29.2	29.0	28.7	28.5	28.2	28.0	27.5	27.2	27.0	26.7	26.5	26.2	26.0
6	39.3	38.7	38.4	37.8	37.2	36.9	36.3	35.7	35.4	35.1	34.8	34.5	34.2	33.9	33.6	33.0	32.7	32.4	32.1	31.8	31.5	31.2
7	45.8	45.1	44.8	44.1	43.4	43.0	42.3	41.6	41.3	40.9	40.6	40.2	39.9	39.5	39.2	38.5	38.1	37.8	37.4	37.1	36.7	36.4
8	52.4	51.6	51.2	50.4	49.6	49.2	48.4	47.6	47.2	46.8	46.4	46.0	45.6	45.2	44.8	44.0	43.6	43.2	42.8	42.4	42.0	41.6
9	58.9	58.0	57.6	56.7	55.8	55.3	54.4	53.5	53.1	52.6	52.2	51.7	51.3	50.8	50.4	49.5	49.0	48.6	48.1	47.7	47.2	46.8

	51	51	50	50	49
1	5.1	5.1	5.0	5.0	4.9
2	10.3	10.2	10.1	10.0	9.9
3	15.4	15.3	15.1	15.0	14.8
4	20.6	20.4	20.2	20.0	19.8
5	25.7	25.5	25.2	25.0	24.7
6	30.9	30.6	30.3	30.0	29.7
7	36.0	35.7	35.3	35.0	34.6
8	41.2	40.8	40.4	40.0	39.6
9	46.3	45.9	45.4	45.0	44.5

Table for passing from Sexagesimal to Circular Measure.

°	Circular Meas.	′	Circular Meas.	″	Circular Meas.
100	1.74 532 9	10	0.00 290 9	10	0.00 004 8
200	3.49 065 8	20	0.00 581 8	20	0.00 009 7
300	5.23 598 8	30	0.00 872 6	30	0.00 014 5
		40	0.01 163 5	40	0.00 019 4
40	0.69 813 1				
50	0.87 266 4	50	0.01 454 4	50	0.00 024 2
60	1.04 719 7				
		6	0.00 174 5	6	0.00 002 9
70	1.22 173 0	7	0.00 203 6	7	0.00 003 4
80	1.39 626 3	8	0.00 232 7	8	0.00 003 9
90	1.57 079 6	9	0.00 261 8	9	0.00 004 3

TABLE X.—NATURAL VERSED SINES AND EXTERNAL SECANTS.

0°–10° **10°–20°**

° ′	Vers.	d.	Exsec.	d.	° ′	Vers.	d.	Exsec.	d.	P. P.
0 0	.00000	0̂	.00000	0̂	10 0	.01519	51	.01542	52	
10	.00000̂	1	.00000̂	1	10	.01570	52	.01595	53	
20	.0000î	2	.0000î	2	20	.01622	52	.01648	54	
30	.00004	3	.00004	3	30	.01674̂	53	.01703	55	
40	.00007	3	.00007	3	40	.01728	54	.01758̂	56	
50	.00010̂	4	.00010̂	4	50	.01782	55	.01814	57	
1 0	.00015	4	.00015	5	11 0	.01837	55	.01871̂	58	110 100 90 80 70 60 50 40
10	.00020̂	5	.00020̂	6	10	.01893	57	.01929	59	1 11 10 9 8 7 6 5 4
20	.00027	6	.00027	7	20	.01950	57	.01988	60	2 22 20 18 16 14 12 10 8
30	.00034̂	7	.00034̂	8	30	.02007	58	.02048	61	3 33 30 27 24 21 18 15 12
40	.00042̂	8	.00042̂	8	40	.02066	59	.02109	62	4 44 40 36 32 28 24 20 16
50	.00051	8	.00051	10	50	.02125	60	.02171̂	62	5 55 50 45 40 35 30 25 20
2 0	.00061	10	.00061	10	12 0	.02185	61	.02234	63	6 66 60 54 48 42 36 30 24
10	.00071̂	10̂	.00071̂	11̂	10	.02246	62	.02297	65	
20	.00083	11̂	.00083	12̂	20	.02308	62	.02362̂	65̂	7 77 70 63 56 49 42 35 28
30	.00095	12	.00095	13	30	.02370̂	63	.02428	66	8 88 80 72 64 56 48 40 32
40	.00108	13̂	.00108	14	40	.02434	64	.02494̂	67	9 99 90 81 72 63 54 45 36
50	.00122	13̂	.00122̂	14̂	50	.02498	65	.02562	68	
3 0	.00137	15	.00137	16	13 0	.02563	66	.02630̂	69	
10	.00152̂	15̂	.00153	16̂	10	.02629	66̂	.02700	70	
20	.00169	16̂	.00169̂	17	20	.02695̂	67	.02770	71̂	30 20 10 9̂ 9 8̂ 8 7̂
30	.00186̂	17̂	.00187	18	30	.02763	68	.02841̂	72̂	1 3 2 1 0.9 0.9 0.8 0.8 0.7
40	.00204	18	.00205	19	40	.02831̂	69	.02914	73	2 6 4 2 1.9 1.8 1.7 1.6 1.5
50	.00223̂	19	.00224	20	50	.02900̂	70	.02987	73	3 9 6 3 2.8 2.7 2.5 2.4 2.2
4 0	.00243	20	.00244	21	14 0	.02970̂	70̂	.03061̂	74	4 12 8 4 3.8 3.6 3.4 3.2 3.0
10	.00264̂	21	.00265	2î	10	.03041	72	.03136	75̂	5 15 10 5 4.7 4.5 4.2 4.0 3.7
20	.00286	2î	.00286̂	2̂	20	.03113	72	.03213	76	6 18 12 6 5.7 5.4 5.1 4.8 4.5
30	.00308	2̂	.00309	2̂	30	.03185	72̂	.03290	77	7 21 14 7 6.6 6.3 5.9 5.6 5.2
40	.00331̂	23	.00332̂	2̂	40	.03258	74	.03368	78	8 24 16 8 7.6 7.2 6.8 6.4 6.0
50	.00355̂	24	.00357	25	50	.03332̂	75	.03447̂	79̂	9 27 18 9 8.5 8.1 7.6 7.2 6.7
5 0	.00380̂	25	.00382	26	15 0	.03407	75	.03527	80	
10	.00406̂	26	.00408	27	10	.03483	76̂	.03609	8î	
20	.00433	26̂	.00435	27̂	20	.03559̂	76	.03691	82	
30	.00460̂	27	.00462̂	28	30	.03637	77	.03774	83	7 6̂ 6 5̂ 5 4̂ 4
40	.00488	28	.00491	29	40	.03715	78	.03858̂	84	1 0.7 0.6 0.6 0.5 0.5 0.4 0.4
50	.00518	29	.00520̂	30	50	.03794	79	.03943̂	85	2 1.4 1.3 1.2 1.1 1.0 0.9 0.8
6 0	.00548	30	.00551	31	16 0	.03874	80	.04030	86̂	3 2.1 1.9 1.8 1.6 1.5 1.3 1.2
10	.00578̂	30̂	.00582	32	10	.03954	8î	.04117	87	4 2.8 2.6 2.4 2.2 2.0 1.8 1.6
20	.00610̂	32	.00614	32̂	20	.04036	82	.04205	88	5 3.5 3.2 3.0 2.7 2.5 2.2 2.0
30	.00643	32̂	.00647	33	30	.04118	83	.04295	89̂	6 4.2 3.9 3.6 3.3 3.0 2.7 2.4
40	.00676̂	33	.00681̂	34	40	.04201	84	.04385̂	90̂	
50	.00710̂	33̂	.00715̂	34̂	50	.04285	84	.04476	91	7 4.9 4.5 4.2 3.8 3.5 3.1 2.8
7 0	.00745	35	.00751	35̂	17 0	.04369̂	85	.04569	92̂	8 5.6 5.2 4.8 4.4 4.0 3.6 3.2
10	.00781̂	36	.00787̂	36̂	10	.04455	86	.04662̂	93	9 6.3 5.8 5.4 4.9 4.5 4.0 3.6
20	.00818	36̂	.00824̂	37	20	.04541	87	.04757̂	95	
30	.00855̂	37	.00863	38	30	.04628̂	87	.04853	95̂	
40	.00894	38	.00902	39	40	.04716	89	.04949̂	96	3̂ 3 2̂ 2 1̂ 1 0̂
50	.00933	39	.00942	40	50	.04805	89	.05047̂	98	1 0.3 0.3 0.2 0.2 0.1 0.1 0.0
8 0	.00973	40	.00983	4î	18 0	.04894̂	90	.05146	98	2 0.7 0.6 0.5 0.4 0.3 0.2 0.1
10	.01014	41	.01024̂	42	10	.04984̂	9î	.05246	100	3 1.0 0.9 0.7 0.6 0.4 0.3 0.1
20	.01056	42	.01067	43̂	20	.05076	9î	.05347	101	4 1.4 1.2 1.0 0.8 0.6 0.4 0.2
30	.01098	42̂	.01110̂	44	30	.05167̂	93	.05449	102	5 1.7 1.5 1.2 1.0 0.7 0.5 0.2
40	.01142	43̂	.01155	45	40	.05260̂	93	.05552̂	103̂	6 2.1 1.8 1.5 1.2 0.9 0.6 0.3
50	.01186̂	44	.01200̂	46	50	.05354	94	.05656̂	105	7 2.4̂ 2.1 1.7 1.4 1.0 0.7 0.3̂
9 0	.01231	45	.01246̂	47	19 0	.05448	95	.05762	106̂	8 2.8 2.4 2.0 1.6 1.2 0.8 0.4
10	.01277	46	.01293̂	47	10	.05543	95	.05868	107	9 3.1̂ 2.7 2.2̂ 1.8 1.3̂ 0.9 0.4
20	.01324	47	.01341̂	48	20	.05639	96	.05976	109	
30	.01371̂	47̂	.01390̂	49	30	.05736	97	.06085	109̂	
40	.01420	48	.01440̂	50̂	40	.05833̂	98	.06194	111	
50	.01469	49	.01491̂	5î	50	.05931̂	98	.06305̂	112̂	
10 0	.01519	50	.01542̂		20 0	.06030̂	99	.06418		
° ′	Vers.	d.	Exsec.	d.	° ′	Vers.	d.	Exsec.	d.	P. P.

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TABLE X.—NATURAL VERSED SINES AND EXTERNAL SECANTS.
20°-30°   30°-40°

° '	Vers.	d.	Exsec.	d.	° '	Vers.	d.	Exsec.	d.
20 0	.0603	10	.0642	11	30 0	.1339̂	15	.1547	19̂
10	.0613	10	.0653	1î	10	.1354	15	.1566̂	19̂
20	.0623	1ô	.0664	1î	20	.1369	14̂	.1586	20
30	.0633̂	10	.0676	12	30	.1383̂	15	.1606	20
40	.0643̂	1ô	.0688	1î	40	.1398	15	.1626	20
50	.0654	10	.0699̂	12	50	.1413̂	15	.1646	2ô
21 0	.0664	1ô	.0711̂	12	31 0	.1428	15	.1666̂	2ô
10	.0674̂	1ô	.0723̂	12	10	.1443̂	15	.1687	2ô
20	.0685	11	.0735̂	1ẑ	20	.1458̂	15	.1707	21
30	.0696̂	1ô	.0748	12	30	.1473̂	15	.1728	21
40	.0706̂	11	.0760	1ẑ	40	.1489	15	.1749	21
50	.0717̂	1ô	.0772̂	13	50	.1504	15	.1770̂	2î
22 0	.0728	11	.0785̂	1ẑ	32 0	.1519̂	15	.1792	2î
10	.0739	11	.0798	13	10	.1535	1ŝ	.1813̂	2î
20	.0750	11	.0811	13	20	.1550̂	1ŝ	.1835	22
30	.0761	1î	.0824	13	30	.1566	16	.1857	22
40	.0772̂	11	.0837	13	40	.1582	1ŝ	.1879	22
50	.0783̂	1î	.0850	1ẑ	50	.1597	16	.1901	2ẑ
23 0	.0795	1î	.0863̂	1ẑ	33 0	.1613̂	1ŝ	.1923̂	23
10	.0806̂	1î	.0877	1ẑ	10	.1629	16	.1946	2ẑ
20	.0818	1î	.0890̂	14	20	.1645	16	.1969	23
30	.0829̂	1î	.0904̂	14	30	.1661	16	.1992	23
40	.0841	12	.0918	14	40	.1677	1ô	.2015	23
50	.0853	1î	.0932̂	14	50	.1693̂	16	.2038̂	24
24 0	.0864̂	12	.0946̂	14	34 0	.1709̂	1ô	.2062	24
10	.0876̂	12	.0960̂	1â	10	.1726	1ô	.2086	24
20	.0888̂	12	.0975	1â	20	.1742̂	16	.2110	24
30	.0900̂	12	.0989̂	1â	30	.1758̂	16	.2134	2â
40	.0912̂	12	.1004	15	40	.1775̂	17	.2158	2â
50	.0924̂	1ẑ	.1019	15	50	.1792	1ô	.2183	2â
25 0	.0937	1ẑ	.1034	15	35 0	.1808̂	1ô	.2207	25
10	.0949	1ẑ	.1049	15	10	.1825	1ô	.2232̂	25
20	.0961̂	12	.1064	1ŝ	20	.1842	17	.2258	25
30	.0974	1ẑ	.1079̂	15	30	.1859	17	.2283̂	25
40	.0986̂	1ẑ	.1094̂	16	40	.1876	17	.2309	25
50	.0999̂	13	.1110̂	1ŝ	50	.1893	17	.2334̂	26
26 0	.1012	1ẑ	.1126	16	36 0	.1910	17	.2360̂	26
10	.1025	13	.1142	16	10	.1927	17	.2387	2ô
20	.1037̂	1ẑ	.1158	16	20	.1944	1ŷ	.2413̂	2ô
30	.1050̂	13	.1174	1ô	30	.1961̂	1ŷ	.2440	27
40	.1063̂	13	.1190̂	16	40	.1979	17	.2467	27
50	.1077	13	.1206̂	17	50	.1996	1ŷ	.2494	27
27 0	.1090	13	.1223̂	1ô	37 0	.2013̂	1ŷ	.2521̂	27
10	.1103	13	.1240	17	10	.2031	17	.2549	27
20	.1116̂	1ẑ	.1257	17	20	.2049	18	.2576	28
30	.1130	1ẑ	.1274	17	30	.2066̂	1ŷ	.2604	28
40	.1143̂	1ẑ	.1291	1ŷ	40	.2084	18	.2633	28
50	.1157	1ẑ	.1308̂	17	50	.2102	18	.2661̂	28
28 0	.1170̂	1ẑ	.1325̂	18	38 0	.2120	18	.2690	29
10	.1184	13	.1343̂	1ŷ	10	.2138	18	.2719	29
20	.1198	14	.1361	18	20	.2156	18	.2748	2ŷ
30	.1212	14	.1379	18	30	.2174	18	.2778	2ŷ
40	.1225̂	1ẑ	.1397	18	40	.2192	18	.2807	30
50	.1239̂	14	.1415	18	50	.2210	1ô	.2837̂	30
29 0	.1254	1â	.1433̂	1ŝ	39 0	.2228̂	18	.2867̂	3ô
10	.1268	14	.1452	1ŝ	10	.2247	18	.2898	3ô
20	.1282	14	.1470̂	19	20	.2265̂	18	.2928̂	31
30	.1296̂	1â	.1489̂	19	30	.2284	1ô	.2959̂	3î
40	.1311	14	.1508̂	19	40	.2302̂	1ô	.2991	3î
50	.1325	1â	.1527̂	19	50	.2321	1ô	.3022̂	3î
30 0	.1339̂		.1547		40 0	.2339̂		.3054	
° '	Vers.	d.	Exsec.	d.	° '	Vers.	d.	Exsec.	d.

P. P.

	31	30	29	28
1	3.1	3.0	2.9	2.8
2	6.2	6.0	5.8	5.6
3	9.3	9.0	8.7	8.4
4	12.4	12.0	11.6	11.2
5	15.5	15.0	14.5	14.0
6	18.6	18.0	17.4	16.8
7	21.7	21.0	20.3	19.6
8	24.8	24.0	23.2	22.4
9	27.9	27.0	26.1	25.2

	27	26	25	24
1	2.7	2.6	2.5	2.4
2	5.4	5.2	5.0	4.8
3	8.1	7.8	7.5	7.2
4	10.8	10.4	10.0	9.6
5	13.5	13.0	12.5	12.0
6	16.2	15.6	15 0	14.4
7	18.9	18.2	17.5	16.8
8	21.6	20.8	20.0	19.2
9	24.3	23.4	22.5	21.6

	23	22	21	20
1	2.3	2.2	2.1	2.0
2	4.6	4.4	4.2	4.0
3	6.9	6.6	6.3	6.0
4	9.2	8.8	8.4	8.0
5	11.5	11.0	10.5	10.0
6	13.8	13.2	12.6	12.0
7	16.1	15.4	14.7	14.0
8	18.4	17.6	16.8	16.0
9	20.7	19.8	18.9	18.0

	19	18	17	16
1	1.9	1.8	1.7	1.6
2	3.8	3.6	3.4	3.2
3	5.7	5.4	5.1	4.8
4	7 6	7.2	6.8	6.4
5	9.5	9.0	8.5	8.0
6	11.4	10.8	10.2	9.6
7	13.3	12.6	11.9	11.2
8	15.2	14.4	13.6	12.8
9	17.1	16.2	15.3	14.4

	15	14	13	12
1	1.5	1.4	1.3	1.2
2	3.0	2.8	2.6	2.4
3	4.5	4.2	3.9	3.6
4	6.0	5.6	5.2	4.8
5	7.5	7.0	6.5	6.0
6	9.0	8.4	7.8	7.2
7	10.5	9.8	9.1	8.4
8	12.●	11.2	10.4	9.6
9	13.5	12.6	11.7	10.8

	11	10	6̂
1	1.1	1.0	0.ô
2	2.2	2.0	0.1
3	3.3	3.0	0.î
4	4.4	4.0	0.2
5	5.5	5.0	0.2̂
6	6.6	6.0	0.3
7	7.7	7.0	0.3̂
8	8.8	8.0	0.4
9	9.9	9.0	0.4̂

TABLE X.—NATURAL VERSED SINES AND EXTERNAL SECANTS.
40°-50°   50°-60°

° ′	Vers.	d.	Exsec.	d.	° ′	Vers.	d.	Exsec.	d.
40 0	.2339	19	.3054	32	50 0	.3572	22	.5557	53
10	.2358	18	.3086	32	10	.3594	22	.5611	54
20	.2377	19	.3118	32	20	.3617	22	.5666	54
30	.2396	19	.3151	32	30	.3639	22	.5721	55
40	.2415	19	.3183	33	40	.3661	22	.5777	55
50	.2434	19	.3217	33	50	.3684	23	.5833	56
41 0	.2453	19	.3250	34	51 0	.3707	22	.5890	56
10	.2472	19	.3284	33	10	.3729	22	.5947	57
20	.2491	19	.3317	34	20	.3752	23	.6005	58
30	.2510	19	.3352	34	30	.3775	22	.6064	58
40	.2529	19	.3386	34	40	.3797	23	.6123	59
50	.2549	19	.3421	35	50	.3820	23	.6182	59
42 0	.2568	19	.3456	35	52 0	.3843	23	.6242	60
10	.2588	19	.3491	36	10	.3866	23	.6303	61
20	.2607	19	.3527	36	20	.3889	23	.6365	61
30	.2627	20	.3563	36	30	.3912	23	.6427	62
40	.2647	19	.3599	37	40	.3935	23	.6489	62
50	.2666	20	.3636	37	50	.3958	23	.6552	63
43 0	.2686	20	.3673	37	53 0	.3982	23	.6616	64
10	.2706	20	.3710	37	10	.4005	23	.6681	64
20	.2726	20	.3748	38	20	.4028	23	.6746	65
30	.2746	20	.3786	38	30	.4052	23	.6811	65
40	.2766	20	.3824	39	40	.4075	23	.6878	66
50	.2786	20	.3863	38	50	.4098	23	.6945	67
44 0	.2806	20	.3901	39	54 0	.4122	23	.7013	68
10	.2827	20	.3941	39	10	.4145	23	.7081	68
20	.2847	20	.3980	39	20	.4169	24	.7156	69
30	.2867	20	.4020	40	30	.4193	23	.7220	70
40	.2888	20	.4060	40	40	.4216	24	.7291	70
50	.2908	20	.4101	41	50	.4240	24	.7362	72
45 0	.2929	20	.4142	41	55 0	.4264	24	.7434	73
10	.2949	20	.4183	41	10	.4288	24	.7507	73
20	.2970	21	.4225	42	20	.4312	24	.7581	74
30	.2991	20	.4267	42	30	.4336	24	.7655	75
40	.3011	21	.4309	43	40	.4360	24	.7730	76
50	.3032	21	.4352	43	50	.4384	24	.7806	77
46 0	.3053	21	.4395	43	56 0	.4408	24	.7883	77
10	.3074	21	.4439	44	10	.4432	24	.7960	78
20	.3095	21	.4483	44	20	.4456	24	.8039	79
30	.3116	21	.4527	44	30	.4480	24	.8118	80
40	.3137	21	.4572	45	40	.4505	24	.8198	81
50	.3159	21	.4617	45	50	.4529	24	.8279	82
47 0	.3180	21	.4663	45	57 0	.4553	24	.8361	82
10	.3201	21	.4708	46	10	.4578	24	.8443	83
20	.3222	21	.4755	47	20	.4602	24	.8527	84
30	.3244	21	.4802	47	30	.4627	24	.8611	85
40	.3265	21	.4849	47	40	.4651	24	.8697	86
50	.3287	21	.4896	48	50	.4676	25	.8783	88
48 0	.3308	22	.4945	48	58 0	.4701	24	.8871	88
10	.3330	21	.4993	48	10	.4725	24	.8959	89
20	.3352	22	.5042	49	20	.4750	25	.9048	90
30	.3374	21	.5091	50	30	.4775	25	.9139	91
40	.3395	22	.5141	50	40	.4800	24	.9230	92
50	.3417	22	.5192	50	50	.4824	25	.9322	93
49 0	.3439	22	.5242	51	59 0	.4849	25	.9416	94
10	.3461	22	.5294	51	10	.4874	25	.9510	95
20	.3483	22	.5345	52	20	.4899	25	.9606	97
30	.3505	22	.5397	53	30	.4924	25	.9703	98
40	.3527	22	.5450	53	40	.4949	25	.9801	99
50	.3550	22	.5503	53	50	.4975	25	.9900	100
50 0	.3572		.5557		60 0	.5000		1.0000	

P. P.

	9	8	7	6	5	4
1	0.9	0.8	0.7	0.6	0.5	0.4
2	1.8	1.6	1.4	1.2	1.0	0.8
3	2.7	2.4	2.1	1.8	1.5	1.2
4	3.6	3.2	2.8	2.4	2.0	1.6
5	4.5	4.0	3.5	3.0	2.5	2.0
6	5.4	4.8	4.2	3.6	3.0	2.4
7	6.3	5.6	4.9	4.2	3.5	2.8
8	7.2	6.4	5.6	4.8	4.0	3.2
9	8.1	7.2	6.3	5.4	4.5	3.6

	3	2	1	9	8	7
1	0.3	0.2	0.1	0.9	0.8	0.7
2	0.6	0.4	0.2	1.9	1.7	1.5
3	0.9	0.6	0.3	2.8	2.5	2.2
4	1.2	0.8	0.4	3.8	3.4	3.0
5	1.5	1.0	0.5	4.7	4.2	3.7
6	1.8	1.2	0.6	5.7	5.1	4.5
7	2.1	1.4	0.7	6.6	5.9	5.2
8	2.4	1.6	0.8	7.6	6.8	6.0
9	2.7	1.8	0.9	8.5	7.6	6.7

	6	5	4	3	2	1
1	0.6	0.5	0.4	0.3	0.2	0 1
2	1.3	1.1	0.9	0.7	0.5	0.3
3	1.9	1.6	1.3	1.0	0.7	0.4
4	2.6	2.2	1.8	1.4	1.0	0.6
5	3.2	2.7	2.2	1.7	1.2	0.7
6	3.9	3.3	2.7	2.1	1.5	0.9
7	4.5	3.8	3.1	2.4	1.7	1.0
8	5.2	4.4	3.6	2.8	2.0	1.2
9	5.8	4.9	4.0	3.1	2.2	1.3

	25	25	24	24	23	23
1	2.5	2.5	2.4	2.4	2.3	2.3
2	5.1	5.0	4.9	4.8	4.7	4.6
3	7.6	7.5	7.3	7.2	7.0	6.9
4	10.2	10.0	9.8	9.6	9.4	9.2
5	12.7	12.5	12.2	12.0	11.7	11.5
6	15.3	15.0	14.7	14.4	14.1	13.8
7	17.8	17.5	17.1	16.8	16.4	16.1
8	20.4	20.0	19.6	19.2	18.8	18.4
9	22.9	22.5	22.0	21.6	21.1	20.7

	22	22	21	21	20	20
1	2.2	2.2	2.1	2.1	2.0	2.0
2	4.5	4.4	4.3	4.2	4.1	4.0
3	6.7	6.6	6.4	6.3	6.1	6.0
4	9.0	8.8	8.6	8.4	8.2	8.0
5	11.2	11.0	10.7	10.5	10.2	10.0
6	13.5	13.2	12.9	12.6	12.3	12.0
7	15.7	15.4	15.0	14.7	14.3	14.0
8	18.0	17.6	17.2	16.8	16.4	16.0
9	20.2	19.8	19.3	18.9	18.4	18.0

	19	19	18
1	1.9	1.0	1.8
2	3.9	3.8	3.7
3	5.8	5.7	5.5
4	7.8	7.6	7.4
5	9.7	9.5	9.2
6	11.7	11.4	11.1
7	13.6	13.3	12.9
8	15.6	15.2	14.8
9	17.5	17.1	16.6

° ′	Vers.	d.	Exsec.	d.	° ′	Vers.	d.	Exsec.	d.	P. P.

TABLE X.—NATURAL VERSED SINES AND EXTERNAL SECANTS.
60°–70° . 70°–80°

° ′	Vers.	d.	Exsec.	d.	° ′	Vers.	d.	Exsec.	d.
60 0	.5000	25	1.0000	101̂	70 0	.6580	27	1.9238	235̂
10	.5025	25	1.0101	102̂	10	.6607	27	1.9473̂	240
20	.5050̂	25	1.0204	103̂	20	.6634	27	1.9713̂	244
30	.5076	25	1.0307	105̂	30	.6662	27	1.9957	248
40	.5101	25	1.0413	106	40	.6689̂	27	2.0205̂	253
50	.5126̂	25	1.0519	107̂	50	.6717	27	2.0458	257
61 0	.5152	25	1.0626̂	109	71 0	.6744̂	27	2.0715̂	262
10	.5177̂	26	1.0735̂	110̂	10	.6772	27	2.0977̂	267
20	.5203	25	1.0846̂	111̂	20	.6799̂	27	2.1244	271̂
30	.5228̂	26	1.0957̂	113	30	.6827	28	2.1515	277
40	.5254	25	1.1070̂	114	40	.6854̂	28	2.1792	281̂
50	.5279̂	26	1.1184̂	116	50	.6882	28	2.2073	287
62 0	.5305̂	25	1.1300̂	117̂	72 0	.6910	27	2.2360̂	292̂
10	.5331	25	1.1418	118̂	10	.6937̂	28	2.2653	298
20	.5356̂	26	1.1536̂	120̂	20	.6965	28	2.2951	304
30	.5382̂	26	1.1657	121̂	30	.6993	27	2.3255	310
40	.5408̂	25̂	1.1778̂	123̂	40	.7020̂	28	2.3565	316
50	.5434	26	1.1902	125	50	.7048	28	2.3881	322
63 0	.5460	26	1.2027	126̂	73 0	.7076	28	2.4203	328
10	.5486	26	1.2153̂	128	10	.7104	28	2.4531̂	335̂
20	.5512	26	1.2281̂	130	20	.7132	28	2.4867	342̂
30	.5538	26	1.2411̂	131̂	30	.7160	29	2.5209	349
40	.5564	26	1.2543	133̂	40	.7189̂	27	2.5558̂	356̂
50	.5590	26	1.2676̂	135	50	.7215	28	2.5915̂	364
64 0	.5616̂	26	1.2811̂	137	74 0	.7243̂	28	2.6279	372
10	.5642̂	26	1.2948̂	139	10	.7271	28	2.6651̂	380
20	.5668̂	26̂	1.3087̂	140̂	20	.7299̂	28	2.7031̂	388̂
30	.5695	26	1.3228	143	30	.7327	28	2.7420	396
40	.5721	26	1.3371	144̂	40	.7355	28	2.7816̂	406
50	.5747̂	26̂	1.3515̂	146	50	.7383̂	28	2.8222̂	414̂
65 0	.5774	26	1.3662	148	75 0	.7412	28	2.8637	424
10	.5800	26	1.3810̂	151	10	.7440	28	2.9061	434
20	.5826̂	26	1.3961̂	152̂	20	.7468	28	2.9495	444̂
30	.5853	26̂	1.4114	155	30	.7496	28	2.9939	454
40	.5879̂	26̂	1.4269	157̂	40	.7524̂	28	3.0394	465
50	.5906	26̂	1.4426̂	159̂	50	.7552̂	28	3.0859	476
66 0	.5932̂	26	1.4586	161̂	76 0	.7581	28	3.1335	488̂
10	.5959	27	1.4747̂	164̂	10	.7609	28	3.1824	500
20	.5986	26̂	1.4912	166̂	20	.7637̂	28	3.2324	512̂
30	.6012̂	26̂	1.5078	169	30	.7665̂	28	3.2836	525̂
40	.6039	27	1.5247	171̂	40	.7694	28	3.3362	539
50	.6066	26̂	1.5419	174	50	.7722	28	3.3901	553
67 0	.6092̂	27	1.5593	177	77 0	.7750̂	28	3.4454	567̂
10	.6119̂	27	1.5770	179	10	.7779	28	3.5021̂	582̂
20	.6146̂	26̂	1.5949	182	20	.7807	28	3.5604	598
30	.6173	27	1.6131̂	185	30	.7835̂	28	3.6202̂	614
40	.6200	27	1.6316	188	40	.7864	28	3.6816̂	631̂
50	.6227	27	1.6504	190̂	50	.7892̂	28	3.7448	649
68 0	.6254	27	1.6694̂	194	78 0	.7921	28	3.8097	667
10	.6281	27	1.6888	196̂	10	.7949̂	28	3.8765	686̂
20	.6308	27	1.7085	200	20	.7978	28	3.9451̂	707
30	.6335	27	1.7285	203	30	.8006̂	28	4.0158	728
40	.6362	27	1.7488	206̂	40	.8035	28	4.0886̂	749̂
50	.6389	27	1.7694̂	210	50	.8063̂	28	4.1636	772̂
69 0	.6416̂	27	1.7904̂	213	79 0	.8092	28	4.2408	796̂
10	.6443̂	27	1.8117	216̂	10	.8120̂	28	4.3205	821̂
20	.6470̂	27	1.8334	220̂	20	.8149	28	4.4026̂	847
30	.6498	27	1.8554̂	224	30	.8177̂	29	4.4874	873̂
40	.6525	27	1.8778	227	40	.8206	28	4.5749	904
50	.6552̂	27	1.9006̂	232	50	.8235	28	4.6653̂	934
70 0	.6580		1.9238		80 0	.8263̂		4.7587̂	

° ′	Vers.	d.	Exsec.	d.	° ′	Vers.	d.	Exsec.	d.

P. P.

	9	8	7	6	5	4
1	0.9	0.8	0.7	0.6	0.5	0.4
2	1.8	1.6	1.4	1.2	1.0	0.8
3	2.7	2.4	2.1	1.8	1.5	1.2
4	3.6	3.2	2.8	2.4	2.0	1.6
5	4.5	4.0	3.5	3.0	2.5	2.0
6	5.4	4.8	4.2	3.6	3.0	2.4
7	6.3	5.6	4.9	4.2	3.5	2.8
8	7.2	6.4	5.6	4.8	4.0	3.2
9	8.1	7.2	6.3	5.4	4.5	3.6

	3	2	1	9̂	8̂	7̂
1	0.3	0.2	0.1	0.9̂	0.8̂	0.7̂
2	0.6	0.4	0.2	1.9	1.7	1.5
3	0.9	0.6	0.3	2.8̂	2.5̂	2.2̂
4	1.2	0.8	0.4	3.8	3.4	3.0
5	1.5	1.0	0.5	4.7̂	4.2̂	3.7̂
6	1.8	1.2	0.6	5.7	5.1	4.5
7	2.1	1.4	0.7	6.6̂	5.9̂	5.2̂
8	2.4	1.6	0.8	7.6	6.8	6.0
9	2.7	1.8	0.9	8.5̂	7.6̂	6.7̂

	6̂	5̂	4̂	3̂	2̂	1̂
1	0.6̂	0.5̂	0.4̂	0.3̂	0.2̂	0.1̂
2	1.3	1.1	0.9	0.7	0.5	0.3
3	1.9̂	1.6̂	1.3̂	1.0̂	0.7̂	0.4̂
4	2.6	2.2	1.8	1.4	1.0	0.6
5	3.2̂	2.7̂	2.2̂	1.7̂	1.2̂	0.7̂
6	3.9	3.3	2.7	2.1	1.5	0.9
7	4.5̂	3.8̂	3.1̂	2.4̂	1.7̂	1.0̂
8	5.2	4.4	3.6	2.8	2.0	1.2
9	5.8̂	4.9̂	4.0̂	3.1̂	2.2̂	1.3̂

	29	28̂	28	27̂
1	2.9	2.8̂	2.8	2.7̂
2	5.8	5.7	5.6	5.5
3	8.7	8.5̂	8.4	8.2̂
4	11.6	11.4	11.2	11.0
5	14.5	14.2̂	14.0	13.7̂
6	17.4	17.1̂	16.8	16.5
7	20.3	19.9̂	19.6	19.2̂
8	23.2	22.8	22.4	22.0
9	26.1	25.6̂	25.2	24.7̂

	27	26̂	26	25̂
1	2.7	2.6̂	2.6	2.5̂
2	5.4	5.3	5.2	5.1
3	8.1	7.9̂	7.8	7.6̂
4	10.8	10.6	10.4	10.2
5	13.5	13.2̂	13.0	12.7̂
6	16.2	15.9̂	15.6	15.3
7	18.9	18.5̂	18.2	17.8̂
8	21.6	21.2̂	20.8	20.4
9	24.3	23.8̂	23.4	22.9̂

TABLE X.—NATURAL VERSED SINES AND EXTERNAL SECANTS.
80°–85°   **85°–90°**

° ′	Vers.	d.	Exsec.	d.	° ′	Vers.	d.	Exsec.	d.	P. P.		
80 0	.8263̂	28̂	4.7587̂		85 0	.9128̂	29	10.4737				
10	.8292	29	4.8554	966̂	10	.9157̂	29	10.8683̂	.3946̂			
20	.8321	28̂	4.9553̂	999̂	20	.9186̂	29	11.2912̂	.4229			
30	.8349̂	28̂	5.0588̂	1035	30	.9215̂	29	11.7455	.4542̂			
40	.8378	29	5.1660̂	1072	40	.9244̂	29	12.2347	.4892			
50	.8407	28̂	5.2772	1111̂	50	.9273̂	29	12.7631	.5284			
81 0	.8435̂	29	5.3924̂	1152̂	86 0	.9302̂	29	13.3356	.5725			
10	.8464̂	28̂	5.5121	1196̂	10	.9331̂	29	13.9579	.6223			
20	.8493	29	5.6363̂	1242̂	20	.9360̂	29	14.6368	.6789	29̂	29	28̂
30	.8522	28̂	5.7654̂	1291	30	.9389̂	29	15.3804	.7436	1 2.9̂	2.9	2.8̂
40	.8550̂	29	5.8998	1343̂	40	.9418̂	29	16.1984̂	.8186̂	2 5.9	5.8	5.7
50	.8579̂	29	6.0396̂	1398̂	50	.9447̂	29	17.1026	.9041̂	3 8.8	8.7	8.5
82 0	.8608̂	29	6.1853	1456̂	87 0	.9476̂	29	18.1073	1.0047	4 11.8	11.6	11.4
10	.8637	28̂	6.3372	1519	10	.9505̂	29	19.2303	1.1230	5 14.7	14.5	14.2̂
20	.8666	29	6.4957	1585	20	.9534̂	29̂	20.4937	1.2634	6 17.7	17.4	17.1
30	.8694̂	28̂	6.6613	1656	30	.9564̂	29	21.9256	1.4319	7 20.6̂	20.3	19.9̂
40	.8723̂	29	6.8344̂	1731̂	40	.9593̂	29	23.5621	1.6365	8 23.6	23.2	22.8
50	.8752̂	29	7.0156̂	1812	50	.9622̂	29	25.4505	1.8884	9 26.5̂	26.1	25.6̂
83 0	.8781̂	29	7.2055	1898̂	88 0	.9651̂	29	27.6537	2.2032			
10	.8810	28̂	7.4046̂	1991̂	10	.9680	29	30.2576	2.6039			
20	.8839	29	7.6138	2091̂	20	.9709	29	33.3823	3.1247			
30	.8868	29	7.8336	2198̂	30	.9738̂	29̂	37.2015̂	3.8192̂			
40	.8897	29	8.065̂1	2315	40	.9767̂	29	41.9757	4.7741̂			
50	.8926	29	8.309̂1	2440	50	.9796̂	29	48.1146̂	6.1383̂			
84 0	.8954̂	28̂	8.5667	2576	89 0	.9825̂	29	56.2987	8.1846̂			
10	.8983̂	29	8.8391	2723̂	10	.9854̂	29	67.7573̂				
20	.9012̂	29	9.1275	2884	20	.9883̂	29	84.9456				
30	.9041̂	29	9.4334	3059̂	30	.9912̂	29	113.5930				
40	.9070̂	29	9.7585	3250̂	40	.9942̂	29̂	170.8883				
50	.9099̂	29	10.1045̂	3466̂	50	.9971	29	342.7752				
85 0	.9128̂		10.4737	3691̂	90 0	1.0000		∞				
° ′	Vers.	d.	Exsec.	d.	° ′	Vers.	d.	Exsec.	d.			

TABLE XI.—USEFUL TRIGONOMETRICAL FORMULÆ.

$$1 \quad \sin a \quad = \frac{1}{\operatorname{cosec} a} = \frac{\tan a}{\sqrt{1 + \tan^2 a}} = \sqrt{\frac{1 - \cos 2a}{2}} = \frac{1}{\sqrt{1 + \cot^2 a}}$$

$$= \cos a \tan a = \sqrt{1 - \cos^2 a} = 2 \sin \tfrac{1}{2}a \cos \tfrac{1}{2}a$$

$$= \frac{1 + \cos a}{\cot \tfrac{1}{2}a} = \frac{2 \tan \tfrac{1}{2}a}{1 + \tan^2 \tfrac{1}{2}a} = \operatorname{vers} a \cot \tfrac{1}{2}a.$$

$$2 \quad \cos a \quad = \frac{1}{\sec a} = \frac{\cot a}{\sqrt{1 + \cot^2 a}} = \frac{1}{\sqrt{1 + \tan^2 a}}$$

$$= 1 - \operatorname{vers} a = \sin a \cot a = \sqrt{1 - \sin^2 a} = 2 \cos^2 \tfrac{1}{2}a - 1$$

$$= \sin a \cot \tfrac{1}{2}a - 1 = \cos^2 \tfrac{1}{2}a - \sin^2 \tfrac{1}{2}a = 1 - 2 \sin^2 \tfrac{1}{2}a.$$

$$3 \quad \tan a \quad = \frac{1}{\cot a} = \frac{\sin a}{\cos a} = \frac{\sec a}{\operatorname{cosec} a} = \frac{1}{\sqrt{\operatorname{cosec}^2 a - 1}}$$

$$= \operatorname{vers} 2a \operatorname{cosec} 2a = \cot a - 2 \cot 2a = \sin a \sec a$$

$$= \frac{\sin 2a}{1 + \cos 2a} = \operatorname{exsec} a \cot \tfrac{1}{2}a = \operatorname{exsec} 2a \cot 2a.$$

$$4 \quad \cot a \quad = \frac{1}{\tan a} = \frac{\cos a}{\sin a} = \frac{\sin 2a}{1 - \cos 2a} = \frac{1 + \cos 2a}{\sin 2a}$$

$$= \sqrt{\operatorname{cosec}^2 a - 1} = \cot \tfrac{1}{2}a - \operatorname{cosec} a.$$

$$5 \quad \operatorname{vers} a \quad = 1 - \cos a = \sin a \tan \tfrac{1}{2}a = 2 \sin^2 \tfrac{1}{2}a = \cos a \operatorname{exsec} a.$$

$$6 \quad \operatorname{exsec} a \quad = \sec a - 1 = \tan a \tan \tfrac{1}{2}a = \operatorname{vers} a \sec a.$$

$$7 \quad \sin \tfrac{1}{2}a \quad = \sqrt{\frac{\operatorname{vers} a}{2}} = \frac{\sin a}{2 \cos \tfrac{1}{2}a} = \frac{\operatorname{vers} a \cos \tfrac{1}{2}a}{\sin a}.$$

$$8 \quad \cos \tfrac{1}{2}a \quad = \sqrt{\frac{1 + \cos a}{2}} = \frac{\sin a}{2 \sin \tfrac{1}{2}a} = \frac{\sin a \sin \tfrac{1}{2}a}{\operatorname{vers} a}.$$

$$9 \quad \tan \tfrac{1}{2}a \quad = \operatorname{vers} a \operatorname{cosec} a = \operatorname{cosec} a - \cot a = \frac{\tan a}{1 + \sec a}.$$

$$10 \quad \cot \tfrac{1}{2}a \quad = \frac{1 + \cos a}{\sin a} = \operatorname{cosec} a + \cot a = \frac{\tan a}{\operatorname{exsec} a} = \frac{1}{\operatorname{cosec} a - \cot a}.$$

$$11 \quad \operatorname{vers} \tfrac{1}{2}a \quad = 1 - \sqrt{\tfrac{1}{2}(1 + \cos a)}.$$

$$12 \quad \operatorname{exsec} \tfrac{1}{2}a \quad = \frac{1}{\sqrt{\tfrac{1}{2}(1 + \cos a)}} - 1.$$

TABLE XI.—USEFUL TRIGONOMETRICAL FORMULÆ.

13	$\sin 2a \quad = 2 \sin a \cos a = \dfrac{2 \tan a}{1 + \tan^2 a}.$
14	$\cos 2a \quad = \cos^2 a - \sin^2 a = 1 - 2 \sin^2 a = 2 \cos^2 a - 1$ $= \dfrac{1 - \tan^2 a}{1 + \tan^2 a}.$
15	$\tan 2a \quad = \dfrac{2 \tan a}{1 - \tan^2 a}.$
16	$\cot 2a \quad = \tfrac{1}{2} \cot a - \tfrac{1}{2} \tan a = \dfrac{\cot^2 a - 1}{2 \cot a} = \dfrac{1 - \tan^2 a}{2 \tan a}.$
17	$\text{vers } 2a \quad = 2 \sin^2 a = 1 - \cos 2a = 2 \sin a \cos a \tan a.$
18	$\text{exsec } 2a = \dfrac{\tan 2a}{\cot a} = \dfrac{2 \tan^2 a}{1 - \tan^2 a} = \dfrac{2 \sin^2 a}{1 - 2 \sin^2 a}.$
19	$\sin (a \pm b) = \sin a \cos b \pm \cos a \sin b.$
20	$\cos (a \pm b) = \cos a \cos b \mp \sin a \sin b.$
21	$\sin a + \sin b = 2 \sin \tfrac{1}{2}(a + b) \cos \tfrac{1}{2}(a - b).$
22	$\sin a - \sin b = 2 \sin \tfrac{1}{2}(a - b) \cos \tfrac{1}{2}(a + b).$
23	$\cos a + \cos b = 2 \cos \tfrac{1}{2}(a + b) \cos \tfrac{1}{2}(a - b).$
24	$\cos a - \cos b = -2 \sin \tfrac{1}{2}(a + b) \sin \tfrac{1}{2}(a - b).$

Call the sides of any triangle A, B, C, and the opposite angles a, b, and c. Call $s = \tfrac{1}{2}(A + B + C)$.

25	$\tan \tfrac{1}{2}(a - b) = \dfrac{A - B}{A + B} \quad \tan \tfrac{1}{2}(a + b) = \dfrac{A - B}{A + B} \cot \tfrac{1}{2}c.$
26	$C = (A + B) \dfrac{\cos \tfrac{1}{2}(a + b)}{\cos \tfrac{1}{2}(a - b)} = (A - B) \dfrac{\sin \tfrac{1}{2}(a + b)}{\sin \tfrac{1}{2}(a - b)}.$
27	$\sin \tfrac{1}{2}a = \sqrt{\dfrac{(s - B)(s - C)}{BC}}.$
28	$\cos \tfrac{1}{2}a = \sqrt{\dfrac{s(s - A)}{BC}}.$
29	$\text{vers } A = \dfrac{2(s - B)(s - C)}{BC}.$
30	$\text{Area} = \sqrt{s(s - A)(s - B)(s - C)} = A^2 \dfrac{\sin b \sin c}{2 \sin a}.$

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